gauge-Higgs 統一理論の構築と現象論

「質量起源」研究会@つくば (3/7/05-3/8/05)

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「質量起源」

↓

「Higgs」の起源

||

gauge 場じゃあなかろうか？

何が嬉しいかというと･･･

☆Higgs massはfinite
☆Yukawaも出てくる=gauge相互作用

Plan of talk

1. Introduction
2. gauge-Higgs unification

   -----$SU(3)\times SU(3)$, $SU(6)$ models-----

3. dynamical EW symmetry breaking I

   NH, Y. Hosotani, Y. Kawamura and T. Yamashita, Phys.Rev.D70:015010, 2004

   NH and T. Yamashita, JHEP 0402:059,2004

4. dynamical EW symmetry breaking II

   NH and T. Yamashita, JHEP 0404 (2004) 016

5. Higgs mass and phenomenology

   NH, K.Takenaga and T.Yamashita, Phys.Rev.D71:025006,2005
   NH, K.Takenaga and T.Yamashita, hep-ph/0411250

6. summary and discussion

1. Introduction

1-1. motivation

1-2. notation

1-1: motivation

☆ motivation of introducing extra dimension

1. string theory
   (compactification, brane world, AdS/CFT, …..)
2. weakness of gravity
   → large extra dimension (ADD)
3. solution of GUT problems
   → extra dimension GUT (Kawamura)
4. KK theory (unification of gravity and EM)
5. origin of adjoint Higgs
   → "Hosotani mech."

1-1: motivation

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5. origin of doublet Higgs
   → “Hosotani mech.”

1-1: motivation

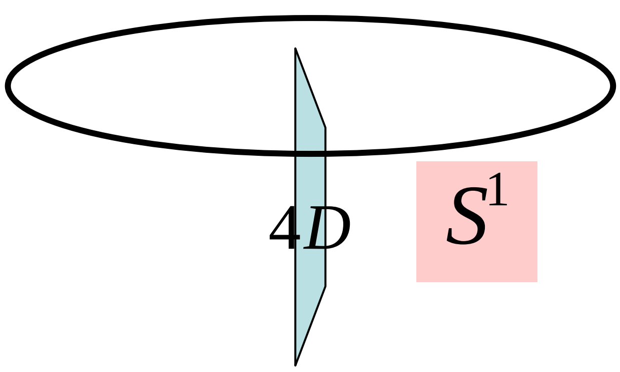
☆ higher dimensional gauge theory:

scalar A_5 in 4D effective theory ← extraD component

→ **identify Higgs field**

ex.

$5D \ SU(5) \ GUT$



$A_M \ (= \underbrace{0\sim3}_{\mu},5)$

$A_5 = \Sigma_{24}$

Origin of adjoint Higgs which break SU(5)

dynamics of 5D gauge theory $\rightarrow \langle \Sigma_{24} \rangle$ "Hosotani mech."

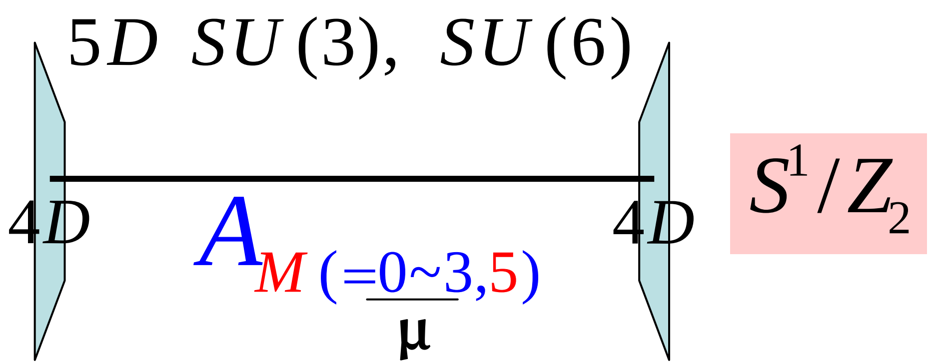
Gauge-Higgs unification

"Higgs doublet" mass is finite! (~1/R)

↑ **5D gauge inv.**

→ **identify Higgs field**

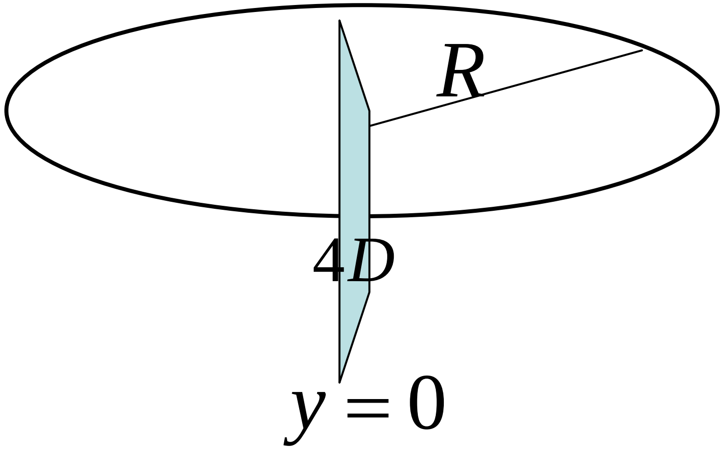
ex.



- $H_D \subset A_5(\Sigma_{8,35})$ ⇒ **origin of Higgs doublets**
- $g_5\, \underline{\psi^c_{5D}}_R\, A_5\, \underline{\psi_{5D}}_L$ ⇒ **origin of Yukawa int.**

1-2: notation

$$(1):\quad M^4 \otimes S^1$$



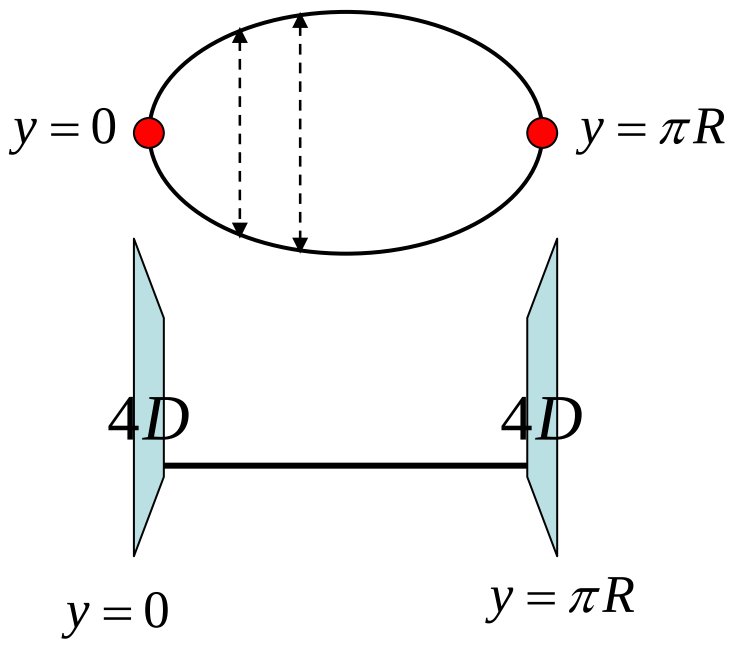
$$T:\ \phi(x^\mu, y+2\pi R) = T\phi(x^\mu, y)$$

$$[T \in U(N)]$$

$$\phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu)\, e^{i\frac{n}{R}y}$$

1-2: notation

(2): $M^4 \otimes S^1 / Z^2$ — $y = -y$



$$P: \ \phi(x^\mu, -y) = P\phi(x^\mu, y)$$

$$[P^2 = 1 \ \because \phi(y) = P\phi(-y) = P^2\phi(y)]$$

$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu) \ \cos(\frac{ny}{R})$$

$$\phi_-(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \ \sin(\frac{ny}{R})$$

1-2: notation

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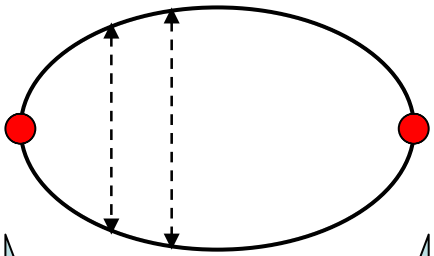
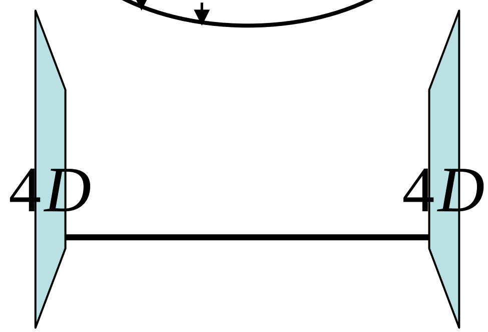
$y = -y$





$$A_\mu(x^\mu,-y) = PA_\mu(x^\mu,y)P$$

$$A_5(x^\mu,-y) = -PA_5(x^\mu,y)P$$

$$\psi_L(x^\mu,-y) = P\psi_L(x^\mu,y)$$

$$\psi_R(x^\mu,-y) = -P\psi_R(x^\mu,y)$$

$$[\psi(x^\mu,-y) = Pi\gamma^y\psi(x^\mu,y)]$$

$$5D:\ \gamma^M = (\gamma^\mu, i\gamma^5)$$

$$P:\ \phi(x^\mu,-y) = P\phi(x^\mu,y)$$

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2. gauge-Higgs unification

--- $SU(3)\times SU(3)$ model & $SU(6)$ model---

2-1. $SU(3)\times SU(3)$ model
2-2. $SU(6)$ model
2-3. SUSY

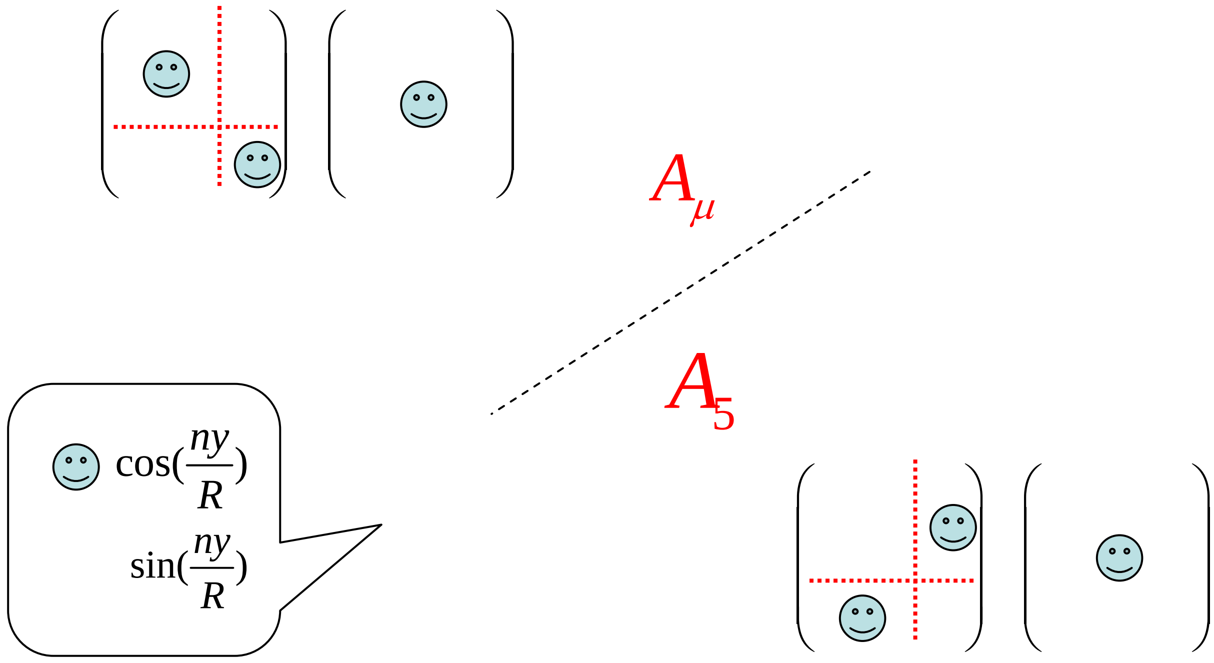
2-1. $SU(3)_c \times SU(3)_W$ model

(Kubo,Lim,Yamashita, Hall,Nomura,Smith, Burdman,Nomura,….)

