Neutrino Frontier Workshop 2014 (Fuji Calm, 2014.12.21)

Matter Effect in Long Baseline Neutrino Oscillation

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working with
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Introduction

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Neutrino Mixings: Achievements

- From discovery to precision measurements

PHYSICAL REVIEW D **89,** 093018 (2014)

Status of three-neutrino oscillation parameters, circa 2013

F. Capozzi,[1,2] G. L. Fogli,[1,2] E. Lisi,[2] A. Marrone,[1,2] D. Montanino,[3,4] and A. Palazzo[5]

Parameter	Best fit	1σ range	2σ range
$\delta m^2/10^{-5}\mathrm{eV}^2$ (NH or IH)	7.54	7.32–7.80	7.15–8.00
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91–3.25	2.75–3.42
$\Delta m^2/10^{-3}\mathrm{eV}^2$ (NH)	2.43	2.37–2.49	2.30–2.55
$\Delta m^2/10^{-3}\mathrm{eV}^2$ (IH)	2.38	2.32–2.44	2.25–2.50
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15–2.54	1.95–2.74
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18–2.59	1.98–2.79
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14–4.70	3.93–5.52
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24–5.94	4.00–6.20
δ/π (NH)	1.39	1.12–1.77	$0.00 - 0.16 \oplus 0.86 - 2.00$
δ/π (IH)	1.31	0.98–1.60	$0.00 - 0.02 \oplus 0.70 - 2.00$

Neutrino Mixings: Challenges

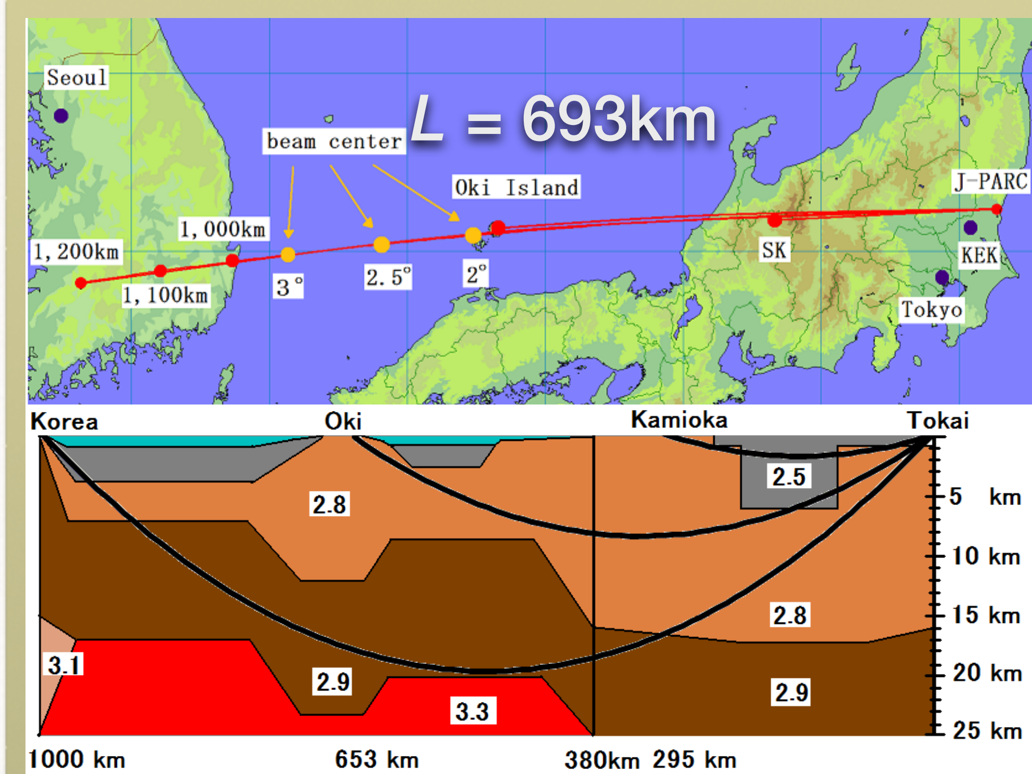
- Mass Hierarchy $\delta m^2_{31} \gtrless 0$?
- Octant Degeneracy $\theta_{23} \gtrless \pi/4$?
- Leptonic CP Violation $\sin \delta_{CP} = 0$?

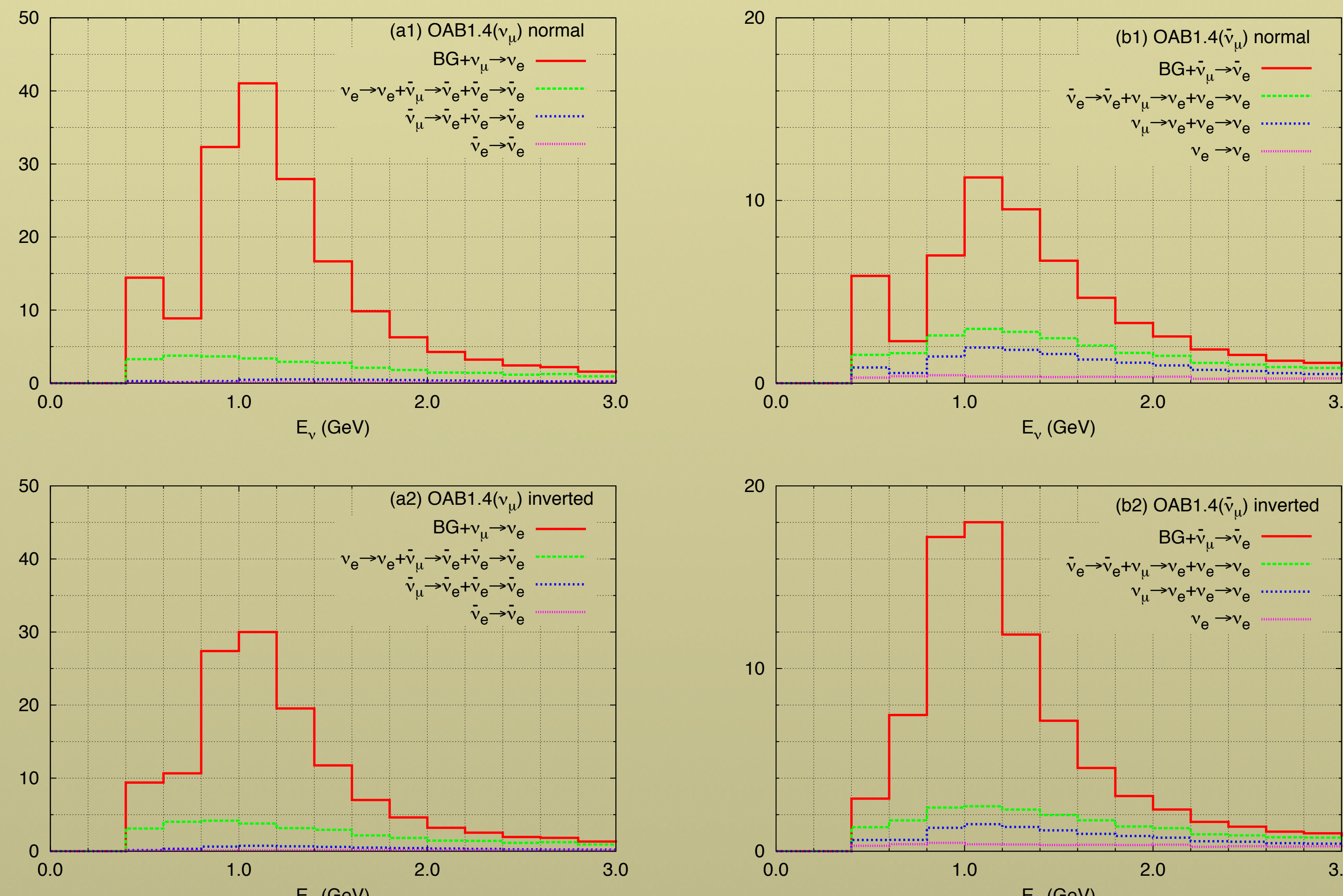
  - Oscillation experiments with very long baseline (1000~10000 km)
  - Exploiting the matter effect

Evaluating the Matter Effect

K. Hagiwara, T. Kiwanami, N. Okamura, K.-i. Senda (2013)

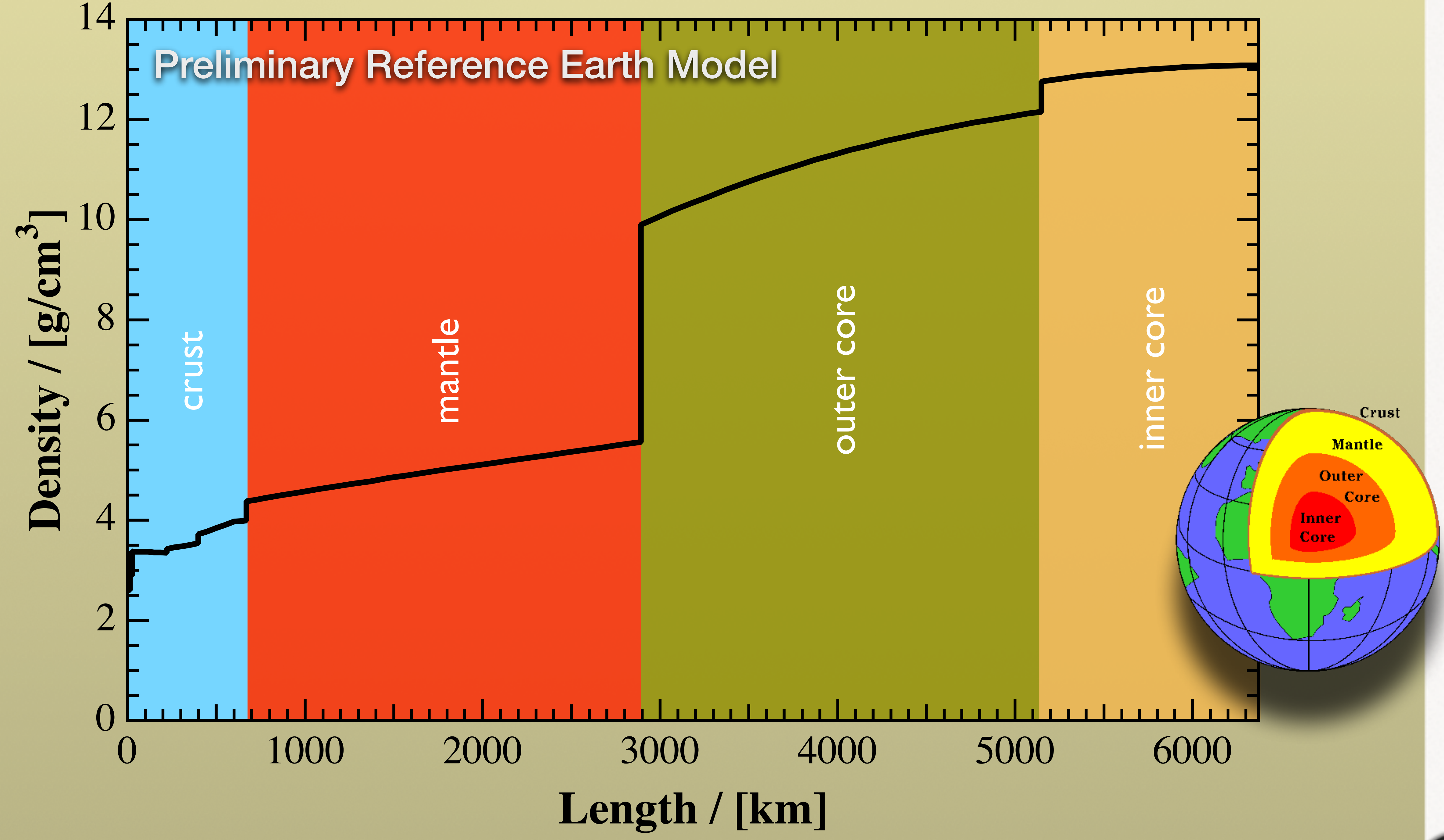
Physics potential of neutrino oscillation experiment
with a far detector in Oki Island
along the T2K baseline





