

1. Chirality 固有状態は, 0でない質量を持ってないことを示せ.

($\gamma^5\psi = \psi$, もしくは $\gamma^5\psi = -\psi$ のとき Dirac 方程式 $(i\gamma^\mu\partial_\mu - m)\psi = 0$ から $m=0$ を導け.)

2. C(荷電共役)変換において Left-handed chirality は, Right-handed chirality となることを確かめよ.

$$\gamma^5\psi_L = -\psi_L \xrightarrow{C} \gamma^5(\psi_L)^C = (\psi_L)^C$$

where $\psi^C = C\bar{\psi}^T = i\gamma^2\psi^*$

3. 反ニュートリノの Dirac spinor は, z 軸スピン固有状態の2成分spinor を χ として

$$v_{\bar{\nu}} \propto \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi \\ \chi \end{pmatrix}$$

と書ける. 今 $m=0$ とし, 運動方向を $+z$ 方向とすると $(1-\gamma^5)v_{\bar{\nu}}$ では反ニュートリノのスピンが $+z$ 方向, すなわちhelicity (+)のみが許されることを示せ.

$$\text{反粒子の2成分spinor は } +z\text{方向スピン } \chi_{\uparrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, -z\text{方向スピン } \chi_{\downarrow} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

4. μ 粒子の崩壊($\mu^-(p) \rightarrow e^-(p') + \bar{\nu}_e(k') + \nu_\mu(k)$)における行列要素

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{G_F^2}{2} \text{Tr}[\gamma^\mu(1-\gamma^5)(\not{p}+m)\gamma^\nu(1-\gamma^5)\not{k}] \times \text{Tr}[\gamma_\mu(1-\gamma^5)\not{k}'\gamma_\nu(1-\gamma^5)\not{p}']$$

を計算すると

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{G_F^2}{2} \times 256(k \cdot p')(k' \cdot p)$$

となることを示せ. (以下の公式を使う)

$$\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4[g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}]$$

$$\text{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4i\varepsilon^{\mu\nu\rho\sigma}$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu}{}^{\gamma\delta} = -2(g^{\rho\gamma}g^{\sigma\delta} - g^{\rho\delta}g^{\sigma\gamma})$$

1. Prove that a particle in the chirality eigenstate can not have non-zero mass. i.e. if you assume $\gamma^5\psi = \psi$, or $\gamma^5\psi = -\psi$, derivate $m=0$ from the Dirac equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$.

2. Prove that a Dirac spinor projected onto its left-handed chirality component changes its chirality to right-handed under a charge conjugation (C) transformation.

$$\gamma^5\psi_L = -\psi_L \xrightarrow{C} \gamma^5(\psi_L)^C = (\psi_L)^C$$

where $\psi^C = C\bar{\psi}^T = i\gamma^2\psi^*$

3. A Dirac spinor of an anti-neutrino can be described as below using a two-component spinor χ .

$$v_{\bar{\nu}} \propto \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi \\ \chi \end{pmatrix}$$

Suppose a particle with $m=0$ and moving along $+z$ direction, then prove that the spin of the anti-neutrino is pointing to $+z$ direction, i.e. the helicity of the anti-neutrino is positive for $(1 - \gamma^5)v_{\bar{\nu}}$. (Mind that a two-component spinor of an anti-particle is pointing oppositely to one of a particle.)

4. Calculate the following matrix element of a muon decay $(\mu^-(p) \rightarrow e^-(p') + \bar{\nu}_e(k') + \nu_\mu(k))$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{G_F^2}{2} \text{Tr}[\gamma^\mu(1 - \gamma^5)(\not{p} + m)\gamma^\nu(1 - \gamma^5)\not{k}] \times \text{Tr}[\gamma_\mu(1 - \gamma^5)\not{k}'\gamma_\nu(1 - \gamma^5)\not{p}']$$

to be

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{G_F^2}{2} \times 256(k \cdot p')(k' \cdot p)$$

using the following formulas

$$\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4[g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}]$$

$$\text{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4i\epsilon^{\mu\nu\rho\sigma}$$

$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu}{}^{\gamma\delta} = -2(g^{\rho\gamma}g^{\sigma\delta} - g^{\rho\delta}g^{\sigma\gamma})$$