素粒子物理学授業(後半) 公式集 2016/6/21

γ行列 (Dirac 表示)

$$\gamma^0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \qquad \gamma^i=\begin{pmatrix}0 & \sigma_i\\ -\sigma_i & 0\end{pmatrix} \qquad \gamma^5=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad 1-\gamma^5=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\{\gamma^\mu,\gamma^\nu\}=2\,g^{\mu\nu} \qquad g^{\mu\nu}=\begin{pmatrix}1 & \\ & -\mathbf{1}\end{pmatrix} \qquad g_{\mu\nu}g^{\mu\nu}=4$$

$$\gamma^5\equiv i\,\gamma^0\gamma^1\gamma^2\gamma^3 \qquad (\gamma^5)^2=1 \qquad \{\gamma^5,\gamma^\mu\}=0 \qquad (1\pm\gamma^5)^2=2\,(1\pm\gamma^5)$$

$$\gamma^{0\dagger}=\gamma^0 \qquad \gamma^{i\dagger}=-\gamma^i \qquad \gamma^{5\dagger}=\gamma^5 \qquad \gamma^{\mu\dagger}=\gamma^0\gamma^\mu\gamma^0$$

$$\gamma_\mu\gamma^\mu=4 \qquad \gamma_\mu\not{a}\gamma^\mu=-2\not{a} \qquad \gamma_\mu\not{a}\not{b}\gamma^\mu=4a\cdot b \qquad \gamma_\mu\not{a}\not{b}\not{c}\gamma^\mu=-2\not{c}\not{b}\not{a}$$

$$P_L\equiv\frac{1}{2}(1-\gamma^5) \qquad P_R\equiv\frac{1}{2}(1+\gamma^5) \qquad P_L+P_R=1 \qquad P_L^2=P_L \qquad P_R^2=P_R$$

$$\gamma^5P_L=-P_L \qquad \gamma^5P_R=P_R \qquad \gamma^\mu P_L=P_R\gamma^\mu \qquad \gamma^\mu P_R=P_L\gamma^\mu$$

$$\sigma^{\mu\nu}\equiv\frac{i}{2}[\gamma^\mu,\gamma^\nu]=\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$$

Dirac spinor

$$\text{粒子}\ \ u_s(p)=N\begin{pmatrix}\phi_s\\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\phi_s\end{pmatrix} \qquad \text{反粒子}\ \ v_s(p)=N\begin{pmatrix}\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi_s\\ \chi_s\end{pmatrix} \qquad N=\sqrt{E+m}$$

2成分spinorの3軸方向のスピン固有状態

$$\phi_\uparrow=\begin{pmatrix}1\\ 0\end{pmatrix} \quad \phi_\downarrow=\begin{pmatrix}0\\ 1\end{pmatrix} \qquad \chi_\uparrow=\begin{pmatrix}0\\ 1\end{pmatrix} \quad \chi_\downarrow=\begin{pmatrix}-1\\ 0\end{pmatrix}$$

$$\bar{\psi}\equiv\psi^\dagger\gamma^0 \qquad (\bar{\psi})^\dagger=\gamma^0\psi$$

$$u_s^\dagger u_r=v_s^\dagger v_r=2E\,\delta_{sr} \qquad \bar{u}_su_r=-\bar{v}_sv_r=2m\,\delta_{sr} \qquad \bar{u}_sv_r=0$$

$$\sum_{s=\uparrow\downarrow}u_s(p)\bar{u}_s(p)=\not{p}+m \qquad \sum_{s=\uparrow\downarrow}v_s(p)\bar{v}_s(p)=\not{p}-m$$

$$(\not{p}-m)u=0 \qquad (\not{p}+m)v=0 \qquad \bar{u}\,(\not{p}-m)=0 \qquad \bar{v}\,(\not{p}+m)=0$$

Charge conjugation

$$C=i\gamma^2\gamma^0=\begin{pmatrix}0 & -i\sigma_2\\ -i\sigma_2 & 0\end{pmatrix} \qquad C=C^*=-C^{-1}=-C^T=-C^\dagger$$

$$\psi^C=C\bar{\psi}^T=i\gamma^2\psi^* \qquad \bar{\psi}^C=-\psi^TC^{-1}=-i\bar{\psi}^*\gamma^2$$

Matrix Element 計算

$$|\bar{\psi}_f\,(\Gamma)\,\psi_i|^2=\bar{\psi}_f\,(\Gamma)\,\psi_i\,\bar{\psi}_i\,(\gamma^0\,\Gamma^\dagger\,\gamma^0)\,\psi_f=\mathrm{Tr}[(\Gamma)\,\psi_i\,\bar{\psi}_i\,(\gamma^0\,\Gamma^\dagger\,\gamma^0)\,\psi_f\bar{\psi}_f]$$