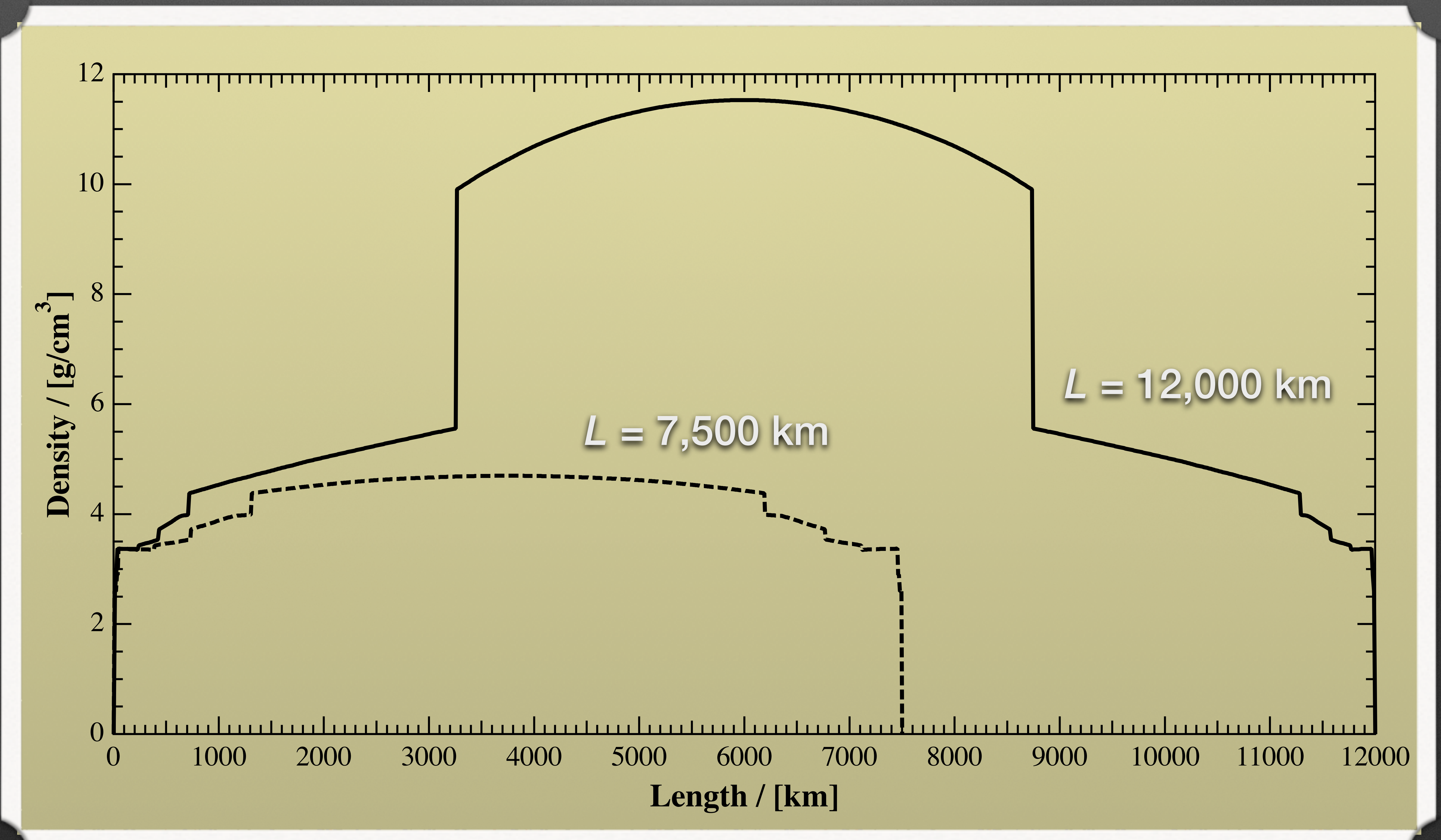
Event number/[2.5x10^{21} POT] vs E_ν/[GeV]

Earth Model

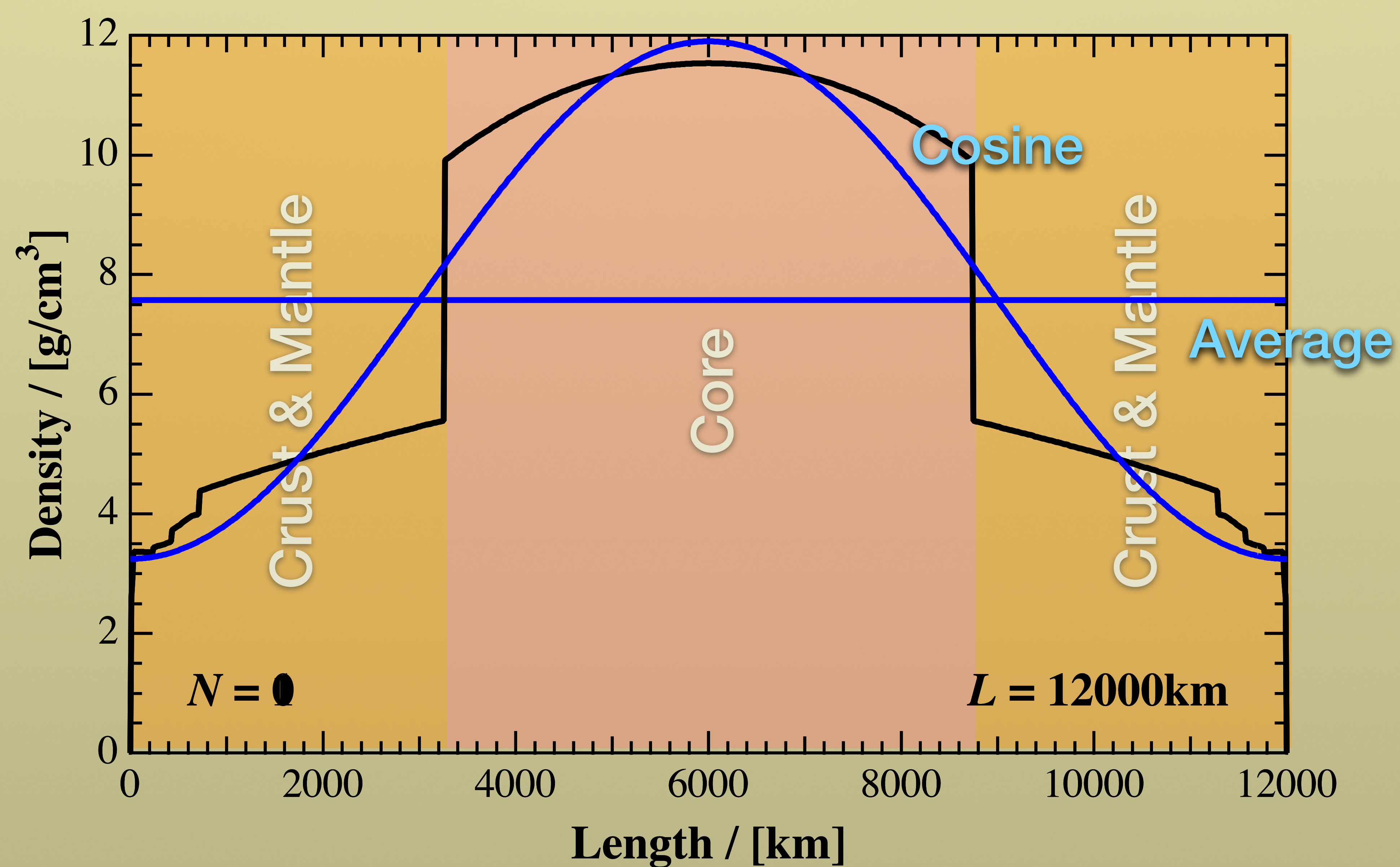


Preliminary Reference Earth Model
crust
mantle
outer core
inner core
Density / [g/cm^3]
Length / [km]
0
2
4
6
8
10
12
14
0
1000
2000
3000
4000
5000
6000
Crust
Mantle
Outer
Core
Inner
Core