$$P = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

in base of

$$SU(3)_W \supset SU(2)_L \times U(1)_Y$$



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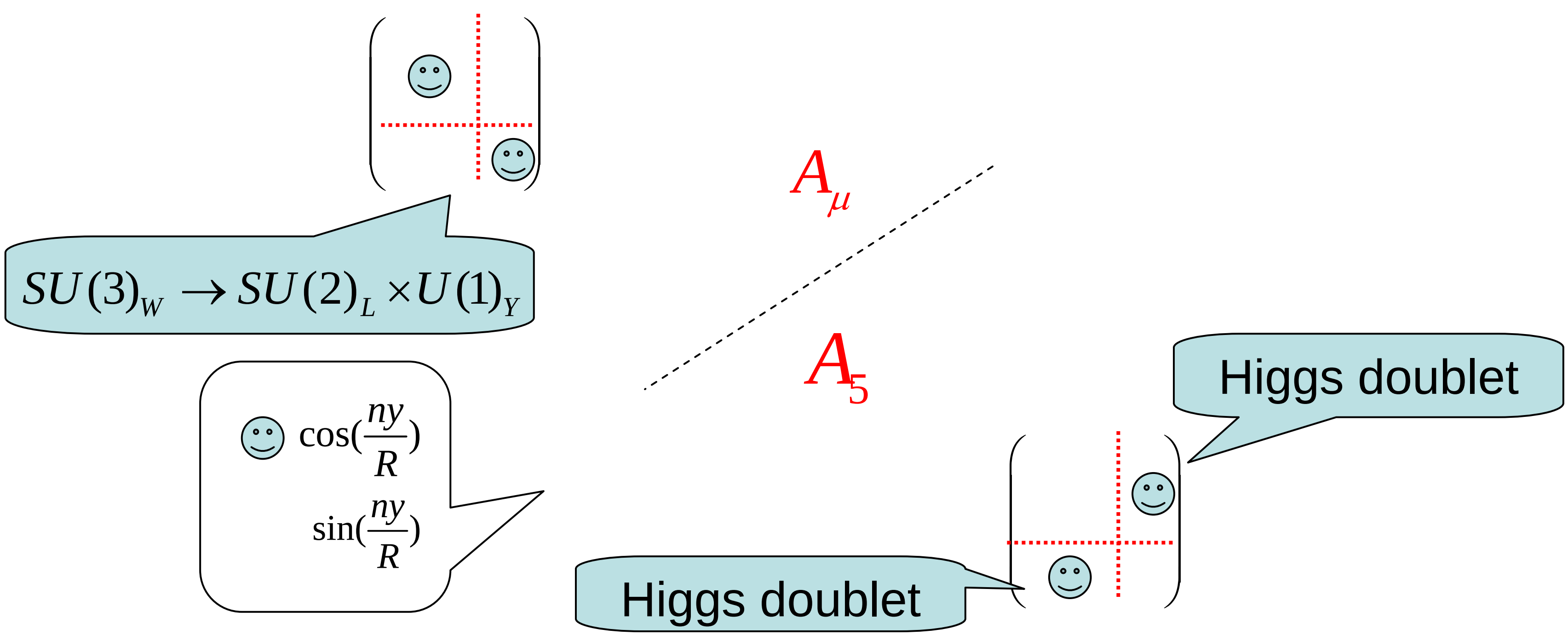
in base of $SU(3)_W \supset SU(2)_L \times U(1)_Y$

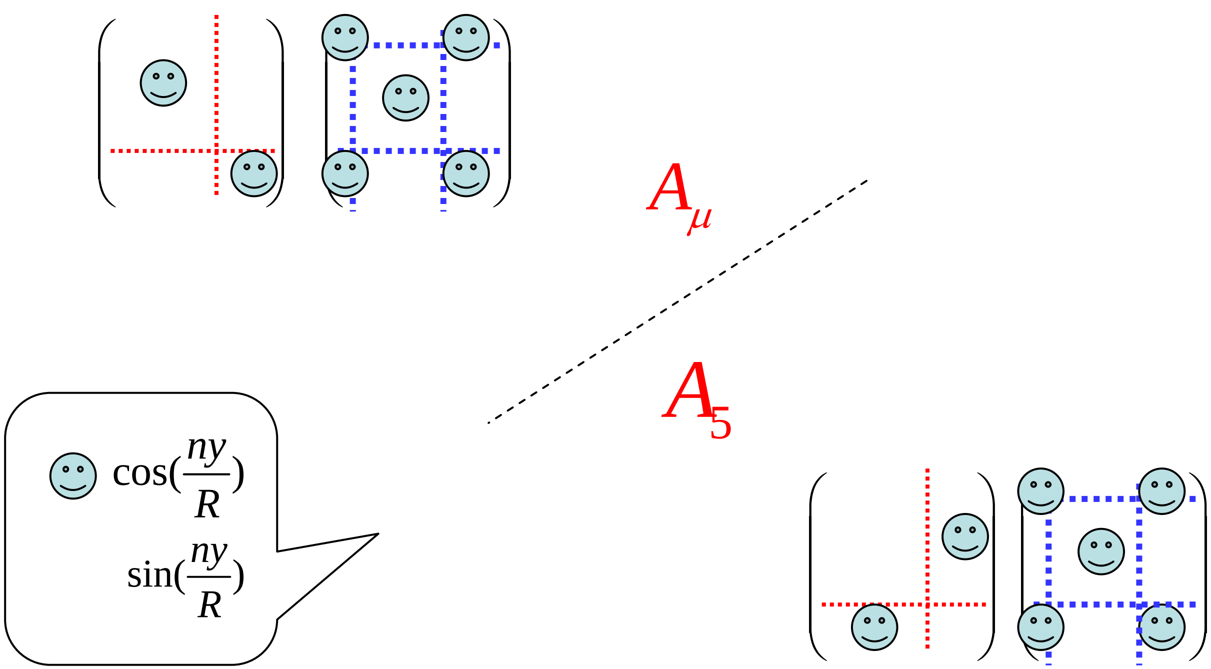


2-2. $SU(6)$ GUT

(Hall, Nomura, Smith, Burdman, Nomura)

$$P = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}$$

in base of $SU(6)$



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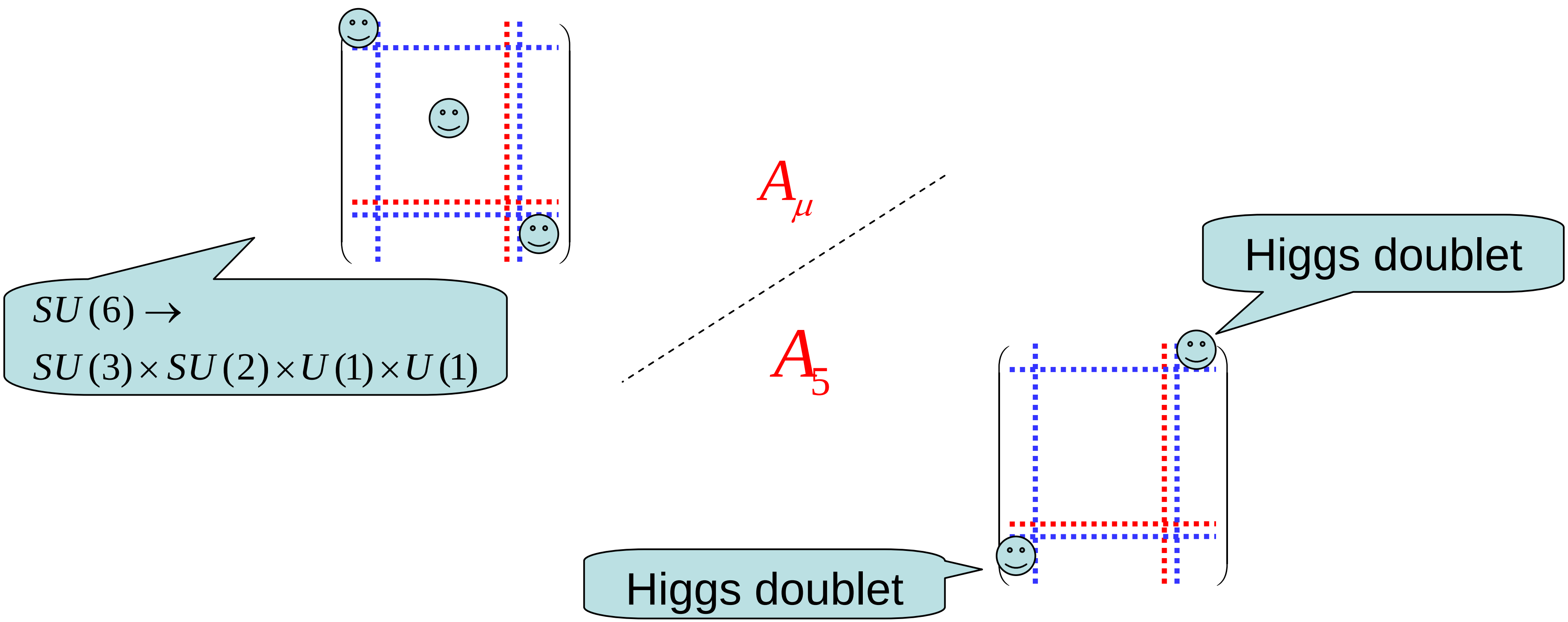
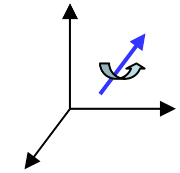
in base of $SU(6)$

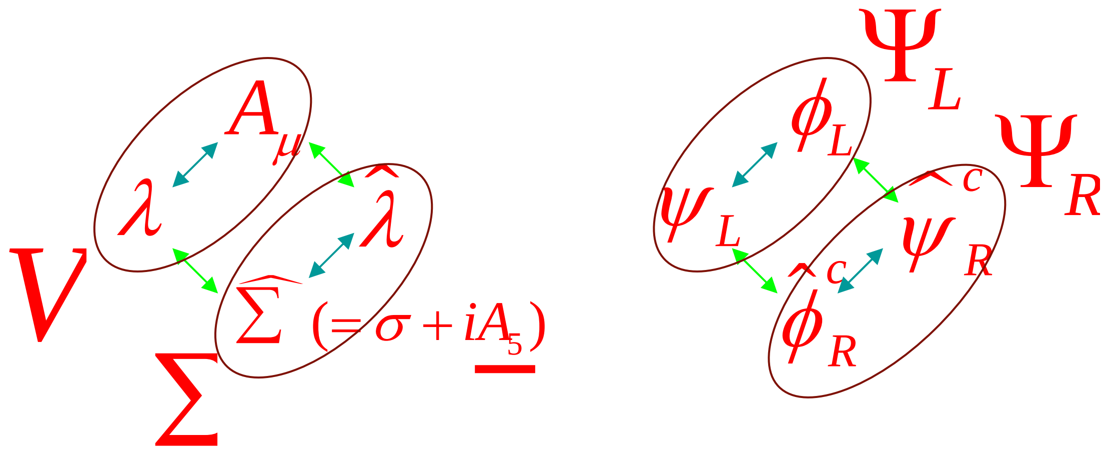


2-3. 5D N=1 SUSY



odd dim.=vector-like

5D N=1 SUSY



⇔ ***4D N=2 SUSY***

$$S_{5D}^{hyp.} = \int d^4x dy [\int d^4\theta (\Psi_R e^V \overline{\Psi}_R + \Psi_L e^V \overline{\Psi}_L)$$

$$+\{\int d^2\theta (\Psi_R (\partial_y - g_5 \Sigma) \Psi_L) + h.c.\}]$$

Yukawa int. !!