$\psi_{f,i}$: Dirac spinor　　Γ : γ行列　　$X=(x_i)$, $Y=(y_i)$ $\Longrightarrow X^TY=\mathrm{Tr}[YX^T]$

$$\gamma^0(1-\gamma^5)^\dagger\gamma^0=1+\gamma^5 \qquad \gamma^0[\gamma^\mu(1-\gamma^5)]^\dagger\gamma^0=\gamma^\mu(1-\gamma^5)$$

γ行列のトレース計算公式

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu]=4g^{\mu\nu} \qquad \mathrm{Tr}[ABC]=\mathrm{Tr}[BCA] \qquad \mathrm{Tr}[S^{-1}AS]=\mathrm{Tr}\,A \qquad \mathrm{Tr}\mathbf{1}=4$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=4[g^{\mu\nu}g^{\rho\sigma}+g^{\mu\sigma}g^{\nu\rho}-g^{\mu\rho}g^{\nu\sigma}]$$

$$\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=4i\,\varepsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}[\gamma^1\gamma^2\cdots\gamma^{2n+1}]=0$$

$$\mathrm{Tr}[\gamma^5]=\mathrm{Tr}[\gamma^5\gamma^\mu]=\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu]=\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho]=0$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu}{}^{\gamma\delta}=-2(g^{\rho\gamma}g^{\sigma\delta}-g^{\rho\delta}g^{\sigma\gamma})$$

$$\gamma^\mu\gamma^\nu\gamma^\rho=g^{\mu\nu}\gamma^\rho-g^{\mu\rho}\gamma^\nu+g^{\nu\rho}\gamma^\mu+i\gamma^5\varepsilon^{\mu\nu\rho\sigma}\gamma_\sigma$$

ローレンツ不変な位相体積 Lorentz Invariant Phase Space (LIPS)

$$d^4P\delta(M^2-P^2)\theta(P^0)=\frac{d^3P}{2E}=\frac{P}{2}dEd\Omega \qquad E=\sqrt{\vec{P}^2+M^2}$$

$$d^3P=P^2dPd\Omega=PEdEd\Omega$$

n体の位相体積（normalization $\int\rho\,dV=2E$）

$$d\Phi_n(P;P_1,\cdots,P_n)=(2\pi)^4\delta^4\left(P-\sum_{i=1}^n P_i\right)\prod_{i=1}^n\frac{d^3P_i}{(2\pi)^3 2E_i}$$

$M(\mathrm{rest})\to m_1+m_2$

$$d\Phi_2(M;P_1,P_2)=\frac{1}{16\pi^2}\frac{|\vec{P}_1|}{M}d\Omega_1$$

$M(\mathrm{rest})\to m_1+m_2+m_3$（始状態の粒子に特別な方向が無い場合）

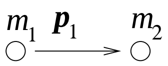
$$d\Phi_3(M;P_1,P_2,P_3)=\frac{1}{4(2\pi)^3}dE_1dE_2=\frac{1}{16(2\pi)^3M^2}dm_{12}^2dm_{23}^2$$

$$\boxed{d\Gamma=\frac{|\mathfrak{M}|^2}{2M}d\Phi_n(P;P_1,\cdots,P_n)}$$

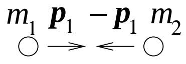
2-bodies decay $\quad d\Gamma=\frac{1}{32\pi^2}|\mathfrak{M}|^2\frac{|\vec{P}_1|}{M^2}d\Omega_1$

3-bodies decay $\quad d\Gamma=\frac{1}{(2\pi)^3}\frac{1}{8M}|\mathfrak{M}|^2dE_1dE_2$

$$\boxed{d\sigma=\frac{|\mathfrak{M}|^2}{4\sqrt{(P_1\cdot P_2)^2-m_1^2m_2^2}}d\Phi_n(P_1+P_2;P_3,\cdots,P_{n+2})}$$



$$\sqrt{(P_1\cdot P_2)^2-m_1^2m_2^2}=m_2P_1$$



$$\sqrt{(P_1\cdot P_2)^2-m_1^2m_2^2}=P_1\sqrt{s} \qquad s\equiv(P_1+P_2)^2$$

$$d\Phi_2(P_1+P_2;P_3,P_4)=\frac{1}{16\pi^2}\frac{|\vec{P}_3|}{\sqrt{s}}d\Omega_3 \qquad \frac{d\sigma}{d\Omega}=\frac{1}{64\pi^2 s}\frac{|\vec{P}_3|}{|\vec{P}_1|}|\mathfrak{M}|^2$$