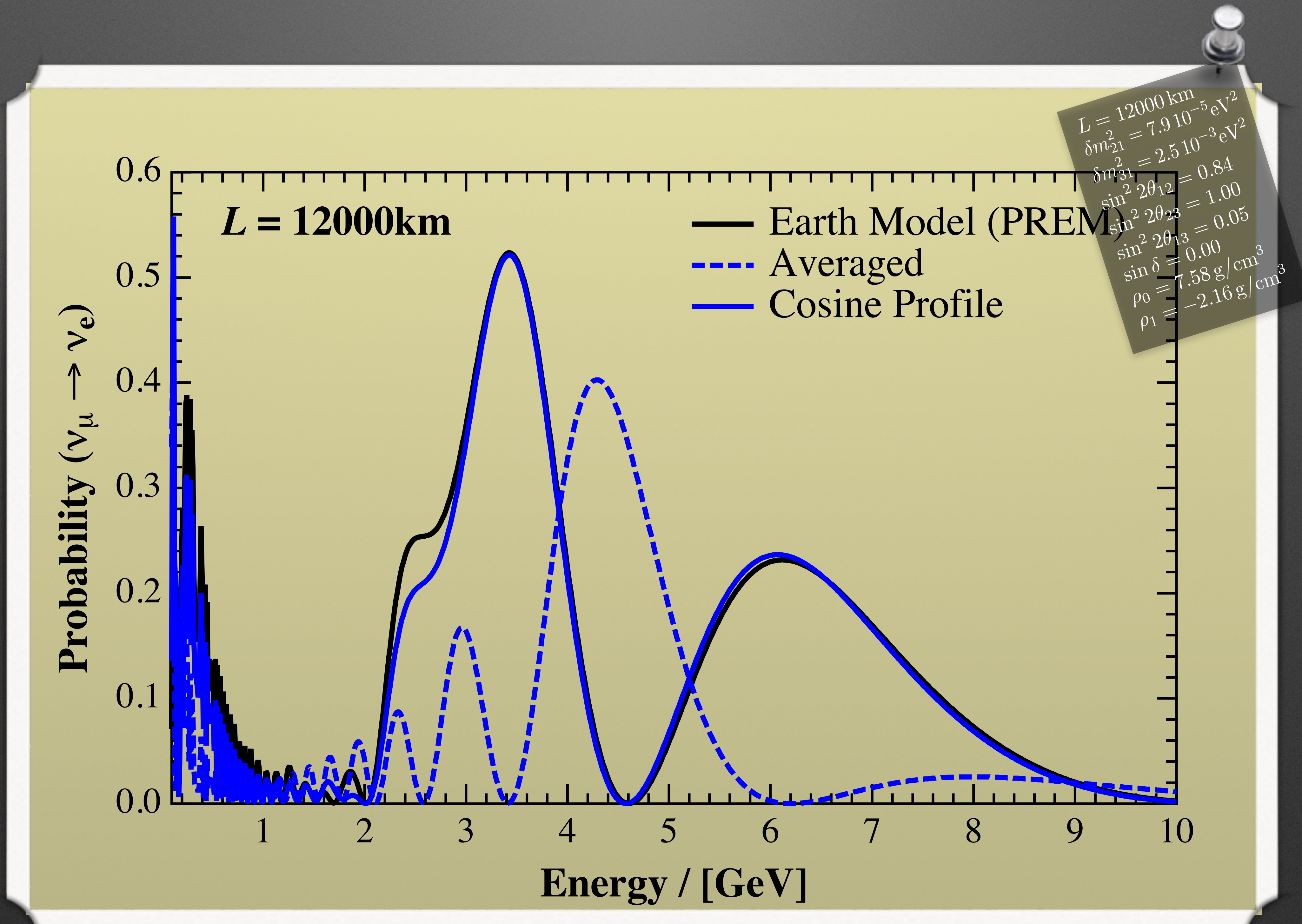
Density Profile on a Baseline



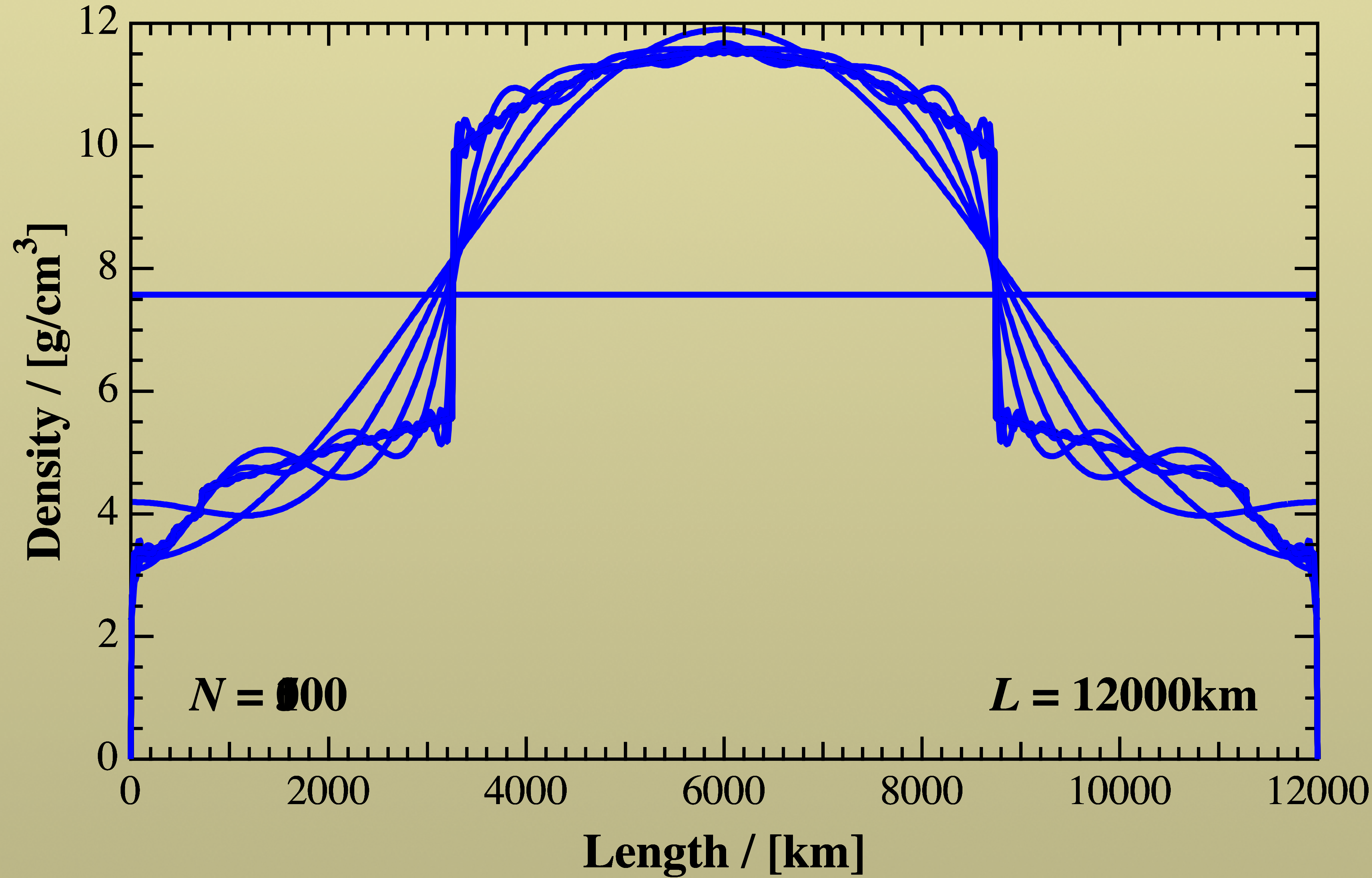
Matter Density Profile



Constant vs. Earth Model



Matter Profile: Fourier Series



Formulation

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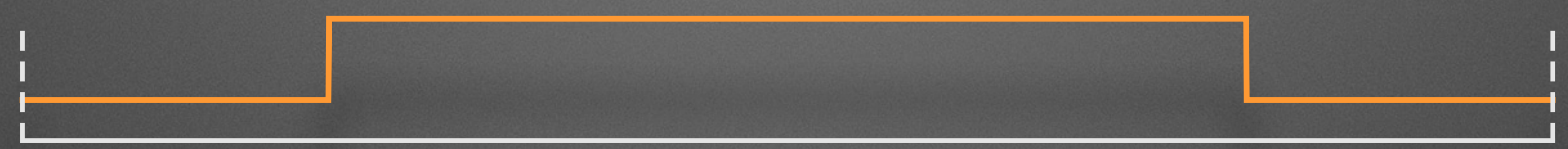
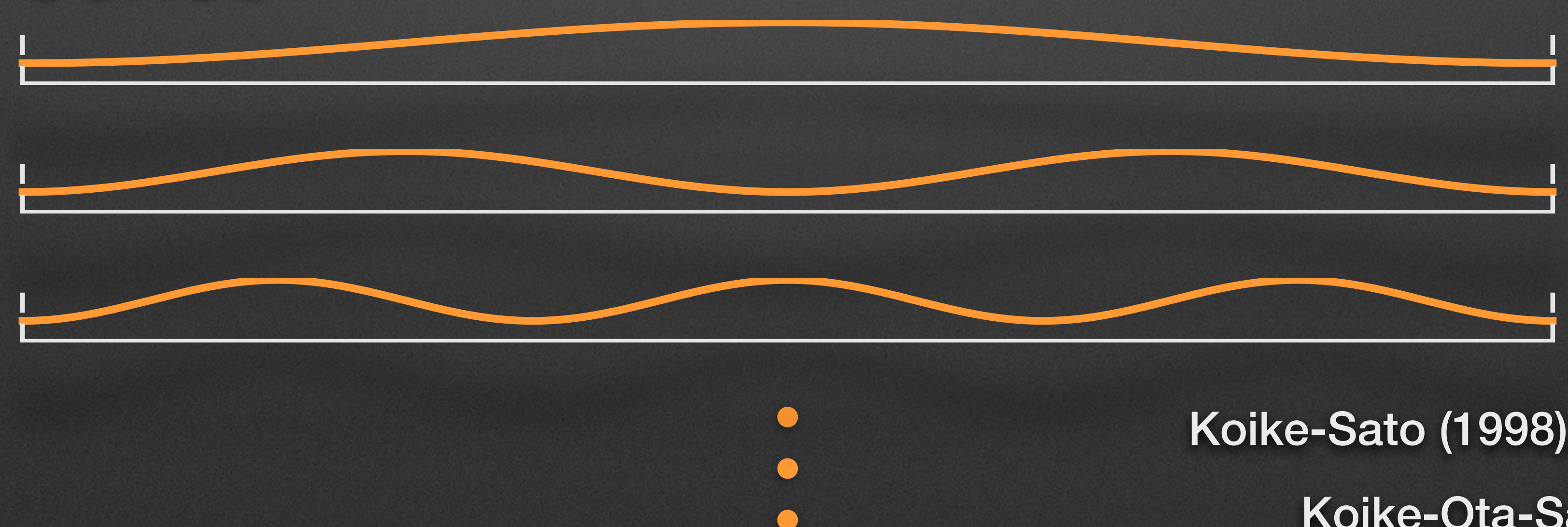
Modeling Density Profiles

- Step function



Akhmedov (1988), Krastev-Smirnov (1989), Krastev-Smirnov (1989),
Liu-Smirnov (1998), Petcov (1998), Chizhov-Petcov (1998), ...,
Akhmedov-Maltoni-Smirnov (2005), ...