g~y_{top}~0.7 at GUT

3. dynamical EW symmetry breaking I

3-1. $SU(3)\times SU(3)$ model
3-2. $SU(6)$ model
3-3. SUSY version

NH, Y. Hosotani, Y. Kawamura and T. Yamashita, Phys.Rev.D70:015010, 2004

NH and T. Yamashita, JHEP 0402:059,2004

Now let us consider the quantum correction! in theory $A_5^{(0)} \equiv H$

calculating one loop effective potential

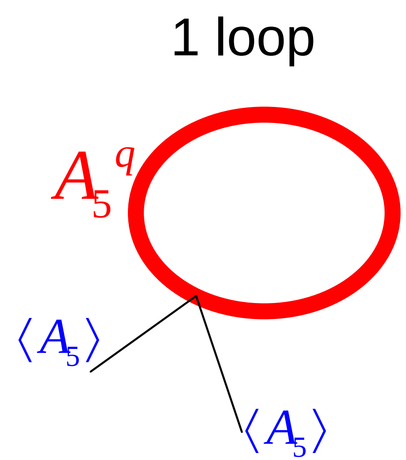
V (p=0) → position independent potential

→ V^{eff} (vev)   (SUSY→zero!)

background field gauge

$$A_5 = \langle A_5 \rangle + A_5{}^q$$

taking $A_5{}^q$'s square term, and integrate it

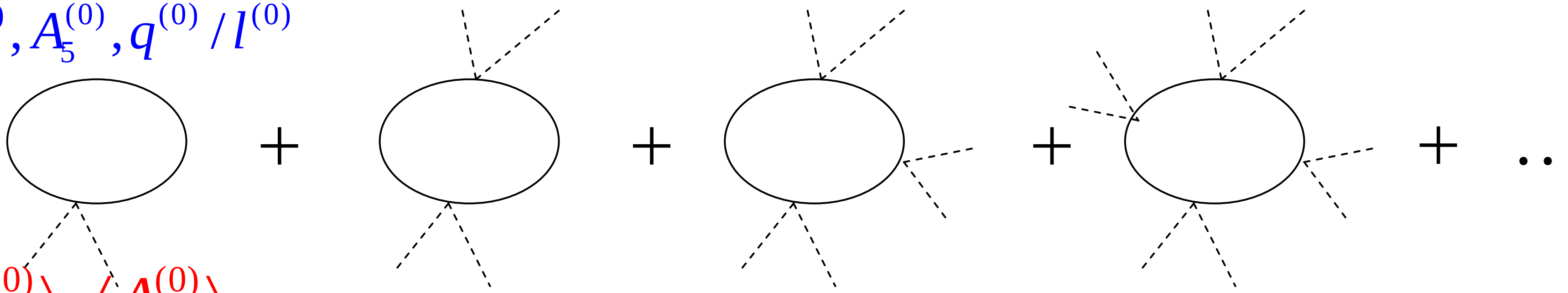


$$V_{eff}(\langle A_5 \rangle)^{g+gh} = -(d-2)\frac{i}{2} Tr\, Ln\ D_\mu(\langle A_5 \rangle) D^\mu(\langle A_5 \rangle)$$

$$V_{eff}(\langle A_5 \rangle)^{f} = i\ Tr\, Ln\ \{i\gamma^\mu D_\mu(\langle A_5 \rangle) - m)$$

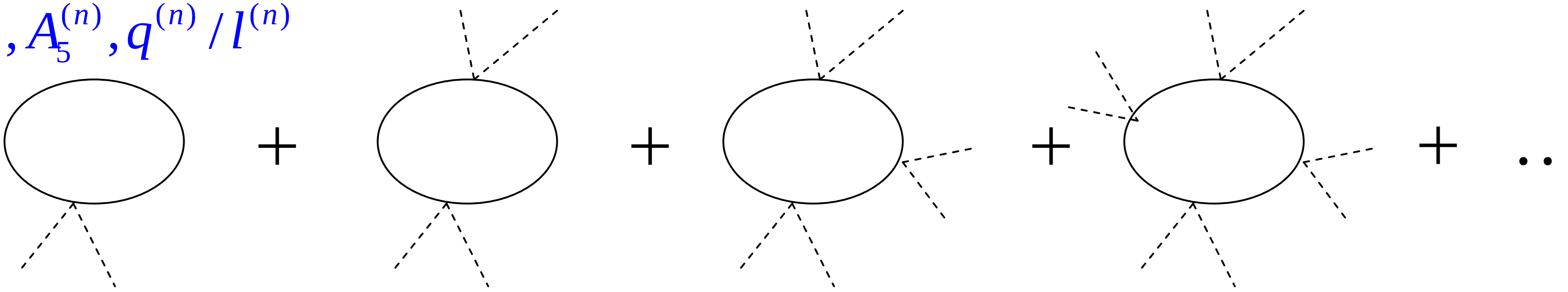
Now let us consider the quantum correction! in theory $A_5^{(0)} \equiv H$

$A_\mu^{(0)}, A_5^{(0)}, q^{(0)} / l^{(0)}$



$\langle A_5^{(0)} \rangle \ \langle A_5^{(0)} \rangle$

$A_\mu^{(n)}, A_5^{(n)}, q^{(n)} / l^{(n)}$



<u>infinite sum of KK mode</u> →effective potential, $V(\langle A_5^{(0)} \rangle)$

(**when SUSY → vanish! → <u>Scherk-Schwarz breaking)</u>**

search min. of $V(\langle A_5^{(0)} \rangle)$ → Higgs VEV $\langle A_5^{(0)} \rangle \neq 0, or = 0$

Q: $\langle A_5^{(0)} \rangle$ is really physical ?

cf. $A_5^{(n)}$ is not, it's gauged away.

$$\langle A_5^{(0)} \rangle = \frac{1}{2\pi g R} \sum_a \theta_a T_a$$

Wilson line d.o.f.

$$\langle A_5^{(0)} \rangle \to \langle A_5^{(0)} \rangle' = \Omega(y) \langle A_5^{(0)} \rangle \Omega^\dagger(y) - \frac{i}{g} \Omega(y) \partial_y \Omega^\dagger(y)$$

$$= \langle A_5^{(0)} \rangle - \frac{1}{2\pi g R} \sum \theta_a T_a \quad = 0$$

$$\Omega(y) = e^{i \sum \theta_a T_a \frac{y}{2\pi R}}$$

$$T \to T' = \Omega(y + 2\pi R)\ T\ \Omega^\dagger(y) \quad = e^{i \sum \theta_a T_a (\frac{y}{2\pi R} + 1)}\ T\ e^{-i \sum \theta_a T_a \frac{y}{2\pi R}}$$

$$P \to P' = \Omega(-y)\ P\ \Omega^\dagger(y) = P$$

$$\langle A_5^{(0)} \rangle \neq 0\ \text{base with}\ T \quad \Leftrightarrow \langle A_5^{(0)} \rangle = 0\ \text{base with}\ T'$$

gauge sym. U_a *remain which*

satisfies $[U_a, T'] = 0, [U_a, W_C] = 0$

$$W_C = P \exp(-ig \int \langle A_5 \rangle dy)$$

cf. Hosotani mech.

3-1. $SU(3)_c \times SU(3)_W$ model

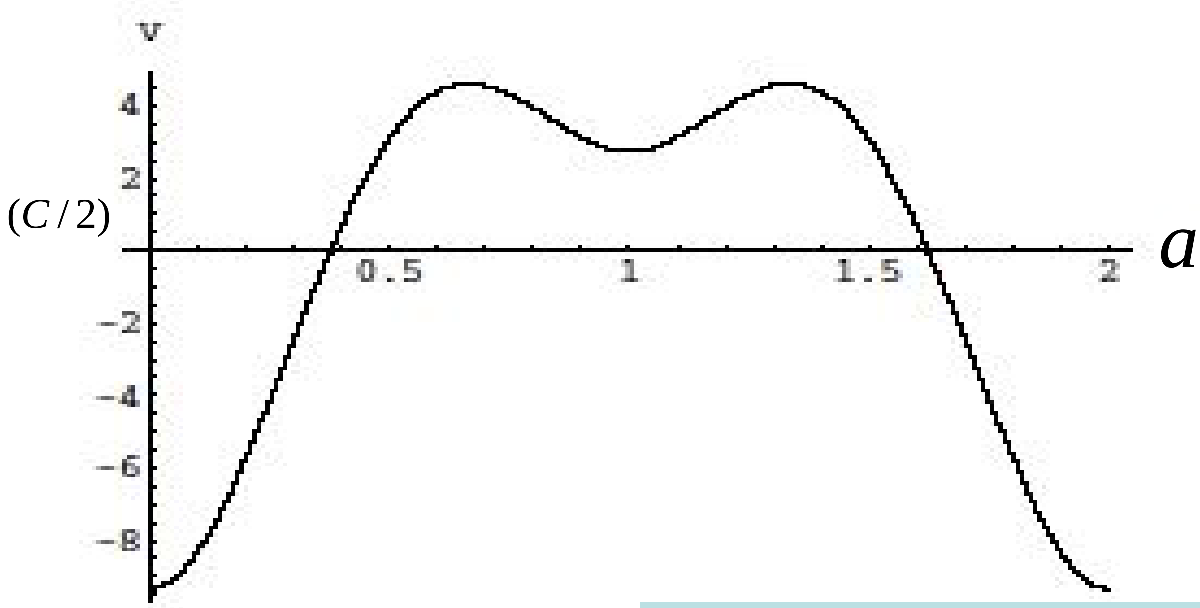
$$\langle A_5^{(0)} \rangle = \frac{1}{2gR}\left(\begin{array}{cc|c} & & a \\ & & a \\ \hline a & a & \end{array}\right)$$

a:dim-less

effective potential:

$$V_{eff}(\langle A_5^{(0)} \rangle)$$

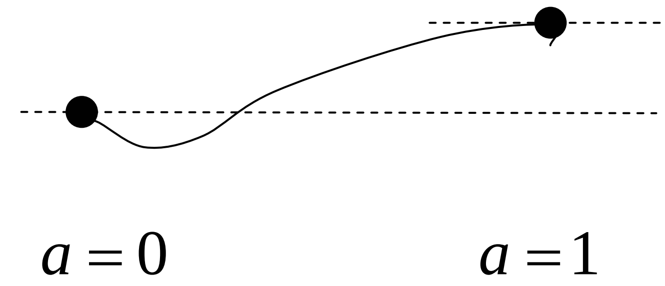
$$V_{eff}^{gauge} = -\frac{3}{2} C \sum_{n=1}^{\infty} \frac{1}{n^5} [\cos(2\pi n a) + \cos(\pi n a)] \qquad C \equiv \frac{3}{64\pi^7 R^5}$$



no sym. breaking!

effective potential: $V_{eff}(\langle A_5^{(0)}\rangle)$

vacuum should be at $0 \le a \ll 1$



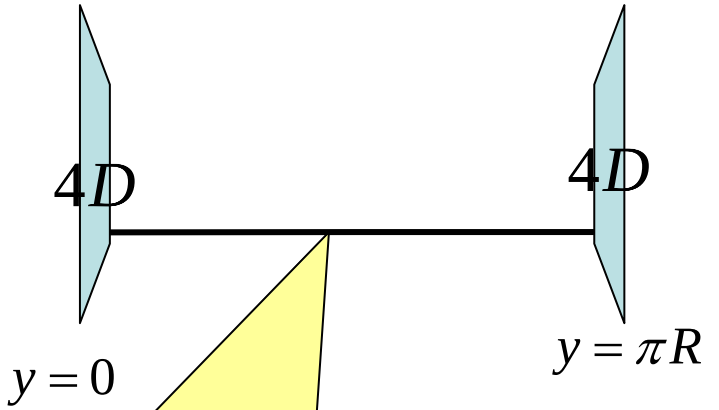
if vacuum is at y=πR

$$
\begin{aligned}
W_C &= \exp\left(ig\int_0^{2\pi R} dy\,\langle\Sigma\rangle\right) \\
&= \exp\left(ig2\pi R\frac{1}{2gR}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix},
\end{aligned}
$$

