

1. 次の等式を確かめよ.

$$\frac{d^3p}{2E} = d^4p \delta(p^2 - m^2) \theta(E) \quad \text{where } p = (E; \vec{p}), E = \sqrt{m^2 + \vec{p}^2}$$

また,

$$\frac{d^3p}{2E} = \frac{|p|}{2} dE d\Omega$$

とも書けることを示せ.

hint:

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|} \quad \text{where } f(x_i) = 0$$

を使えば

$$\delta(m^2 - p^2) \theta(E) = \frac{\delta(E - \sqrt{m^2 + \vec{p}^2})}{2E}$$

となることを示し, dE の積分を実行せよ.

2. 静止した特定の方向を持たない粒子の3体崩壊におけるdLIPSは,

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{4(2\pi)^3} dE_1 dE_2$$

と書けることを示せ.

hint:

前問の等式を使えば3体崩壊のdLIPSは,

$$\begin{aligned} d\Phi_3(m; p_1, p_2, p_3) &= (2\pi)^4 \delta^4(p - p_1 - p_2 - p_3) \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3} \\ &= \frac{1}{(2\pi)^5} \delta^4(p - p_1 - p_2 - p_3) \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} d^4p_3 \theta(E_3) \delta(p_3^2 - m_3^2) \end{aligned}$$

ここで, d^4p_3 の積分を実行すると更に

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{(2\pi)^5} \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \theta(m - E_1 - E_2) \delta((p - p_1 - p_2)^2 - m_3^2)$$

と書ける. 更に前問により,

$$\frac{d^3p_1}{2E_1} = \frac{p_1}{2} dE_1 d\Omega_1 \quad \frac{d^3p_2}{2E_2} = \frac{p_2}{2} dE_2 d(\cos\theta_{12}) d\varphi_2 \quad \text{但し } \theta_{12} \text{ は, } \vec{p}_1 \text{ と } \vec{p}_2 \text{ のなす角}$$

親粒子が特定の方向を持たないとして, $d\Omega_1, d(\cos\theta_{12}), d\varphi_2$ の積分を実行せよ.

3. μ 粒子の崩壊幅

$$\Gamma = \frac{G_F^2 m^5}{192\pi^3}$$

から寿命を計算せよ. $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$, $m = 0.1057 \text{ GeV}$

1. Prove the following equations:

$$\frac{d^3p}{2E} = d^4p \delta(p^2 - m^2) \theta(E) \quad \text{where } p = (E; \vec{p}), E = \sqrt{m^2 + \vec{p}^2}$$

and also

$$\frac{d^3p}{2E} = \frac{|p|}{2} dE d\Omega$$

hint: by using

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|} \quad \text{where } f(x_i) = 0$$

show

$$\delta(m^2 - p^2) \theta(E) = \frac{\delta(E - \sqrt{m^2 + \vec{p}^2})}{2E}$$

and perform the integral on dE .

2. Prove that the dLISP for a three-body decay of a particle with no special direction can be described as follows:

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{4(2\pi)^3} dE_1 dE_2$$

hint: Using the equation given in the previous question, the dLIPS for a three-body decay is to be

$$\begin{aligned} d\Phi_3(m; p_1, p_2, p_3) &= (2\pi)^4 \delta^4(p - p_1 - p_2 - p_3) \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3} \\ &= \frac{1}{(2\pi)^5} \delta^4(p - p_1 - p_2 - p_3) \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} d^4p_3 \theta(E_3) \delta(p_3^2 - m_3^2) \end{aligned}$$

Then the integral on d^4p_3 yields

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{(2\pi)^5} \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \theta(m - E_1 - E_2) \delta((p - p_1 - p_2)^2 - m_3^2)$$

Again, use the following equations which are given in the previous question.

$$\frac{d^3p_1}{2E_1} = \frac{p_1}{2} dE_1 d\Omega_1 \quad \frac{d^3p_2}{2E_2} = \frac{p_2}{2} dE_2 d(\cos\theta_{12}) d\varphi_2$$

where θ_{12} is the angle between $\vec{p}_1$ and $\vec{p}_2$

and perform the integral on $d\Omega_1$, $d(\cos\theta_{12})$, $d\varphi_2$ on the assumption of a particle with no special direction.

3. Obtain the muon lifetime using the muon decay width:

$$\Gamma = \frac{G_F^2 m^5}{192\pi^3} \quad G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}, \quad m = 0.1057 \text{ GeV}$$