- Fourier series



Koike-Sato (1998), Ota-Sato (2003),
Koike-Ota-Saito-Sato (2009)...

Two-Flavor Oscillation

MK-Ota-Saito-Sato, PLB **675**, 69 (2009)

- Evolution equation of the two-flavor neutrino

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\nu_{\mathrm{e}}(x)\\ \nu_{\mu}(x)\end{pmatrix} = \frac{1}{2E}\left[\frac{\delta m^2}{2}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix} + \begin{pmatrix}a(x) & 0\\ 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_{\mathrm{e}}(x)\\ \nu_{\mu}(x)\end{pmatrix}$$

  - Matter effect $a(x) = 2\sqrt{2}G_{\mathrm{F}}n_{\mathrm{e}}(x)E$

- Second-order equation in dimensionless variables

$$z''(\xi) + \frac{1}{4}\Big[\big(\Delta_{\mathrm{m}}(\xi) - \Delta\cos 2\theta\big)^2 + \Delta^2\sin^2 2\theta + 2\mathrm{i}\Delta'_{\mathrm{m}}(\xi)\Big]z(\xi) = 0$$

  - Dimensionless variables:

$$\xi \equiv \frac{x}{L} \qquad \Delta \equiv \frac{\delta m^2 L}{2E} \qquad \Delta_{\mathrm{m}}(\xi) \equiv \frac{a(\xi)L}{2E}$$

Distance — Reciprocal E — Matter effect

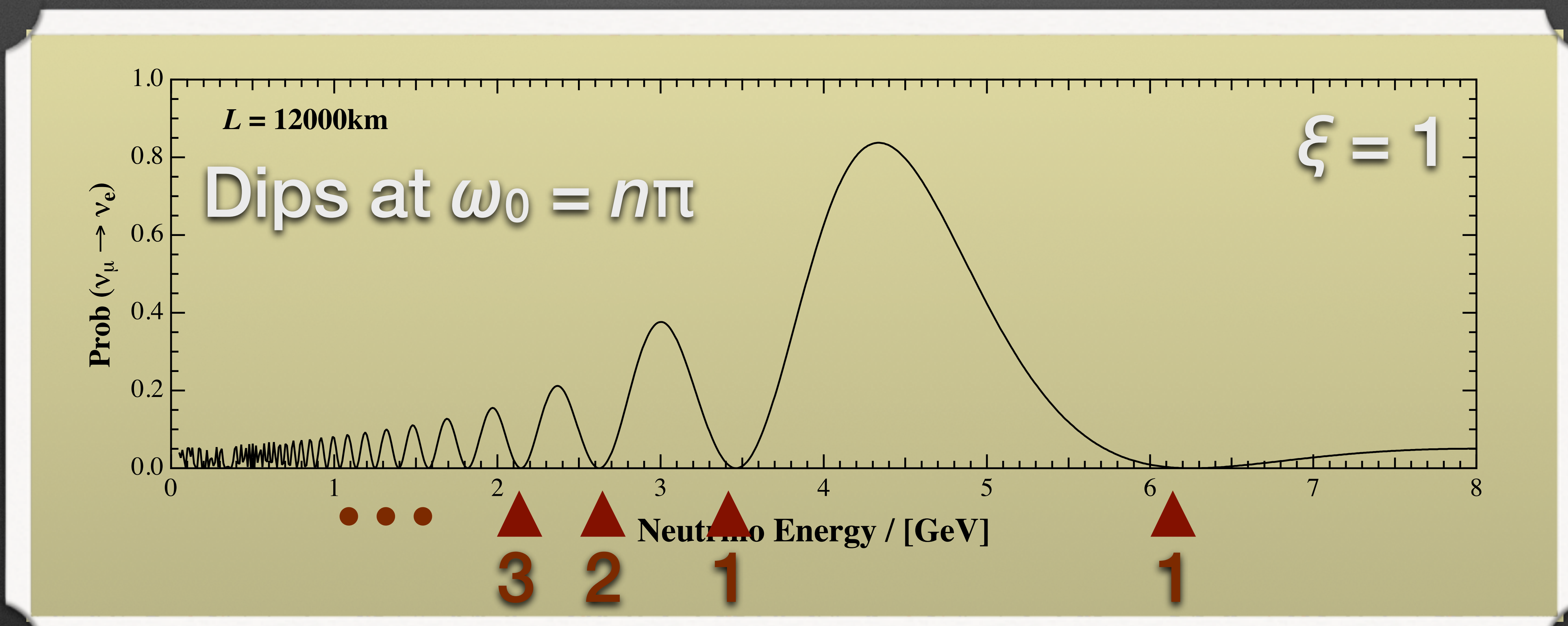
  - $z(\xi) = \nu_{\mathrm{e}}(\xi)\exp\left[\frac{\mathrm{i}}{2}\int_0^{\xi}\mathrm{d}s\,\Delta_{\mathrm{m}}(s)\right]$ $\cdots\ \big|\nu_{\mathrm{e}}(\xi)\big|^2 = \big|z(\xi)\big|^2$
  - Initial conditions $\nu_{\mathrm{e}}(0) = 0,\ \nu_{\mu}(0) = 1 \longrightarrow z(0) = 0\,,\ z'(0) = -\mathrm{i}\frac{\Delta}{2}\sin 2\theta$

Constant-Density Matter

- Constant density: $\Delta_{\mathrm{m}}(\xi) \equiv \Delta_0 = \text{(const.)}$

$$z''(\xi) + \underbrace{\frac{1}{4}\left[\left(\Delta_{\mathrm{m}}(\xi) - \Delta\cos 2\theta\right)^2 + \Delta^2\sin^2 2\theta + 2\mathrm{i}\Delta'_{\mathrm{m}}(\xi)\right]}_{\equiv\,\omega_0^2 \quad \text{(const.)}} z(\xi) = 0$$

$$\longrightarrow \quad \mathrm{Prob}(\nu_\mu \to \nu_{\mathrm{e}}) \propto \sin^2 \omega_0 \xi$$



Inhomogeneous Matter

$$z''(\xi)+\frac{1}{4}\Big[\big(\Delta_{\mathrm{m}}(\xi)-\Delta\cos 2\theta\big)^2+\Delta^2\sin^2 2\theta+2\mathrm{i}\Delta'_{\mathrm{m}}(\xi)\Big]z(\xi)=0$$

- Fourier series of inhomogeneous matter

$$\rho(x)=\sum_{n=0}^{\infty}\rho_n\cos\frac{2n\pi}{L}x\,,\qquad \Delta_{\mathrm{m}}(\xi)=\sum_{n=0}^{\infty}\Delta_{\mathrm{m}n}\cos 2n\pi\xi$$

- Presence of the n-th Fourier mode

$$\rho(x)=\rho_0+\rho_n\cos\frac{2n\pi}{L}x\,,\qquad \Delta_{\mathrm{m}}(\xi)=\Delta_{\mathrm{m}0}+\Delta_{\mathrm{m}n}\cos 2n\pi\xi$$

Mathieu Equation

$$z''(t)+(\omega^2-2\varepsilon\cos t)z(t)=0$$

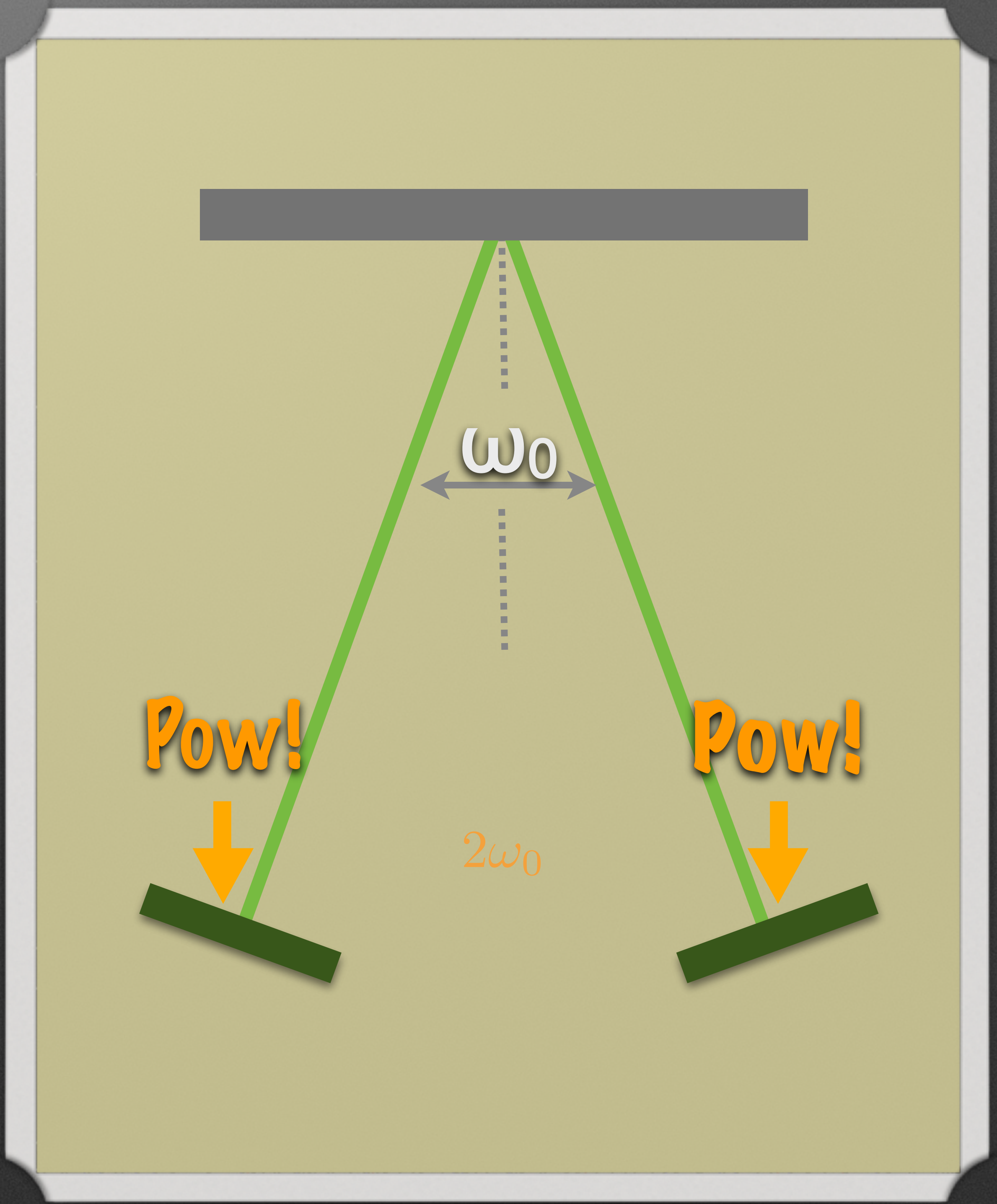
$$z''(\xi)+\big(\omega_0^2+\alpha_n\cos 2n\pi\xi-\mathrm{i}\beta_n\sin 2n\pi\xi+\gamma_n\cos 4n\pi\xi\big)z(\xi)=0$$