$\langle A_5^{(0)}\rangle \sim \dfrac{1}{R}$ $\quad SU(2)\times U(1) \to U(1)\times U(1)$ at ∼1/R

Not Good!

effective potential: $V_{eff}(\langle A_5^{(0)} \rangle)$



introduce
extra fields in bulk

$N_a^{(\pm)}, N_f^{(\pm)}, N_s^{(\pm)}$

fermion (adj. & fund.) scalar (fund.)

$$A_\mu(x^\mu,-y) = P A_\mu(x^\mu,y) P$$

$$A_5(x^\mu,-y) = -P A_5(x^\mu,y) P$$

$$\psi_L(x^\mu,-y) = (\pm) P \psi_L(x^\mu,y)$$

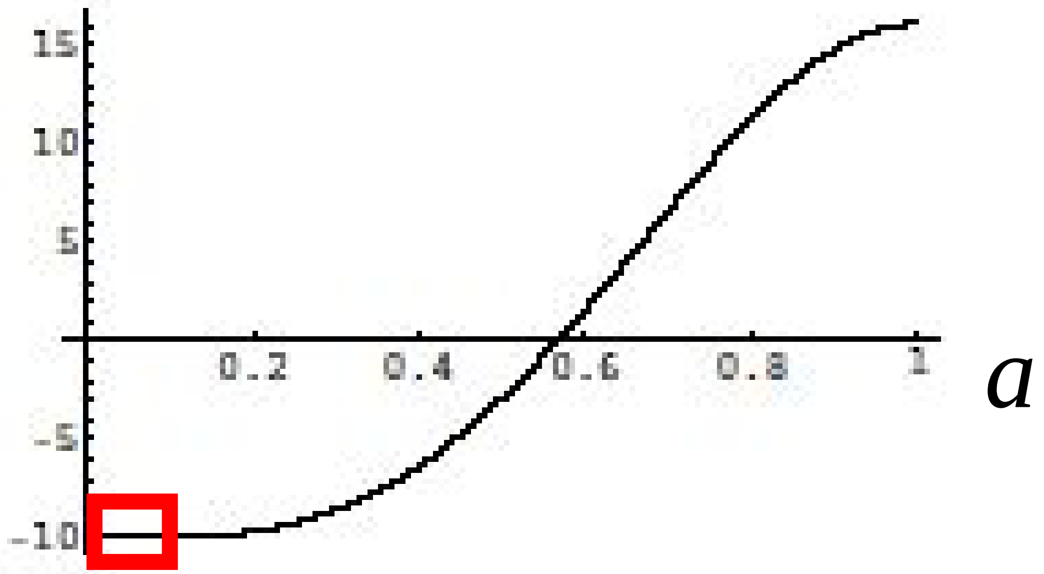
$$\psi_R(x^\mu,-y) = (\pm)(-) P \psi_R(x^\mu,y)$$

$$s(x^\mu,-y) = (\pm) s(x^\mu,y)$$

There is d.o.f. of $(\pm)$

$$\begin{aligned} V_{eff}^m &= C \sum_{n=1}^{\infty} \frac{1}{n^5} [2N_a^{(+)} \cos(2\pi n a) + 2N_a^{(-)} \cos(2\pi n (a - \frac{1}{2})) \\ &+ (4N_a^{(+)} - N_s^{(+)} + 2N_f^{(+)}) \cos(\pi n a) \\ &+ (4N_a^{(-)} - N_s^{(-)} + 2N_f^{(-)}) \cos(\pi n (a-1))]. \end{aligned}$$

effective potential: $V_{eff}(\langle A_5^{(0)} \rangle) = V_{eff}^{gauge} + V_{eff}^{m}$



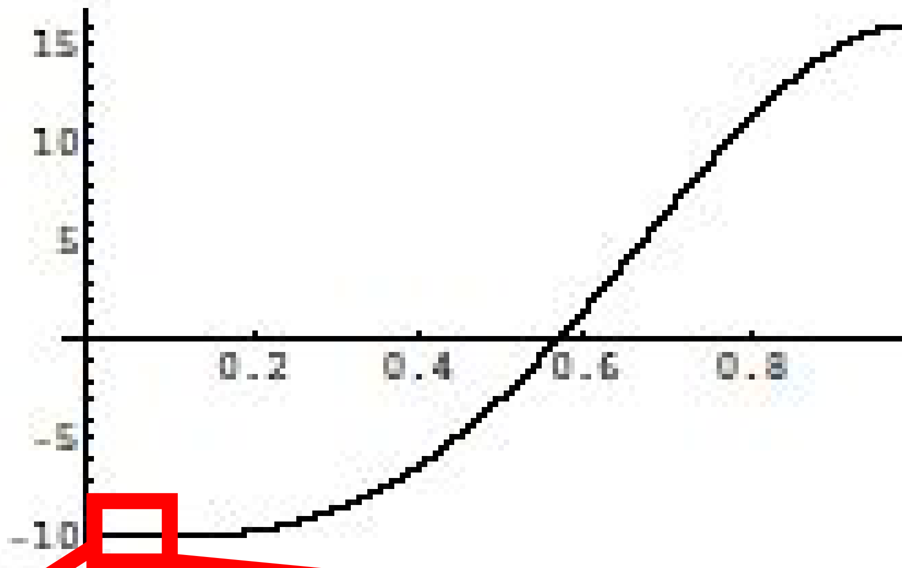
$$N_a^{(+)} = 2, N_f^{(-)} = 8, N_s^{(+)} = 4, N_s^{(-)} = 2, N_a^{(-)} = N_f^{(+)} = 0$$

effective potential: $V_{eff}(\langle A_5^{(0)} \rangle) = V_{eff}^{gauge} + V_{eff}^{m}$

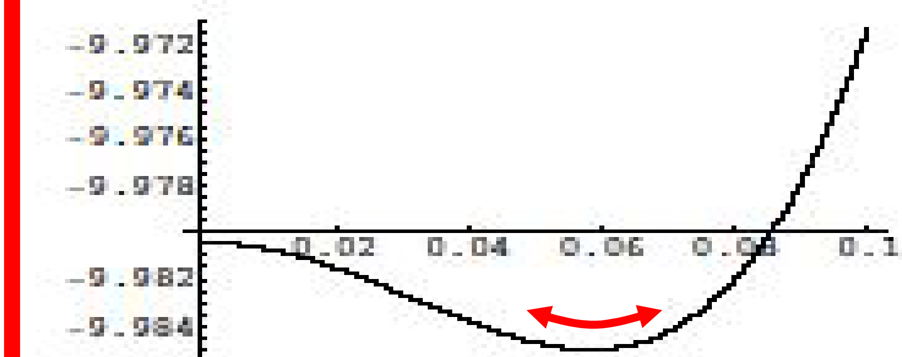
$\frac{1}{R} \sim O(1) \quad TeV$

vev:
O(100)GeV

$SU(2) \times U(1) \downarrow U(1)_{em}$



$N_a^{(+)} = 2$ ... $= N_f^{(+)} = 0$



bulk extra field effect is important!

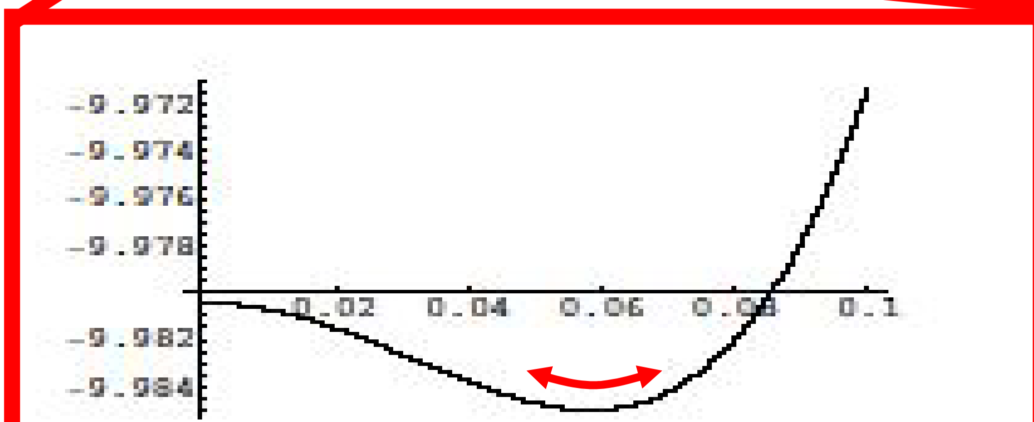
effective potential: $V_{eff}(\langle A_5^{(0)}\rangle) = V_{eff}^{gauge} + V_{eff}^{m}$

$$m_{A_5}^2 = (gR)^2 \frac{\partial^2 V_{eff}}{\partial a^2}\Big|_{\min} \sim \left(\frac{O(100) g_4^2}{R} GeV\right)^2$$

$$\frac{g}{\sqrt{2\pi R}} = g_4 \qquad \frac{\langle A_5^{(0)}\rangle}{\sqrt{2\pi R}} = \frac{a}{g_4 R} \sim 246\ GeV$$

-10

$N_a^{(+)} = 2$

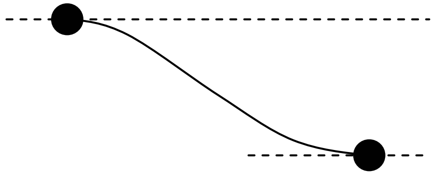


$= N_f^{(+)} = 0$

bulk extra field effect is important!

3-2.$SU(6)$ GUT

$$P = \begin{pmatrix} 1 &&&&& \\ & 1 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& -1 & \\ &&&&& -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 &&&&& \\ & -1 &&&& \\ && -1 &&& \\ &&& -1 && \\ &&&& 1 & \\ &&&&& 1 \end{pmatrix}$$



$a=0$ $\quad$ $a=1$

$$V_{eff}^{gauge} = -\frac{3}{2} C \sum_{n=1}^{\infty} \frac{1}{n^5} [\cos(2\pi n a) + 2\cos(\pi n a) + 6\cos(\pi n (a-1))]$$

$$V_{eff}^{gauge}(a=0) - V_{eff}^{gauge}(a=1) = 12C \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} > 0$$

↓

$$\begin{aligned} W_C &= \exp(ig \int_0^{2\pi R} dy \frac{1}{gR} a \frac{\lambda}{2}) \\ &= \exp(ig \frac{1}{gR} \frac{\lambda}{2} 2\pi R) = \begin{pmatrix} -1 &&&&& \\ & 1 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& 1 & \\ &&&&& -1 \end{pmatrix} \end{aligned}$$

