素粒子物理学 後半 レポート問題 No.4

Jul 14, 2016 Y.Takeuchi

1. Cabibbo-Kobayashi-Maskawa (CKM) 行列

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta_{13}} \\ & 1 & \\ -s_{13}e^{i\delta_{13}} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12} & c_{12} & \\ -c_{12}s_{13}e^{i\delta_{13}} & -s_{12}s_{13}e^{i\delta_{13}} & c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & c_{23}\,c_{13} \end{pmatrix}$$

$\theta_{12} = 13.04 \pm 0.05°$ $\quad \theta_{23} = 2.38 \pm 0.06°$ $\quad \theta_{13} = 0.201 \pm 0.011°$ $\quad \delta_{13} = 1.20 \pm 0.08\,\mathrm{rad}$

に対して次の様に定義する．

$$\lambda \equiv s_{12} \qquad A\lambda^2 \equiv s_{23} \qquad A\lambda^3(\rho - i\eta) \equiv s_{13}e^{-i\delta_{13}}$$

$$\lambda \sim 0.226 \qquad A \sim 0.81 \qquad \rho \sim 0.14 \qquad \eta \sim 0.35$$

CKM行列は，次の様に Wolfenstein 表記で書けることを確かめよ．

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\,\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1 - \lambda^2/2 & A\,\lambda^2 \\ A\,\lambda^3\,(1 - \rho - i\,\eta) & -A\,\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

2. ユニタリティ三角形

CKM行列Vのユニタリティ条件 $V^\dagger V = 1$ から

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$

が言えることを示せ．また，上のWolfenstein表記(λ^3まで)を使ってこの等式を確かめよ．

この条件は，

$$1 + \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} + \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}} = 0$$

と書け，次の 3 つ複素数(β, α, γ)が複素平面上に三角形を作ることを意味する．

$$(\beta, \alpha, \gamma) \equiv \left(1, 1 + \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}}, 1 + \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} + \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}}\right)$$

Wolfenstein表記を表記を使い，(β, α, γ)を複素平面上に図示せよ．この三角形は，ユニタリティ三角形と呼ばれる．

1. Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta_{13}} \\ & 1 & \\ -s_{13}e^{i\delta_{13}} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12} & c_{12} & \\ -c_{12}s_{13}e^{i\delta_{13}} & -s_{12}s_{13}e^{i\delta_{13}} & c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & c_{23}\,c_{13} \end{pmatrix}$$

$\theta_{12} = 13.04 \pm 0.05°$ $\quad\theta_{23} = 2.38 \pm 0.06°$ $\quad\theta_{13} = 0.201 \pm 0.011°$ $\quad\delta_{13} = 1.20 \pm 0.08\,\mathrm{rad}$

For the Cabibbo-Kobayashi-Maskawa (CKM) matrix, if we define variables as follows:

$$\lambda \equiv s_{12} \qquad A\lambda^2 \equiv s_{23} \qquad A\lambda^3(\rho - i\eta) \equiv s_{13}e^{-i\delta_{13}}$$

$$\lambda \sim 0.226 \qquad A \sim 0.81 \qquad \rho \sim 0.14 \qquad \eta \sim 0.35\ ,$$

confirm the CKM matrix can be described as the following Wolfenstein representation:

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\,\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1 - \lambda^2/2 & A\,\lambda^2 \\ A\,\lambda^3\,(1 - \rho - i\,\eta) & -A\,\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

2. Unitarity triangle

Using the unitarity condition of the CKM matrix, show the following identity:

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$

And also prove this identity using the Wolfenstein representation (evaluating up to λ^3) of the CKM matrix.

This identity can be written as

$$1 + \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} + \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}} = 0$$

and this means the following three complex numbers (β, α, γ) form a triangle.

$$(\beta, \alpha, \gamma) \equiv \left(1, 1 + \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}}, 1 + \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} + \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}}\right)$$

Using the Wolfenstein representation, illustrait $(\beta,\ \alpha,\ \gamma)$ on the complex plane. This triangle is called 'Unitarity triangle'.