$$\omega_0^2=\frac{1}{4}(\Delta_{\mathrm{m}0}-\Delta\cos 2\theta)^2+\frac{1}{4}\Delta^2\sin^2 2\theta+\frac{1}{8}\Delta_{\mathrm{m}n}^2\,,$$

$$\alpha_n=\frac{1}{2}(\Delta_{\mathrm{m}0}-\Delta\cos 2\theta)\Delta_{\mathrm{m}n}\,,\quad \beta_n=n\pi\Delta_{\mathrm{m}n}\,,\quad \gamma_n=\frac{1}{8}\Delta_{\mathrm{m}n}^2$$

Parametric Resonance

- Periodic perturbation
  - Twice in a period
  - Grows amplitude of oscillation
- Matter effect as a bunch of periodic perturbations



Ermilova et al. (1986), Akhmedov (1988), Krastev-Smirnov (1989), Liu-Smirnov (1998), Petcov (1998), Chizhov-Petcov (1998), ..., Akhmedov-Maltoni-Smirnov (2005), ...

Resonance Condition

$$z''(\xi) + \left(\omega_0^2 + \alpha_n \cos 2n\pi\xi - \mathrm{i}\beta_n \sin 2n\pi\xi + \gamma_n \cos 4n\pi\xi\right) z(\xi) = 0$$