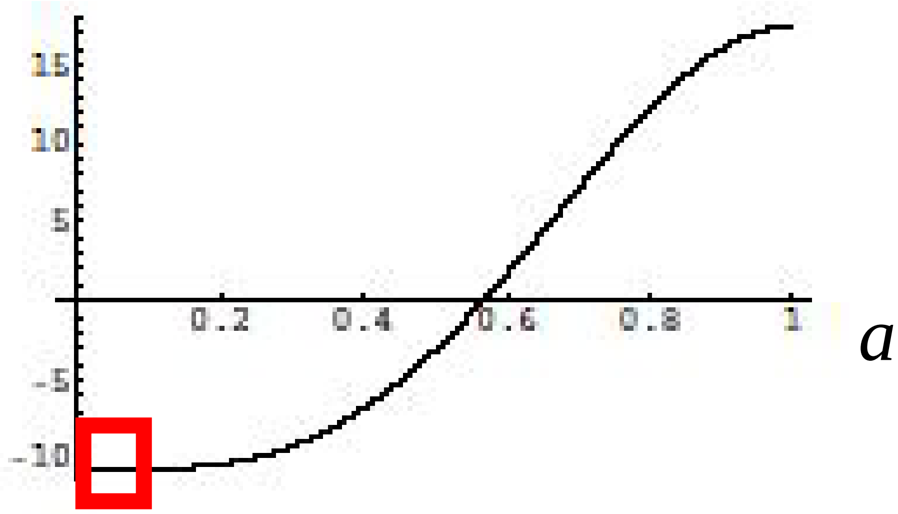
$$SU(2)_L \times U(1)_Y \to U(1)^2$$

$$at \ \langle A_5^{(0)} \rangle \sim \frac{1}{R}$$

not good! → introducing extra fields in bulk

introduce extra bulk fields →

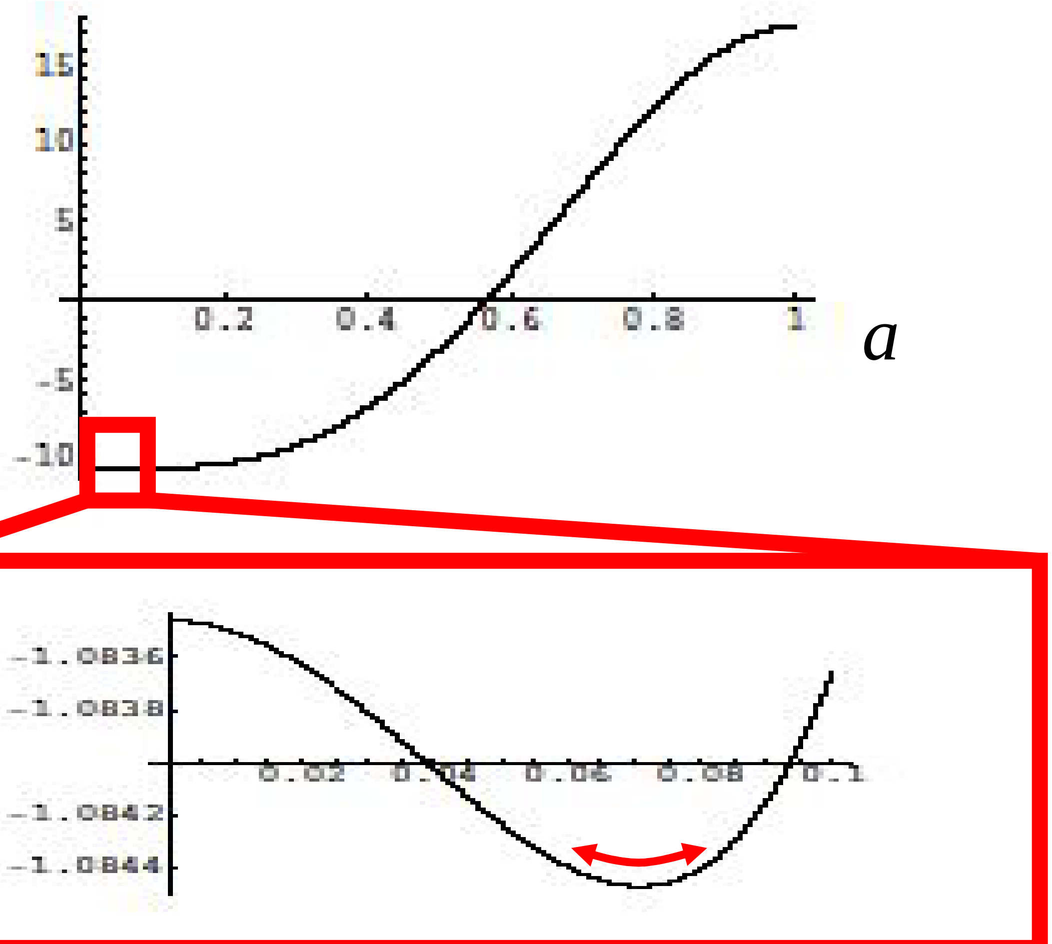
$$
\begin{aligned}
V_{eff}^{m} = & \; C\sum_{n=1}^{\infty}\frac{1}{n^5}[2N_a^{(+)}\cos(2\pi n a) + 2N_a^{(-)}\cos(2\pi n(a-\frac{1}{2})) \\
& +(4N_a^{(+)} + 12N_a^{(-)} + 2N_f^{(+)} - N_s^{(+)})\cos(\pi n a) \\
& +(12N_a^{(+)} + 4N_a^{(-)} + 2N_f^{(-)} - N_s^{(-)})\cos(\pi n(a-1))].
\end{aligned}
$$



$$N_a^{(+)} = N_f^{(-)} = 2, \; other\ Ns = 0$$

introduce extra bulk fields $\rightarrow$

$$
\begin{aligned}
V_{eff}^{m} \;=\; & C\sum_{n=1}^{\infty}\frac{1}{n^5}[2N_a^{(+)}\cos(2\pi n a) + 2N_a^{(-)}\cos(2\pi n(a-\frac{1}{2})) \\
& +(4N_a^{(+)} + 12N_a^{(-)} + 2N_f^{(+)} - N_s^{(+)})\cos(\pi n a) \\
& +(12N_a^{(+)} + 4N_a^{(-)} + 2N_f^{(-)} - N_s^{(-)})\cos(\pi n(a-1))].
\end{aligned}
$$



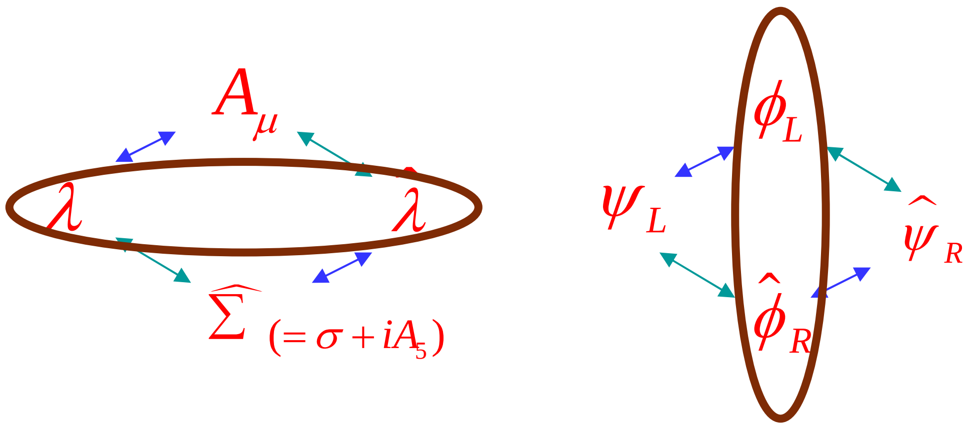
3-3.SUSY version

SUSY must be broken, or V=0!

☆ Scherk-Schwarz SUSY breaking

Imposing different BC for fermion and boson → SUSY br.

twist of $SU(2)_R$



soft mass

$$\rightarrow \quad \frac{\beta}{2R}\lambda\lambda, \quad (\frac{\beta}{R})^2 |\phi|^2$$

☆ soft scalar mass for bulk fields (later)

SUSY version 5D N=1 (→4D N=2) : (Scherk-Schwarz SUSY breaking)

$\beta \sim 0.1$

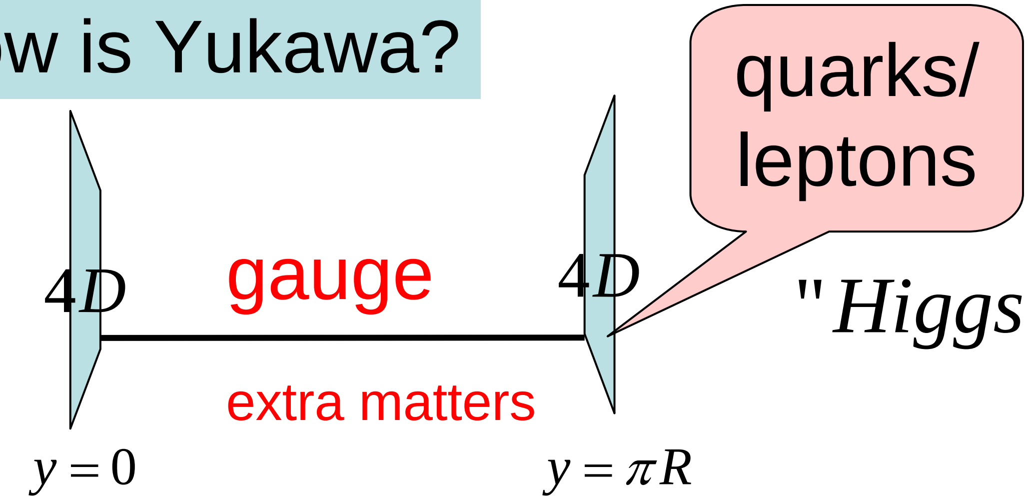
bulk matter: $N_f^{(\pm)}$ fund. & $N_a^{(\pm)}$ adjo. super-feilds

examples:

$$SU(3)\times SU(3): \rightarrow N_a^{(+)} = N_a^{(-)} = 2,\ N_f^{(-)} = 4,\ N_f^{(+)} = 0$$

$$SU(6): \rightarrow N_a^{(+)} = 2,\ N_a^{(-)} = N_f^{(+)} = 0,\ N_f^{(-)} = 10$$

How is Yukawa?



$$"Higgs":\ P\exp(\int A_5\ dy)$$

$$(\Sigma \rightarrow e^{\Lambda}(\Sigma - \sqrt{2}\partial_y)e^{-\Lambda})$$

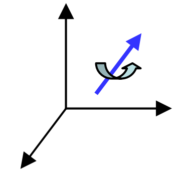
quarks/leptons in bulk $\rightarrow$ $L = [\psi_{5D}^c \Sigma_5\ \psi_{5D}]_{\theta^2} + h.c.$

Yukawa from 5D gauge int.

4. dynamical EW symmetry breaking II

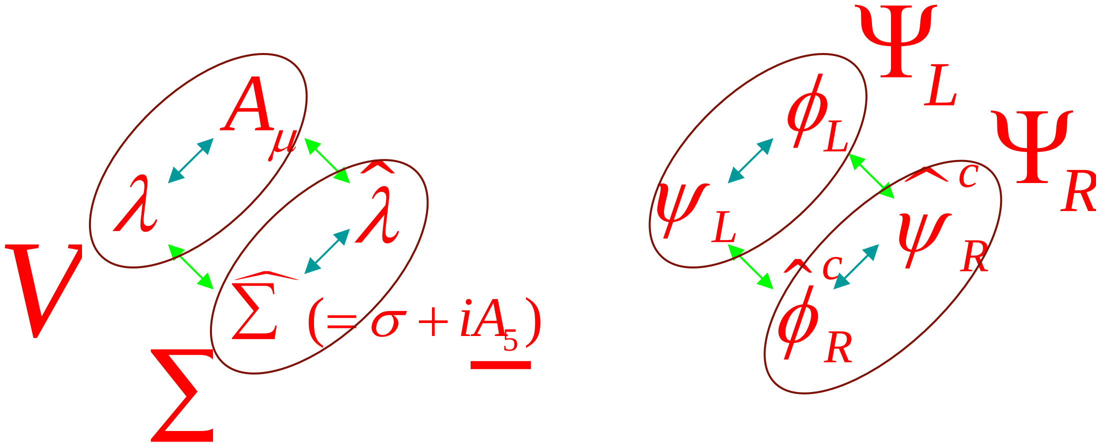
4-1. $SU(3) \times SU(3)$ model
4-2. result

