素粒子物理学 後半 レポート問題 No.5

Jul 20, 2016 Y.Takeuchi

1. 中性K中間子二粒子系$K(t)$の時間発展の方程式を

$$i\frac{d}{dt}K(t)=HK(t)=\left(M-\frac{i}{2}\Gamma\right)K(t)$$

$$\text{where } K(t)=\begin{pmatrix}|K^0\rangle\\|\bar{K}^0\rangle\end{pmatrix}\qquad M=\begin{pmatrix}M_{11}&M_{12}\\M_{12}^*&M_{22}\end{pmatrix}\qquad \Gamma=\begin{pmatrix}\Gamma_{11}&\Gamma_{12}\\\Gamma_{12}^*&\Gamma_{22}\end{pmatrix}$$

と書く．二粒子系の状態をCPの固有状態のbase

$$K(t)=\begin{pmatrix}|K_1\rangle\\|K_2\rangle\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{2}}(|K^0\rangle+|\bar{K}^0\rangle)\\\frac{1}{\sqrt{2}}(|K^0\rangle-|\bar{K}^0\rangle)\end{pmatrix}$$

で書き直したとき，以下のように書けることを示せ.

$$H=M-\frac{i}{2}\Gamma=\begin{pmatrix}M_1&im'+\delta'\\-im'+\delta'&M_2\end{pmatrix}-\frac{i}{2}\begin{pmatrix}\Gamma_1&i\gamma'+\lambda'\\-i\gamma'+\lambda'&\Gamma_2\end{pmatrix}$$

where $\quad M_1=\dfrac{M_{11}+M_{22}}{2}+\mathrm{Re}\,M_{12}\qquad M_2=\dfrac{M_{11}+M_{22}}{2}-\mathrm{Re}\,M_{12}$

$$\Gamma_1=\frac{\Gamma_{11}+\Gamma_{22}}{2}+\mathrm{Re}\,\Gamma_{12}\qquad \Gamma_2=\frac{\Gamma_{11}+\Gamma_{22}}{2}-\mathrm{Re}\,\Gamma_{12}$$

$$m'=-\mathrm{Im}\,M_{12}\qquad \delta'=\frac{M_{11}-M_{22}}{2}\qquad \gamma'=-\mathrm{Im}\,\Gamma_{12}\qquad \lambda'=\frac{\Gamma_{11}-\Gamma_{22}}{2}$$

次にCPTを破る項である δ', λ' はないものとして

$$H=M-\frac{i}{2}\Gamma=\begin{pmatrix}M_1&im'\\-im'&M_2\end{pmatrix}-\frac{i}{2}\begin{pmatrix}\Gamma_1&i\gamma'\\-i\gamma'&\Gamma_2\end{pmatrix}$$

を対角化し，中性K中間子系の質量・寿命の固有状態は，

$$\begin{pmatrix}|K_S\rangle\\|K_L\rangle\end{pmatrix}=\frac{1}{\sqrt{1+|\epsilon|^2}}\begin{pmatrix}1&\epsilon\\\epsilon&1\end{pmatrix}\begin{pmatrix}|K_1\rangle\\|K_2\rangle\end{pmatrix}\qquad \text{where } \epsilon=\frac{-i(m'-i\gamma'/2)}{(M_1-M_2)-i(\Gamma_1-\Gamma_2)/2}$$

となることを示せ．(但し m', γ' は M_i, Γ_i に比べて小さいとする．)

1. Let us suppose the time evolution equation of the system of two neutral K mesons as follows:

$$i\frac{d}{dt}K(t) = HK(t) = \left(M - \frac{i}{2}\Gamma\right)K(t)$$

$$\text{where } K(t) = \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix} \qquad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} \qquad \Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}.$$

When we represent the system of two particles in the basis of CP eigen-states as

$$K(t) = \begin{pmatrix} |K_1\rangle \\ |K_2\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \\ \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \end{pmatrix},$$

show the Hamiltonian to be

$$H = M - \frac{i}{2}\Gamma = \begin{pmatrix} M_1 & im' + \delta' \\ -im' + \delta' & M_2 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_1 & i\gamma' + \lambda' \\ -i\gamma' + \lambda' & \Gamma_2 \end{pmatrix},$$

where

$$M_1 = \frac{M_{11} + M_{22}}{2} + \mathrm{Re}\, M_{12}, \qquad M_2 = \frac{M_{11} + M_{22}}{2} - \mathrm{Re}\, M_{12},$$

$$\Gamma_1 = \frac{\Gamma_{11} + \Gamma_{22}}{2} + \mathrm{Re}\, \Gamma_{12}, \qquad \Gamma_2 = \frac{\Gamma_{11} + \Gamma_{22}}{2} - \mathrm{Re}\, \Gamma_{12},$$

$$m' = -\mathrm{Im}\, M_{12}, \qquad \delta' = \frac{M_{11} - M_{22}}{2}, \qquad \gamma' = -\mathrm{Im}\, \Gamma_{12}, \qquad \text{and } \lambda' = \frac{\Gamma_{11} - \Gamma_{22}}{2}.$$

Next, on the assumption that there are no terms of δ' nor λ', which violate the CPT symmetry,

$$H = M - \frac{i}{2}\Gamma = \begin{pmatrix} M_1 & im' \\ -im' & M_2 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_1 & i\gamma' \\ -i\gamma' & \Gamma_2 \end{pmatrix}$$

by diagonalizing the above, show that the two neutral K mesons in terms of the mass and lifetime eigen-state are to be

$$\begin{pmatrix} |K_S\rangle \\ |K_L\rangle \end{pmatrix} = \frac{1}{\sqrt{1 + |\epsilon|^2}}\begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix}\begin{pmatrix} |K_1\rangle \\ |K_2\rangle \end{pmatrix},$$

$$\text{where } \epsilon = \frac{-i(m' - i\gamma'/2)}{(M_1 - M_2) - i(\Gamma_1 - \Gamma_2)/2},$$

here m' and γ' are sufficiently small comparing to M_i or Γ_i.