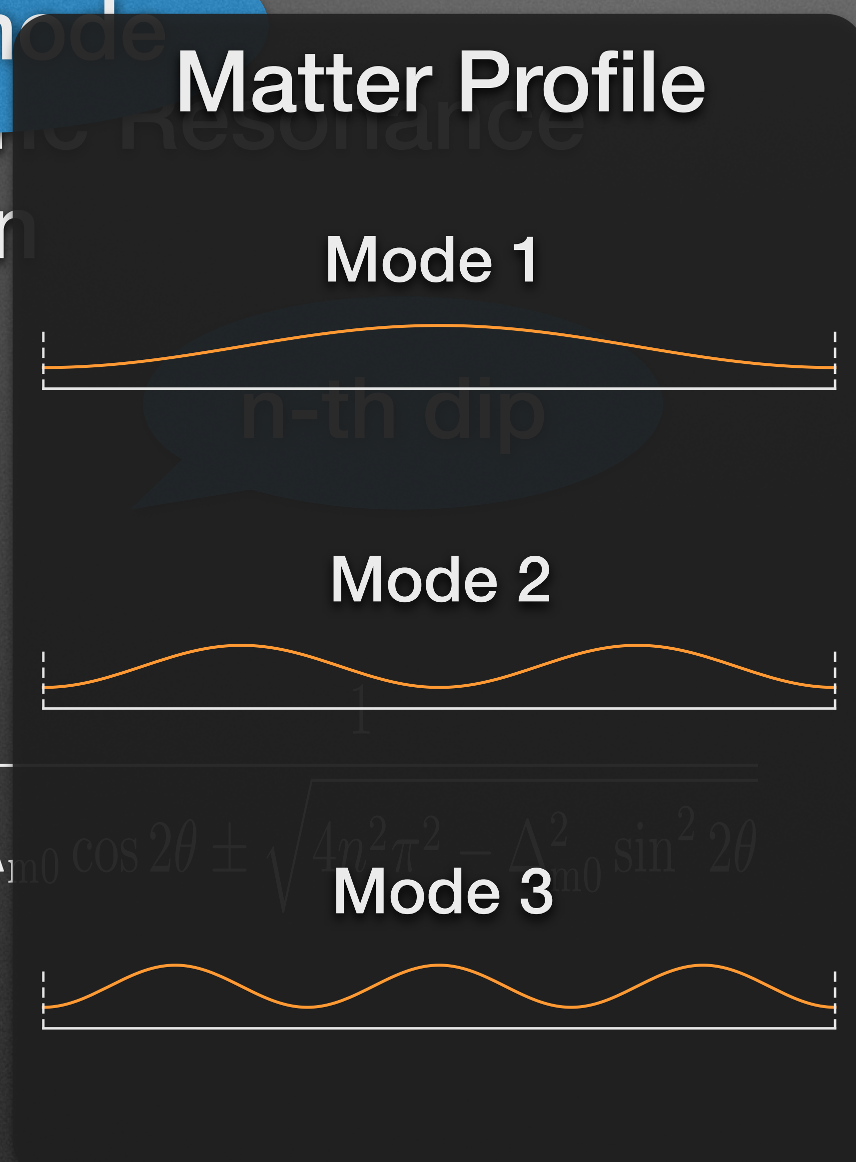
n-th mode

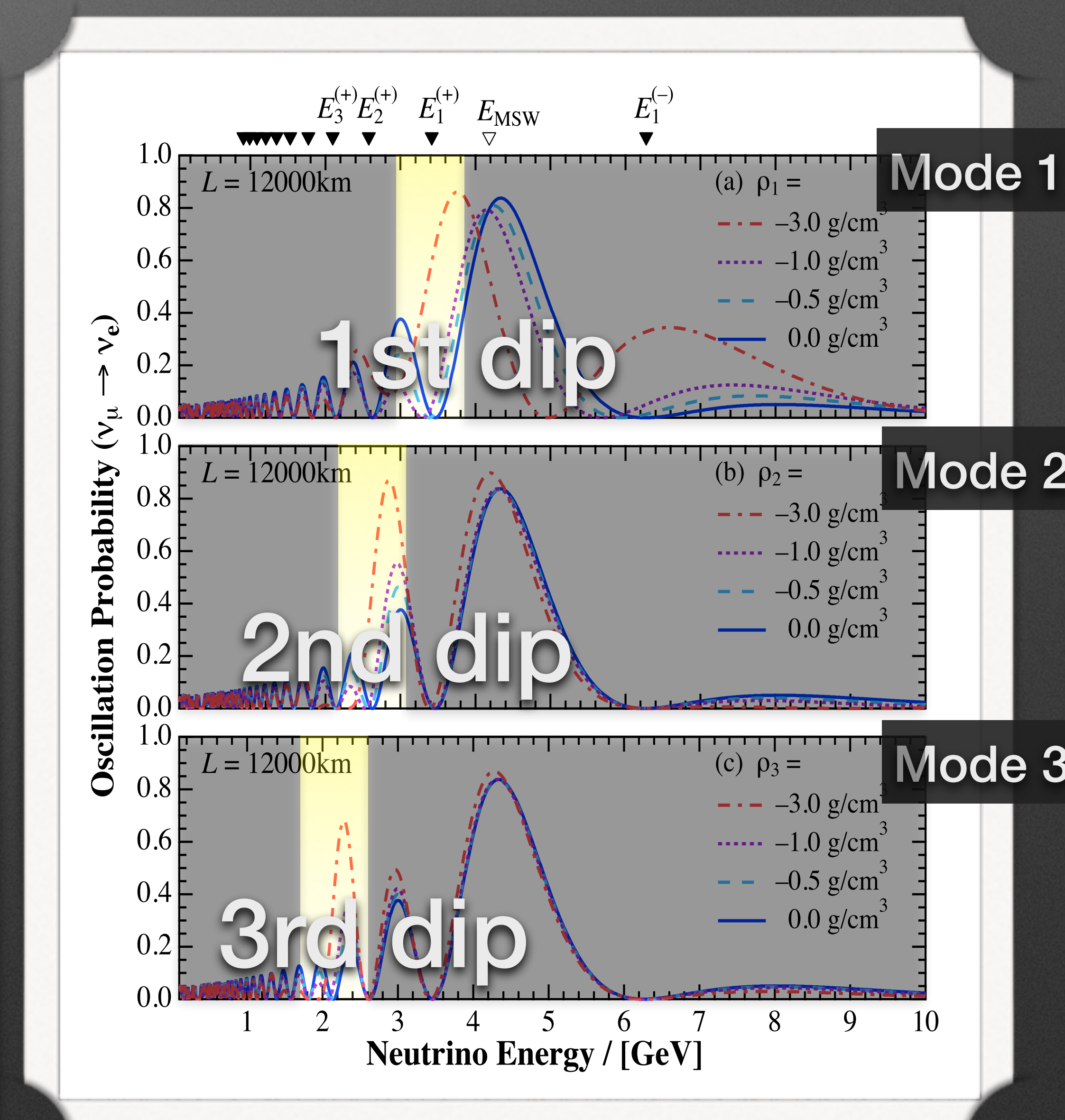
- Parametric Resonance Condition

n-th dip

$$\omega_0 = n\pi$$

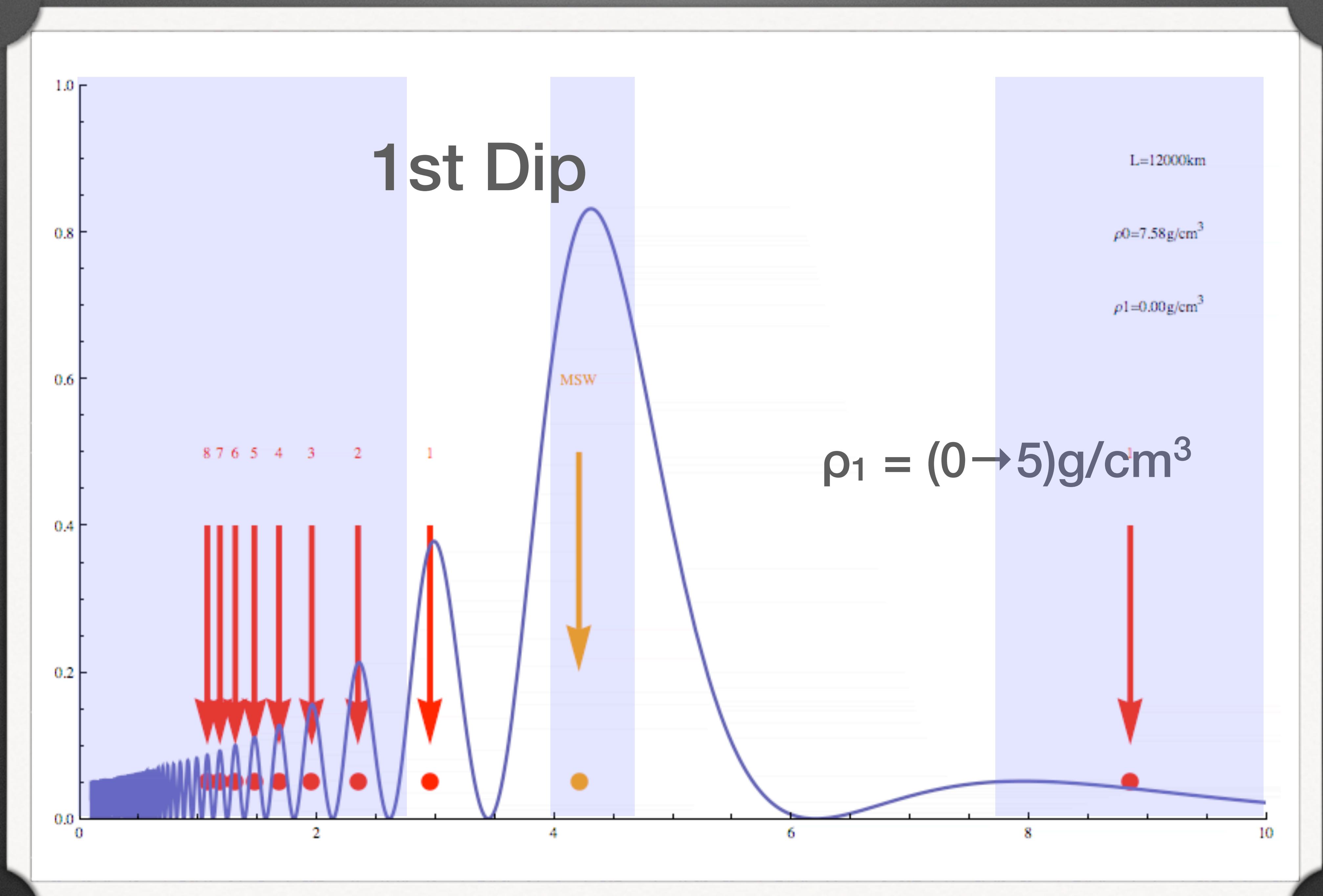
$$E = E_n^{(\pm)} \equiv \frac{\delta m^2 L}{2} \frac{1}{\Delta_{m0} \cos 2\theta \pm \sqrt{4n^2\pi^2 - \Delta_{m0}^2 \sin^2 2\theta}}$$