NH and T. Yamashita, JHEP 0404 (2004) 016



5D N=1 SUSY

odd dim.=vector-like



⇔ 4D N=2 SUSY

$$S^{hyp.}_{5D} = \int d^4x dy \left[\int d^4\theta \left(\Psi_R e^V \overline{\Psi}_R + \Psi_L e^V \overline{\Psi}_L \right) + \left\{ \int d^2\theta \left(\underline{\Psi_R} (\partial_y - g_5 \underline{\Sigma}) \underline{\Psi_L} \right) + h.c. \right\} \right]$$

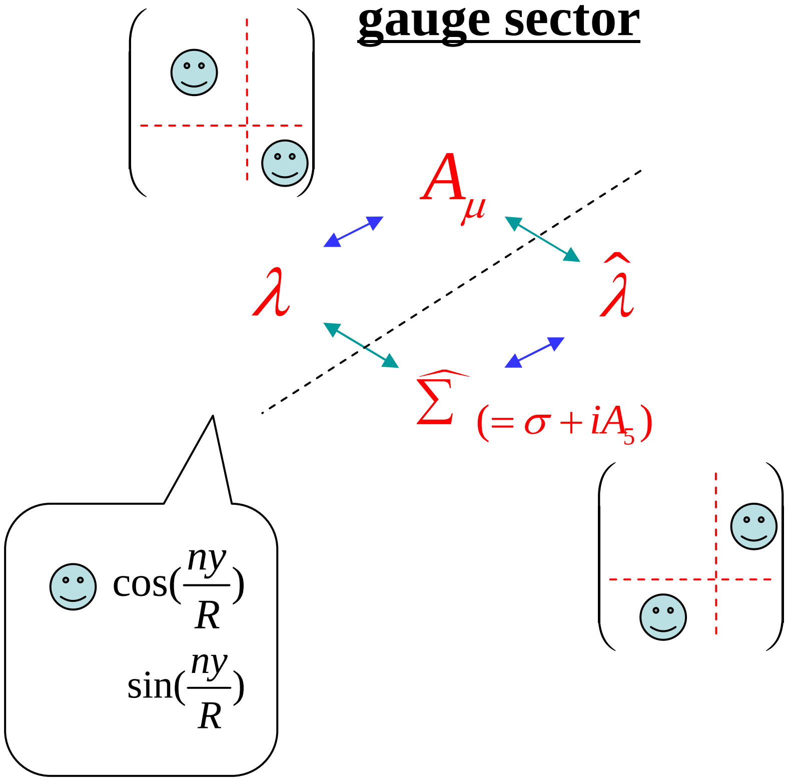
Yukawa int. !!

g∼y_{top}∼0.7 at GUT

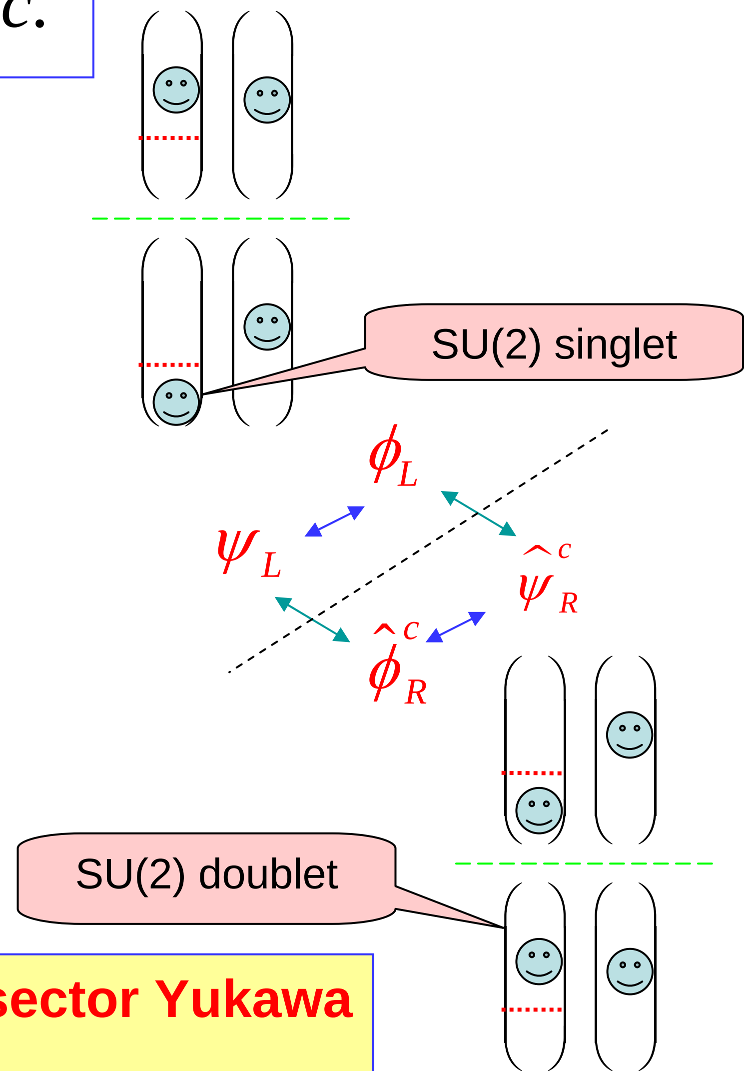
4-1. $SU(3)_c \times SU(3)_W$ model

$$\text{Yukawa} \supset L = [\psi^c_{5D} \Sigma_5 \ \psi_{5D}]_{\theta^2} + h.c.$$

gauge sector



fund. rep. bulk matter



fund. rep. → only down-sector Yukawa
6 → up-sector
10 → charged lepton sector
8 → ν -sector

(Burdman-Nomura)

effective potential: $V(\langle A_5^{(0)} \rangle)$

$$V_{\rm eff}^{q/l} = 2N_g C \sum_{n=1}^{\infty} \frac{1}{n^5}(1-\cos(2\pi n\beta)) \times [3f_u(a)+3f_d(a)+f_e(a)+f_\nu(a)],$$

$$f_u(a) = \cos(2\pi na) + \cos(\pi na),$$

$$f_d(a) = \cos(\pi na),$$

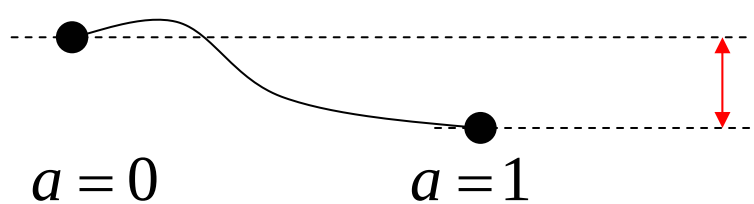
$$f_e(a) = \cos(3\pi na) + \cos(2\pi na) + 2\cos(\pi na),$$

$$f_\nu(a) = \cos(2\pi na) + 2\cos(\pi na),$$

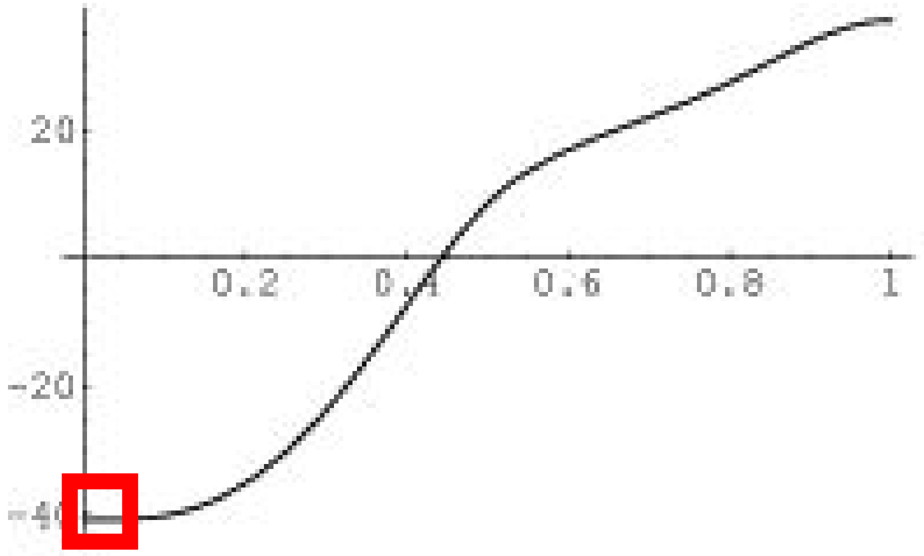
$$V_{\rm eff} = V_{\rm eff}^{gauge} + V_{\rm eff}^{q/l} = 2C \sum_{n=1}^{\infty} \frac{1}{n^5}(1-\cos(2\pi n\beta)) \times [N_g\cos(3\pi na) + (5N_g-1)\cos(2\pi na) + (10N_g-2)\cos(\pi na)]$$

$$V_{\rm eff}(a=0) - V_{\rm eff}(a=1) = 4(11N_g-2)C \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}(1-\cos(2\pi(2n-1)\beta)).$$

