素粒子物理学 後半 レポート問題 No.1

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1. Chirality 固有状態は，0でない質量を持てないことを示せ．

$\gamma^5\psi=\psi$, もしくは$\gamma^5\psi=-\psi$ のとき Dirac 方程式 $(i\gamma^\mu\partial_\mu-m)\psi=0$ から$m=0$ を導け．

2. C(荷電共役)変換において Left-handed chirality は，Right-handed chirality となることを確かめよ．

$$\gamma^5\psi_L=-\psi_L \xrightarrow{C} \gamma^5(\psi_L)^C=(\psi_L)^C$$

$$\text{where } \psi^C=C\bar{\psi}^T=i\gamma^2\psi^*$$

3. 反ニュートリノの Dirac spinor は，2成分spinor をχとして

$$v_{\bar{\nu}}\propto\begin{pmatrix}\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi\\ \chi\end{pmatrix}$$

と書ける．今$m=0$とし，運動方向を$+z$方向とすると$(1-\gamma^5)v_{\bar{\nu}}$では反ニュートリノのスピンが$+z$方向，すなわちhelicity $(+)$のみが許されることを示せ．(反粒子の 2 成分spinor は粒子の2成分spinorと逆になることに注意せよ．)

4. μ粒子の崩壊($\mu^-(p)\to e^-(p')+\bar{\nu}_e(k')+\nu_\mu(k)$)における行列要素

$$\sum_{\text{spin}}|\mathcal{M}|^2=\frac{G_F^2}{2}\text{Tr}[\gamma^\mu(1-\gamma^5)(\not{p}+m)\gamma^\nu(1-\gamma^5)\not{k}]\times\text{Tr}[\,\gamma_\mu(1-\gamma^5)\not{k}'\gamma_\nu(1-\gamma^5)\not{p}'\,]$$

を計算すると

$$\sum_{\text{spin}}|\mathcal{M}|^2=\frac{G_F^2}{2}\times 256(k\cdot p')(k'\cdot p)$$

となることを示せ．(以下の公式を使う)

$$\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=4[g^{\mu\nu}g^{\rho\sigma}+g^{\mu\sigma}g^{\nu\rho}-g^{\mu\rho}g^{\nu\sigma}]$$

$$\text{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=4i\,\varepsilon^{\mu\nu\rho\sigma}$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu}{}^{\gamma\delta}=-2(g^{\rho\gamma}g^{\sigma\delta}-g^{\rho\delta}g^{\sigma\gamma})$$

1. Prove that a particle in the chirality eigenstate can not have non-zero mass. i.e. if you assume $\gamma^5\psi = \psi$, or $\gamma^5\psi = -\psi$, derivate $m = 0$ from the Dirac equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$.

2. Prove that a Dirac spinor projected onto its left-handed chirality component changes its chirality to right-handed under a charge conjugatation (C) transformation.

$$\gamma^5\psi_L = -\psi_L \xrightarrow{C} \gamma^5(\psi_L)^C = (\psi_L)^C$$
$$\text{where } \psi^C = C\bar{\psi}^T = i\gamma^2\psi^*$$

3. A Dirac spinor of an anti-neutrino can be described as below using a two-component spinor χ.

$$v_{\bar{\nu}} \propto \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi \\ \chi \end{pmatrix}$$

Suppose a particle with $m = 0$ and moving along $+z$ direction, then prove that the spin of the anti-neutrino is pointing to $+z$ direction, i.e. the helicity of the anti-neutrino is positive for $(1 - \gamma^5)v_{\bar{\nu}}$. (Mind that a two-component spinor of an anti-particle is pointing oppositely to one of a particle.)

4. Calculate the following matrix element of a muon decay $(\mu^-(p) \to e^-(p') + \bar{\nu}_e(k') + \nu_\mu(k))$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{G_F^2}{2}\mathrm{Tr}[\gamma^\mu(1-\gamma^5)(\not{p}+m)\gamma^\nu(1-\gamma^5)\not{k}] \times \mathrm{Tr}[\gamma_\mu(1-\gamma^5)\not{k}'\gamma_\nu(1-\gamma^5)\not{p}']$$

to be

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{G_F^2}{2} \times 256(k\cdot p')(k'\cdot p)$$

using the following formulas

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4[g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}]$$
$$\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4i\,\varepsilon^{\mu\nu\rho\sigma}$$
$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu}{}^{\gamma\delta} = -2(g^{\rho\gamma}g^{\sigma\delta} - g^{\rho\delta}g^{\sigma\gamma})$$