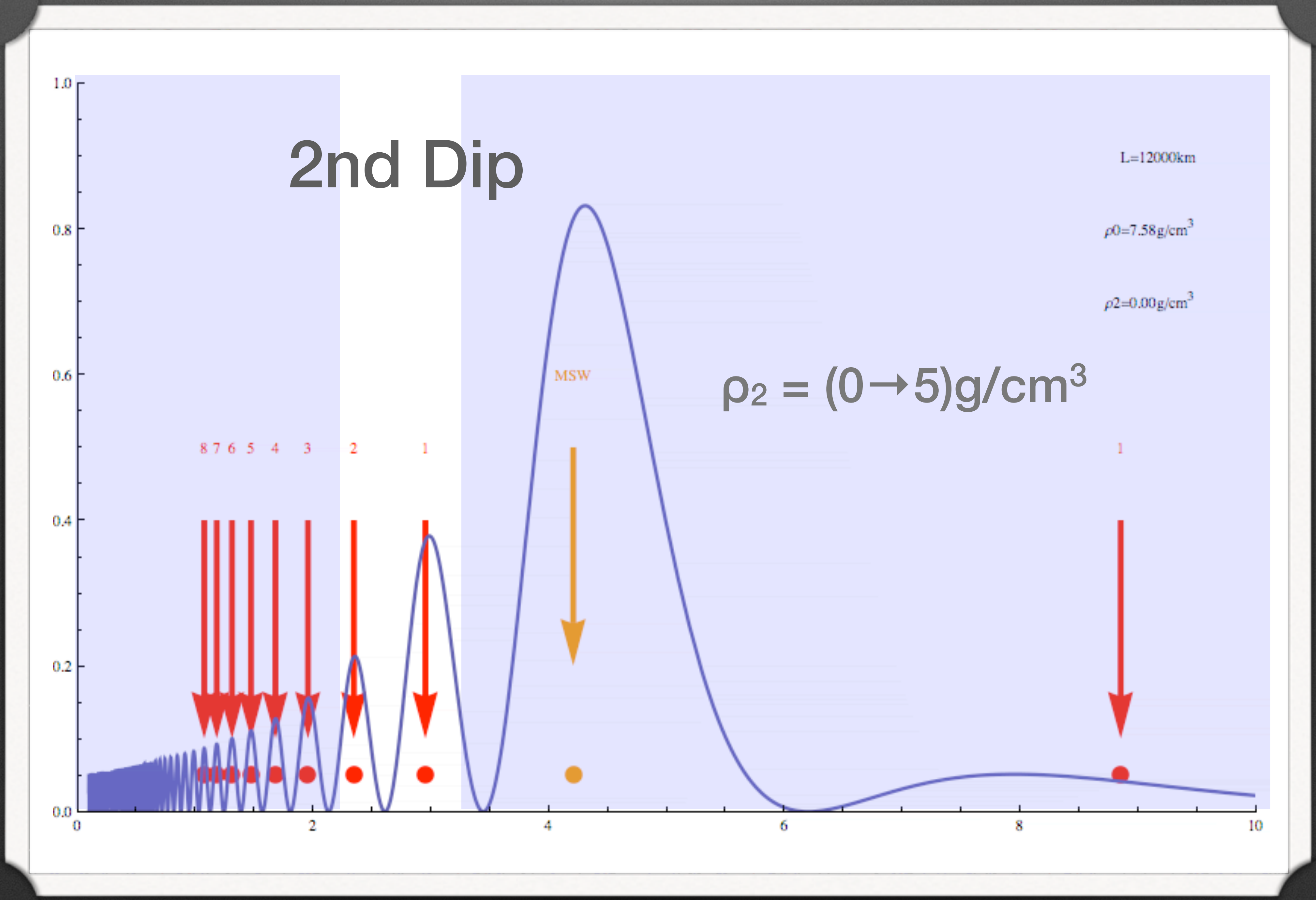


The values of other parameters used in this plot

Effect of the Mode 1



Effect of the Mode 2



Matter-Profile Effects

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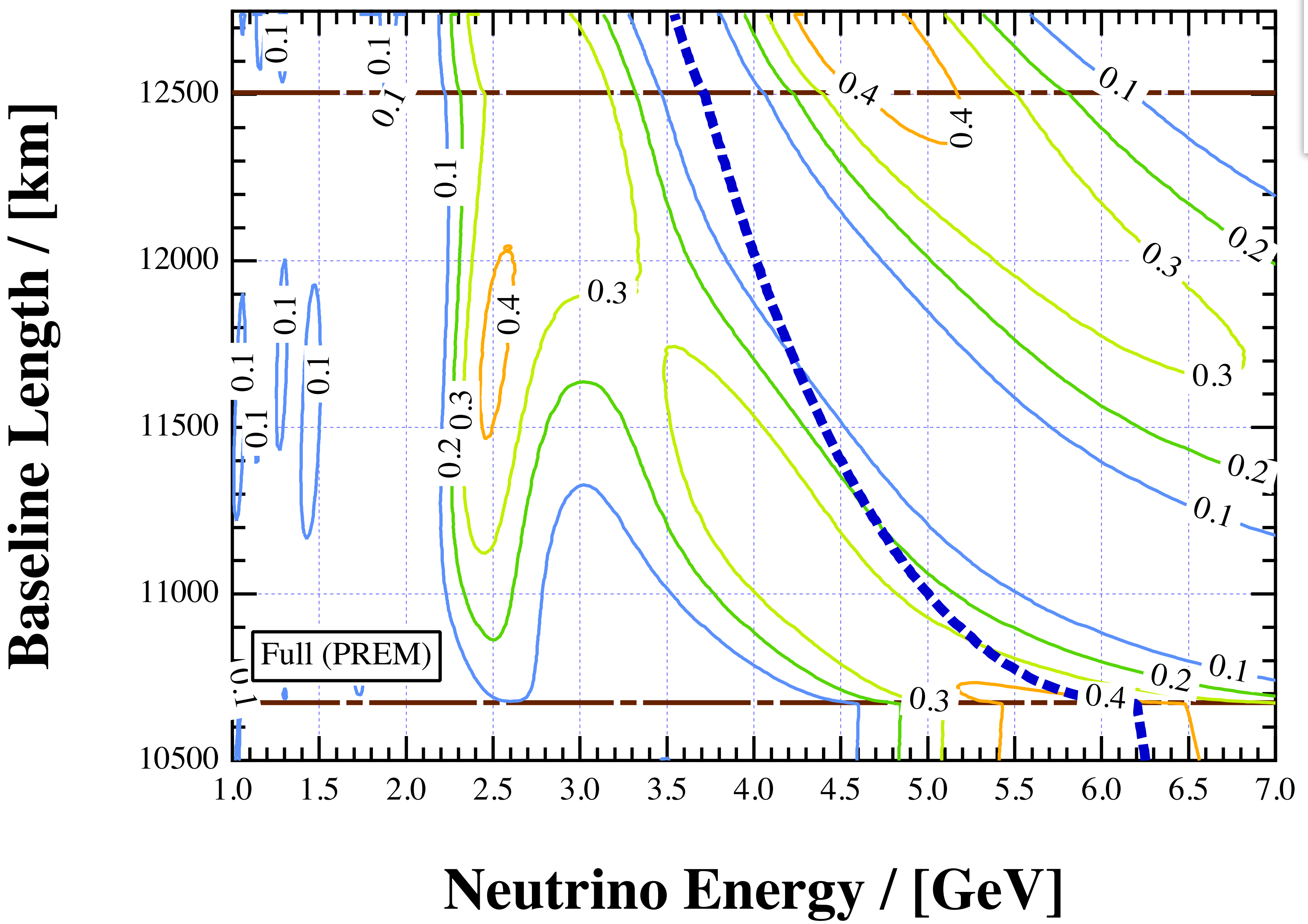
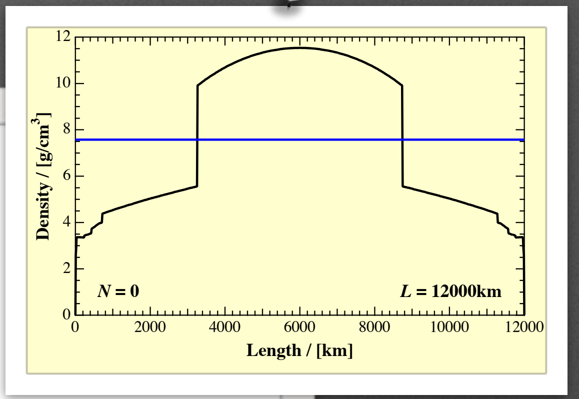
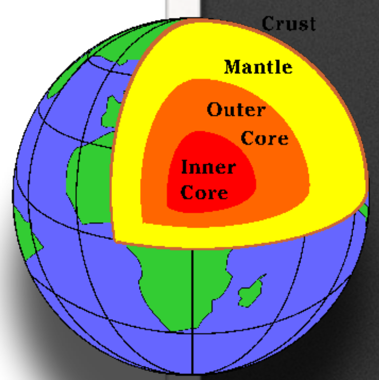
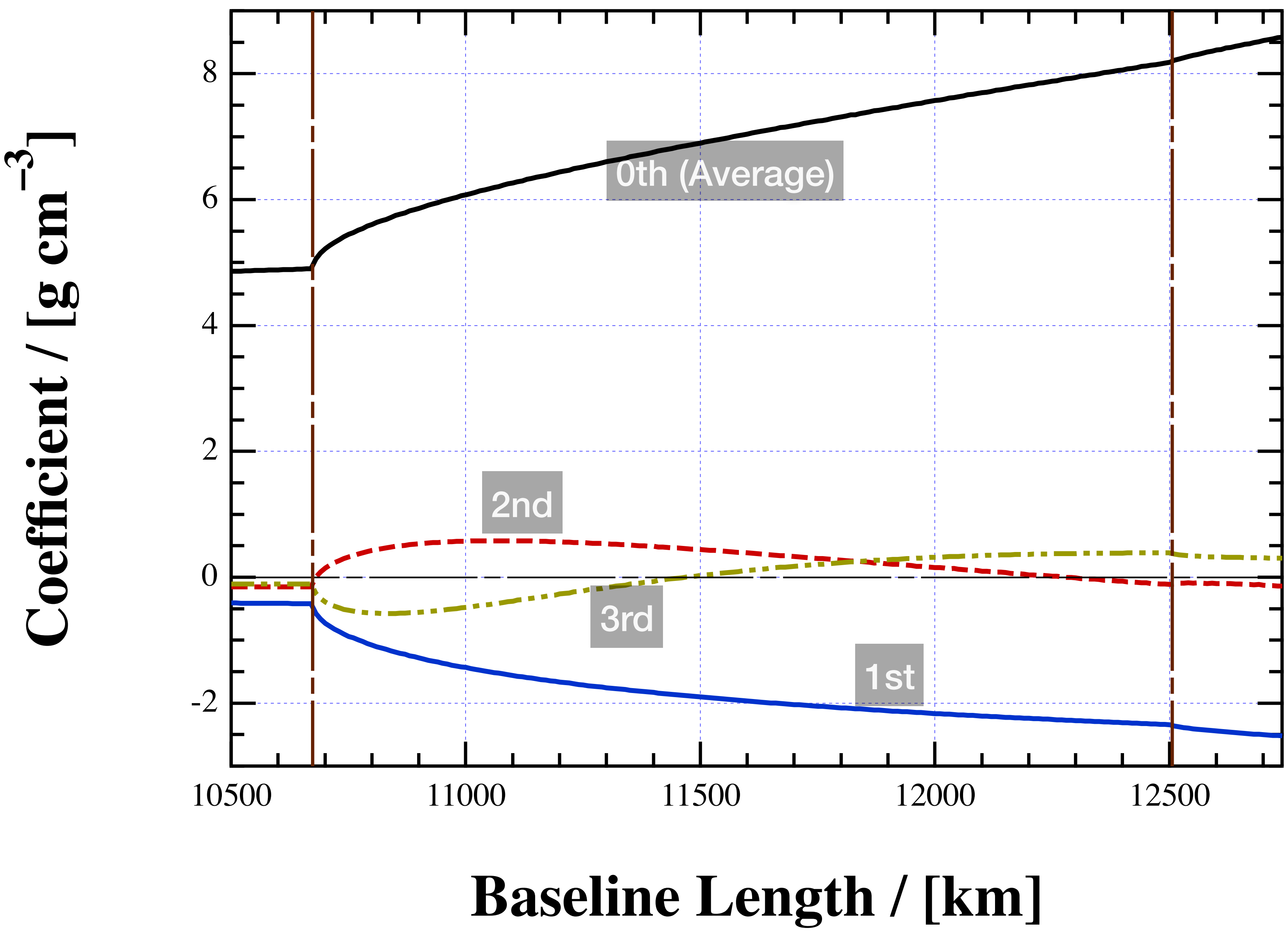
Oscillogram: Full Profile