> 0



effective potential: $V(\langle A_5^{(0)} \rangle)$

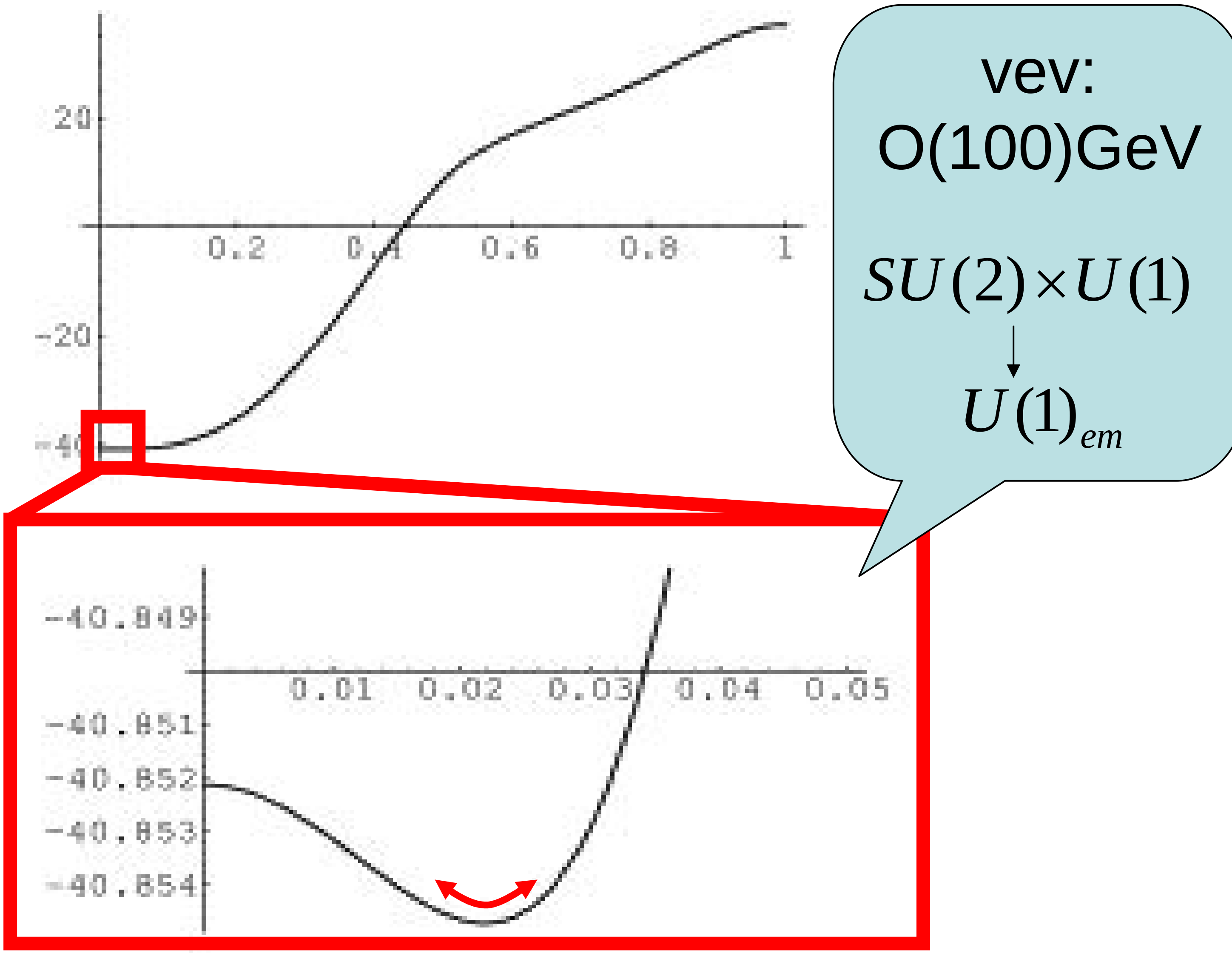


$$N_a^{(+)} = N_f^{(+)} = 0, N_a^{(-)} = 45, N_f^{(-)} = 40 \quad N_g = 3$$

$$\beta = 0.1,$$

$$m_{susy} \sim \frac{\beta}{R} \quad \frac{1}{R} \sim O(1) \quad TeV$$

effective potential: $V(\langle A_5^{(0)} \rangle)$

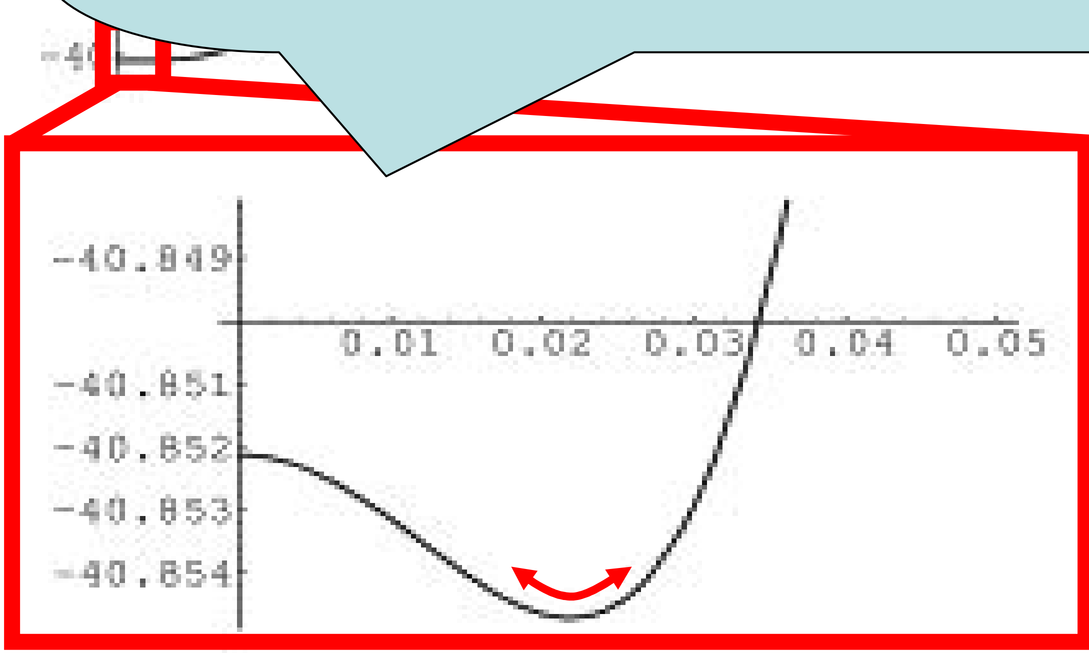


bulk extra field effect is important!

effective potential: $V(\langle A_5^{(0)} \rangle)$

$$m_{A_5}^2 = (gR)^2 \frac{\partial^2 V_{eff}}{\partial a^2}\Big|_{\min} \sim \left(\frac{O(100)g_4^2}{R} GeV\right)^2$$

$$\frac{g}{\sqrt{2\pi R}} = g_4 \qquad \frac{\langle A_5^{(0)} \rangle}{\sqrt{2\pi R}} = \frac{a}{g_4 R} \sim 246\ GeV$$



bulk extra field effect is important!

result:

set-up: all 3-generation quarks/leptons in bulk

$$\frac{1}{R} \sim O(1) \quad TeV \qquad m_{susy} \sim \frac{\beta}{R} \qquad \beta = 0.1,$$

examples:

$SU(3)_c \times SU(3)_W$ model

$$N_a^{(+)} = N_f^{(+)} = 0, N_a^{(-)} = 45, N_f^{(-)} = 40$$

$SU(6)$ GUT

$$N_a^{(+)} = N_f^{(+)} = N_a^{(-)} = 0, N_f^{(-)} = 42$$

5. Higgs mass & phenomenology

5-1. 4D Higgs fields
5-2. 3-point self coupling

NH, K.Takenaga and T.Yamashita, Phys.Rev.D71:025006,2005
NH, K.Takenaga and T.Yamashita, hep-ph/0411250

5-1. 4D Higgs fields

5D gauge kinetic term→4D Higgs kinetic term

$$2\times\int dy\frac{1}{4}F^a_{\mu 5}F^{a\mu 5}=\int dy\frac{1}{2}(\partial A^a_5+igf^a_{bc}A^b_\mu A^c_5)^2=(\partial_\mu+ig_4W^\alpha_\mu\frac{\tau^\alpha}{2}+i\sqrt{3}g_4\frac{B_\mu}{2})H|^2$$

$$A_5=\begin{pmatrix} & \begin{matrix}\frac{A^1_5+iA^2_5}{\sqrt{2}} & \frac{A^4_5+iA^5_5}{\sqrt{2}}\end{matrix} \\ \begin{matrix}\frac{A^1_5-iA^2_5}{\sqrt{2}} \\ \frac{A^4_5-iA^5_5}{\sqrt{2}}\end{matrix} & \end{pmatrix} \qquad (g_4=\frac{g}{\sqrt{2\pi R}})$$

$$\equiv H/\sqrt{2\pi R}$$

- However, $g_Y=\sqrt{3}g_2\rightarrow\sin\theta_W=\sqrt{3}/2$, so we assume wall-localized kinetic terms, $\delta(0)\lambda_0 F^{\mu\nu 2}$, $\delta(\pi R)\lambda_\pi F^{\mu\nu 2}$, which do not respect SU(3) symmetry, are dominant as $g_4^2>\lambda^{-1}$, (we take $g_4\sim 1$), and expect $(W_\mu,B_\mu)\rightarrow(\frac{g_2}{g_4}W_\mu,\frac{g_Y}{\sqrt{3}g_4}B_\mu)$.
- additional U(1)’

$$\sqrt{2\pi R}\ \langle A_5\rangle=\frac{a_0}{g_4R}=v\sim 246\ GeV$$

$cf:[SU(6):\sin\theta_W=\sqrt{3/8}]$

In SU(6), it is the same as SU(5) as $g_2 = \sqrt{5/3}\, g_Y \rightarrow \sin\theta_W = \frac{g_Y}{\sqrt{g_2^2 + g_Y^2}} = \sqrt{3/8}$

SUSY case

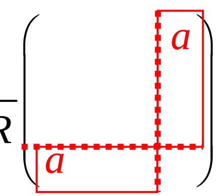
$$
\begin{cases}
H_u = \frac{1}{\sqrt{2}}(\langle A_5^1\rangle - \sigma_5^2 + i(\sigma_5^1 + A_5^2), A_5^4 - \sigma_5^5 + i(\sigma_5^4 + A_5^5)) \\
H_d = \frac{1}{\sqrt{2}}(\langle A_5^1\rangle + \sigma_5^2 + i(\sigma_5^1 - A_5^2), A_5^4 + \sigma_5^5 + i(\sigma_5^4 - A_5^5))
\end{cases}
$$

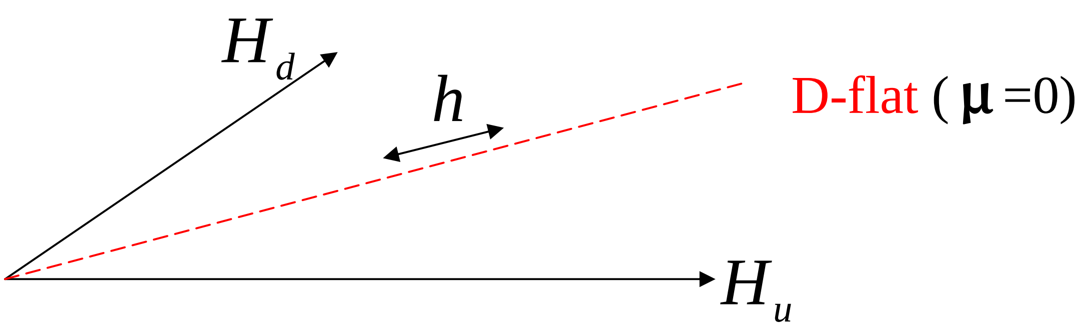
$A_5^1 : massless\ (h)$	$A_5^2 : \chi^0$	$A_5^{4,5} : \chi^\pm$
$\sigma_5^1 : massless\ (A)$	$\sigma_5^2 : M_Z\ (H)$	$\sigma_5^{4,5} : M_W\ (H^\pm)$

NH, K.Takenaga,T.Yamashita, Phys.Rev.D71:025006,2005

$$\langle \sigma \rangle = 0$$

$$\langle A_5^{(0)} \rangle = \frac{1}{2gR}$$





$$\tan\beta = 1$$

SUSY case

$$H_u = \frac{1}{\sqrt{2}}(\langle A_5^1\rangle - \sigma_5^2 + i(\sigma_5^1 + A_5^2), A_5^4 - \sigma_5^5 + i(\sigma_5^4 + A_5^5))$$

$$H_d = \frac{1}{\sqrt{2}}(\langle A_5^1\rangle + \sigma_5^2 + i(\sigma_5^1 - A_5^2), A_5^4 + \sigma_5^5 + i(\sigma_5^4 - A_5^5))$$

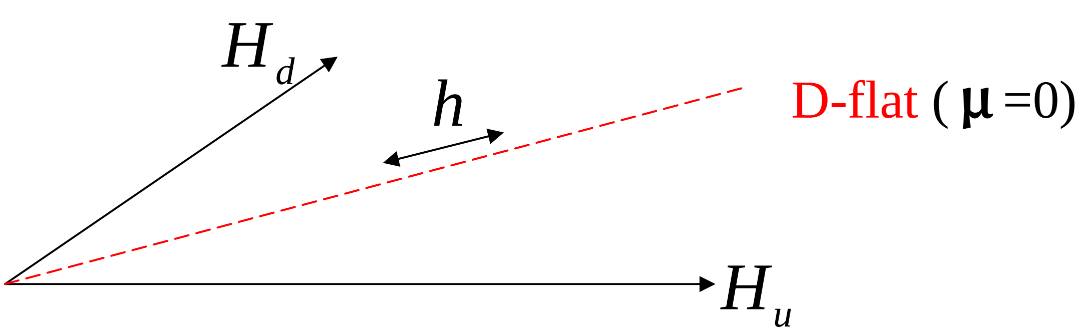
$$Q_{\tau_3}^{+} = A_5^1 + iA_5^2 + i(\sigma_5^1 + i\sigma_5^2)$$ right-up $\tau_1 + i\tau_2$

$$Q_{\tau_3}^{-} = A_5^1 - iA_5^2 + i(\sigma_5^1 - i\sigma_5^2)$$ left-down $\tau_1 - i\tau_2$