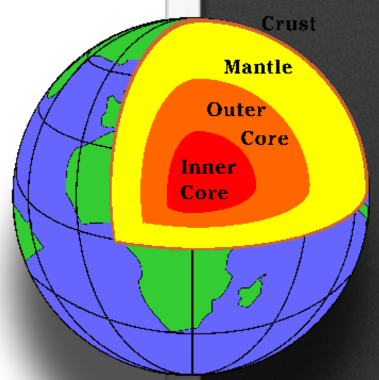




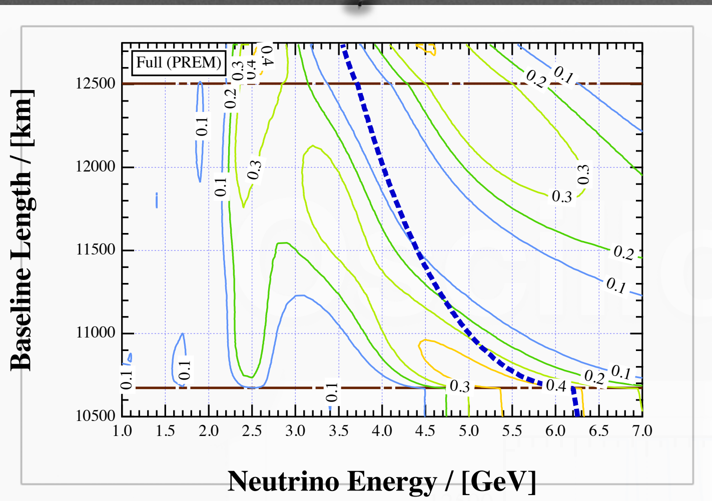


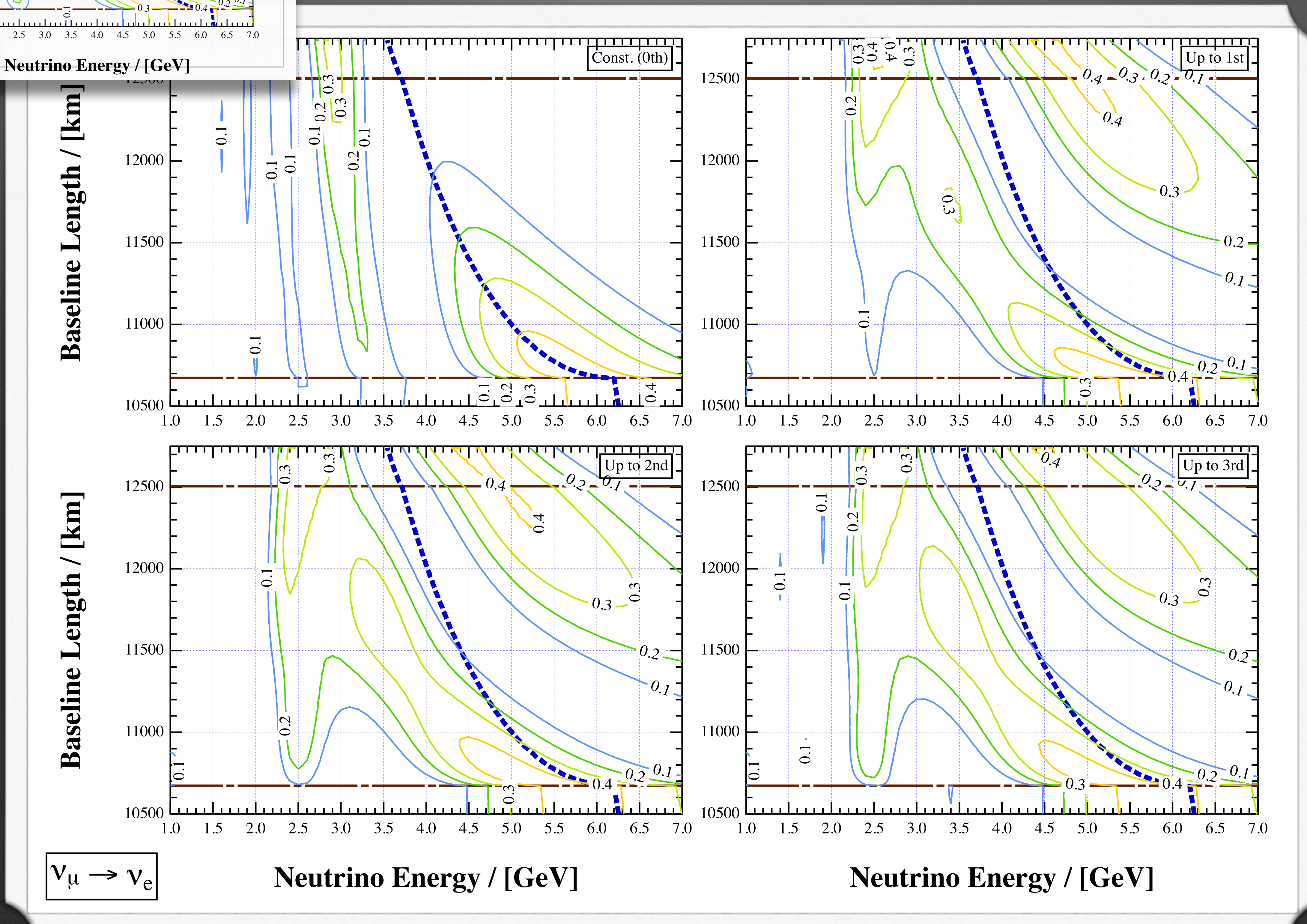
Fourier Coefficients



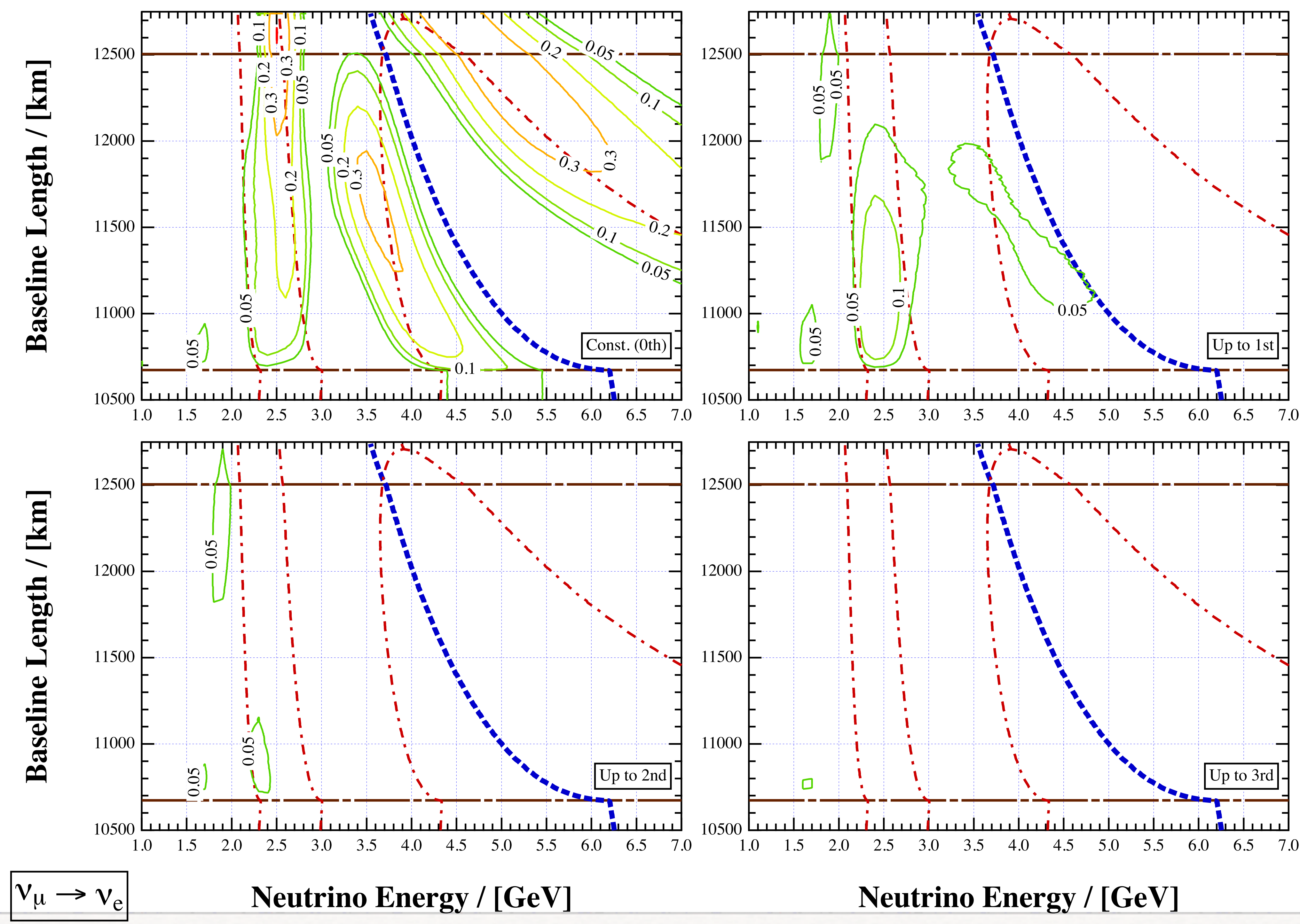


ogram: First Few Modes





Oscillogram: Residues

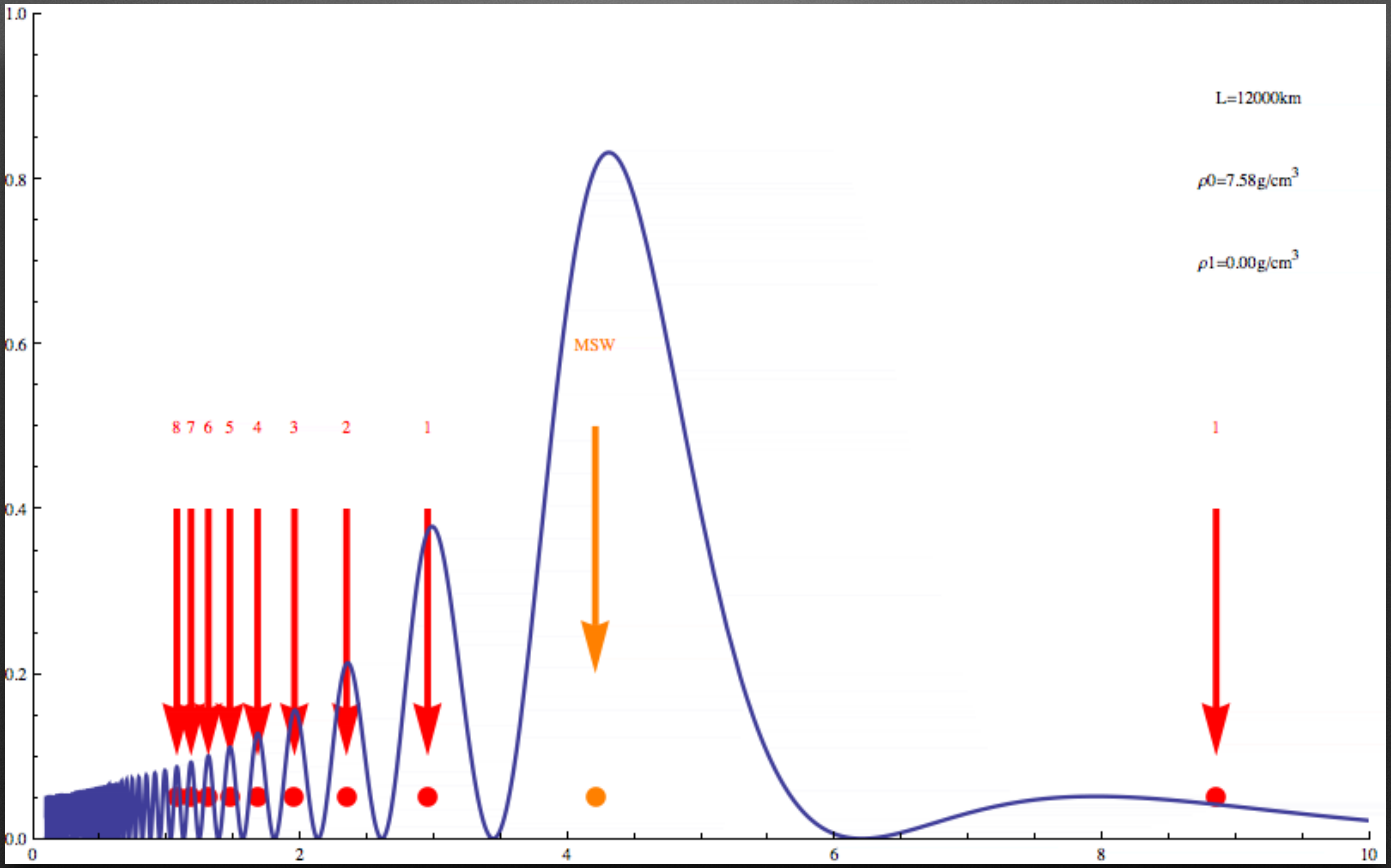


Summary & Outlook

- **Fourier analysis** is powerful to account for the matter-profile effects in neutrino oscillation.
  - *n*-th Fourier mode ↔ *n*-th dip of the appearance probability
  - Inhomogeneity → Parametric resonance
  - Systematic improvement
- Low E_ν ↔ Small-size structure of matter

Backup slides

First-mode effect



Second-mode effect