NH, K.Takenaga,T.Yamashita, Phys.Rev.D71:025006,2005

$$\langle \sigma \rangle = 0$$

$$\langle A_5^{(0)} \rangle = \frac{1}{2gR}\begin{pmatrix} & a \\ a & \end{pmatrix}$$



$$\tan\beta = 1$$

SUSY case

$$\begin{cases} H_u = \dfrac{1}{\sqrt{2}}\left(\langle A_5^1\rangle - \sigma_5^2 + i(\sigma_5^1 + A_5^2),\, A_5^4 - \sigma_5^5 + i(\sigma_5^4 + A_5^5)\right) \\ H_d = \dfrac{1}{\sqrt{2}}\left(\langle A_5^1\rangle + \sigma_5^2 + i(\sigma_5^1 - A_5^2),\, A_5^4 + \sigma_5^5 + i(\sigma_5^4 - A_5^5)\right) \end{cases}$$

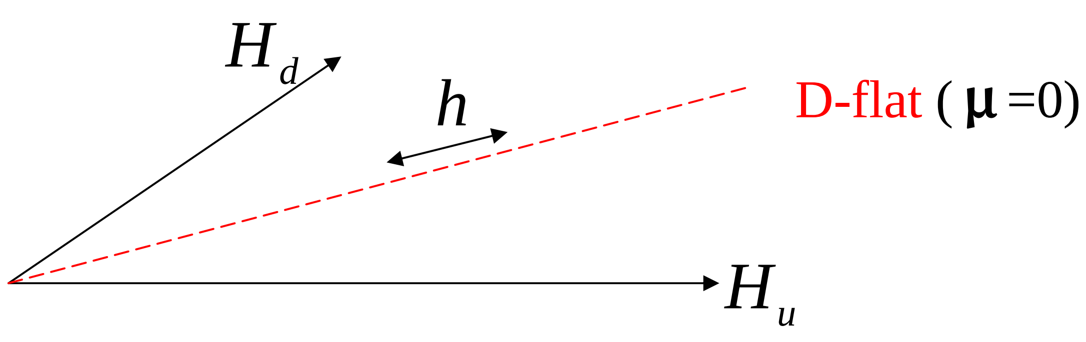
$$\phi_1 = (A_5^1 - iA_5^2,\, A_5^4 - iA_5^5) \qquad H_d = (\phi_1 + i\phi_2)/\sqrt{2}$$

$$\phi_2 = (\sigma_5^1 - i\sigma_5^2,\, \sigma_5^4 - i\sigma_5^5) \qquad H_u^* = (\phi_1 - i\phi_2)/\sqrt{2}$$

NH, K.Takenaga,T.Yamashita, Phys.Rev.D71:025006,2005

$$\langle \sigma \rangle = 0$$

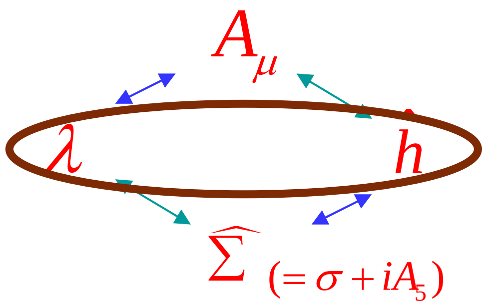
$$\langle A_5^{(0)} \rangle = \frac{1}{2gR}\begin{pmatrix} & & a \\ & & \\ a & & \end{pmatrix}$$



$$\tan\beta = 1$$

Mass Spectrum

twist of $SU(2)_R$

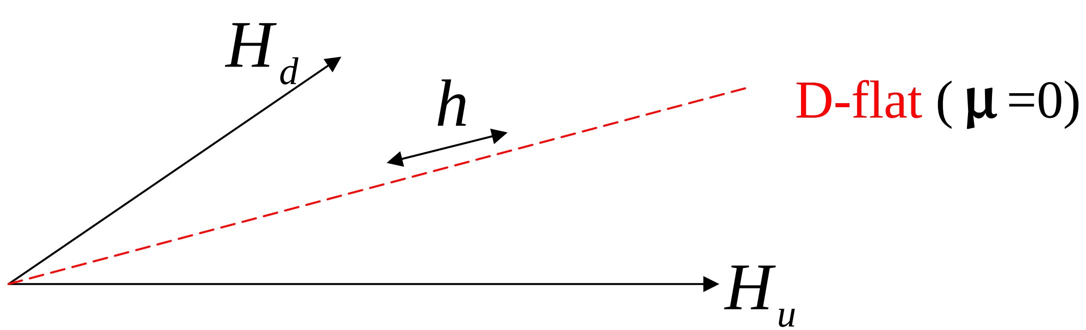


→**gaugino mass〜higgsino mass〜β/R**
(no soft scalar masses)

☆ $h \sim A \sim 100GeV,\ H \sim H^{\pm} \sim M_{W,Z} + 100GeV$

(radiative induced mass〜100GeV)

☆ gauginos mass~higgsinos mass〜β/R

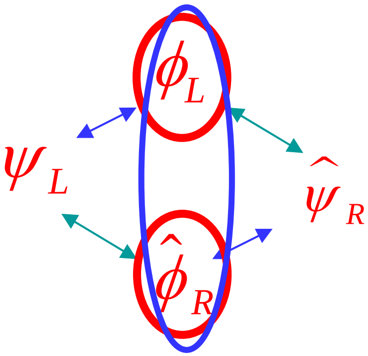


$\tan\beta = 1$

5-2. soft scalar mass

- introducing soft scalar mass, m ($z=mR$) in addition to SS

$SU(3) \times SU(3)$ model



$$V_{eff}^{matter} = 2C\sum_{n=1}^{\infty}\{N_{adj}^{(+)}(I^{(+)}[2a,\beta,z_{adj}^{(+)},n] + 2I^{(+)}[a,\beta,z_{adj}^{(+)},n])$$
$$+N_{adj}^{(-)}(I^{(-)}[2a,\beta,z_{adj}^{(-)},n] + 2I^{(-)}[a,\beta,z_{adj}^{(-)},n])$$
$$+N_{fnd}^{(+)}I^{(+)}[a,\beta,z_{fnd}^{(+)},n] + N_{fnd}^{(-)}I^{(-)}[a,\beta,z_{fnd}^{(-)},n])\}$$

$$I^{(+)}[a,\beta,z,n] \equiv \frac{1}{n^5}\left(1-\left(1+2\pi zn+\frac{(2\pi zn)^2}{3}\right)e^{-2\pi zn}\cos(2\pi n\beta)\right)\cos(\pi na)$$

$$I^{(-)}[a,\beta,z,n] \equiv \frac{1}{n^5}\left(1-\left(1+2\pi zn+\frac{(2\pi zn)^2}{3}\right)e^{-2\pi zn}\cos(2\pi n\beta)\right)\cos(\pi n(a-1))$$

$N_{adj.}^{(+)}$	$N_{adj.}^{(-)}$	$N_{fnd.}^{(+)}$	$N_{fnd.}^{(-)}$	β	$z_{adj.}^{(+)}$	$z_{adj.}^{(-)}$	$z_{fnd.}^{(+)}$	$z_{fnd.}^{(-)}$	a_0	m_H/g_4^2
2	1	0	2	0.1	0	0	-	0	0.2362	42
2	1	0	2	0.1	0.1	0.1	-	1	0.0097	150

(GeV)

similar effect of large β

$SU(3) \times SU(3)$ model

introducing soft scalar mass, m (z=mR)

	$N_{adj.}^{(+)}$	$N_{adj.}^{(-)}$	$N_{fnd.}^{(+)}$	$N_{fnd.}^{(-)}$	β	$z_{adj.}^{(+)}$	$z_{adj.}^{(-)}$	$z_{fnd.}^{(+)}$	$z_{fnd.}^{(-)}$	a_0	m_H/g_4^2
(1)	2	3	0	4	0.05	0.01	0.01	-	0.045	0.0040	164
(2)	2	4	2	6	0.05	0	0	0.05	0.05	0.0037	176
(3)	2	4	0	6	0.025	0.025	0.025	-	0.025	0.0066	129
(4)	2	1	0	2	0.1	0.1	0.1	-	1	0.0097	150
(5)	1	1	0	2	0.01	1	1	-	1	0.0196	125
(6)	2	2	0	2	0.14	0	0	-	0	0.0379	130

$SU(6)$ model

	$N_{adj.}^{(+)}$	$N_{adj.}^{(-)}$	$N_{fnd.}^{(+)}$	$N_{fnd.}^{(-)}$	β	$z_{adj.}^{(+)}$	$z_{adj.}^{(-)}$	$z_{fnd.}^{(+)}$	$z_{fnd.}^{(-)}$	a_0	m_H/g_4^2
(7)	2	0	0	10	0.1	0.05	-	-	0.05	0.0207	139
(8)	2	0	0	6	0.15	0.1	-	-	0.1	0.0268	139
(9)	2	0	0	16	0.04	0	-	-	0.03	0.0021	173
(10)	2	0	0	4	0.07	0.5	-	-	0.5	0.0366	138
(11)	2	0	0	2	0.32	0	-	-	0	0.0594	135

$O(1)$ # bulk fields are OK for DSB

(NH, K.Takenaga and T.Yamashita, hep-ph/0411250)

5-3. 3-point self coupling

☆ higher order operators

$$V \sim \cos a \sim a^n = (g_4 R H)^n$$

$g_4 R \sim$ a few TeV → suppression scale → suppressed enough

☆ effective 3-point coupling

$$\lambda \equiv \frac{3g_4^2}{32\pi^6 R} \frac{\partial^3 \widehat{V}}{\partial a^3}\bigg|_{a=a_0}$$

deviation from SM $\triangle\lambda = \dfrac{\lambda - \lambda_{SM}}{\lambda_{SM}}, \quad \lambda_{SM} = \dfrac{3m_h^2}{v}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\Delta\lambda(\%)$	−8.6	−8.3	−14.0	−10.2	−3.1	−13.7	−12.0	−12.0	−7.6	−11.2	−12.7

tend to be small comparing to SM

6. summary and discussion

summary

origin of Higgs doublets & Yukawa int.

- $H_D \subset A_5$ $\rightarrow$ doublet Higgs
- $\bar{\psi}^c_{5D} A_5 \psi_{5D}$ $\rightarrow$ Yukawa ints.

Higgs mass is finite! (5D gauge invariance)

1 loop effective potential of Higgs doublets (A_5)

in $SU(3)\times SU(3)$ model & $SU(6)$ GUT (q/l: blane & bulk)

↓

EW DSB can be possible by extra bulk matters

(suitable # & rep.)

☆ $\tan\beta = 1$

☆ $h \sim A \sim 100GeV,\ H \sim H^{\pm} \sim M_{W,Z} + 100GeV$

(radiative induced mass～100GeV)

☆ gauginos mass~higgsinos mass～β/R

problems

(1): Winberg angle

← ● small brane-localized Higgs kinetic term

bulk gauge coupling > brane-localized g.c.

$(g_4 \sim O(1),\ (M_* R)^{1/2} \gg 1\ (M_* \gg 1/R))$

● additional $U(1)$'

(2): proton decay suppression for TeV scale compactification

← $U(1)_B$

(3): general soft masses in SUSY etc

← how to calculate deviation from $\tan\beta=1$ (D-flat) ?

related studies

☆ 5D E_6, E_7 GUTs on S^1/Z_2

E_6: bulk matters ⇒ **adjoint & fund.**

E_7: bulk matters ⇒ **adjoint**

fermion mass hierarchy & flavor mixings← wall-localized extra fields effects

(NH and Y. Shimizu, Phys.Rev.D67:095001,2003, Erratum-ibid.D69:059902,2004)

☆ SUSY br.   gaugino ⇔ higgsino

$$M_{\lambda} = \tilde{m}, \quad \mu = -\tilde{m}, \quad \tilde{m}^2_{h_u h_d} = -\tilde{m}^2$$ at tree level at 1/R

radiative br. is possible?
investigate by including gravity effects

another approach of EW symmetry breaking in gauge-Higgs unification:

(Choi, N.H., Jeong, Okumura, Shimizu, Yamaguchi, JHEP 0402:037,2004)