素粒子物理学 後半 レポート問題 No.2 2018/7/2 Y. Takeuchi

1. 次の等式を確かめよ．

$$\frac{d^3p}{2E} = d^4p\delta(p^2 - m^2)\theta(E) \qquad \text{where } p = (E; \vec{p}),\ E = \sqrt{m^2 + \vec{p}^{\,2}}$$

また，

$$\frac{d^3p}{2E} = \frac{|p|}{2} dE d\Omega$$

とも書けることを示せ．

hint:

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|} \qquad \text{where } f(x_i) = 0$$

を使えば

$$\delta(m^2 - p^2)\theta(E) = \frac{\delta\left(E - \sqrt{m^2 + \vec{p}^{\,2}}\right)}{2E}$$

となることを示し，dE の積分を実行せよ．

2. 静止した特定の方向を持たない粒子の3体崩壊におけるdLIPSは，

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{4(2\pi)^3} dE_1 dE_2$$

と書けることを示せ．

hint:

前問の等式を使えば3体崩壊のdLIPSは，

$$d\Phi_3(m; p_1, p_2, p_3) = (2\pi)^4\delta^4(p - p_1 - p_2 - p_3)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}$$

$$= \frac{1}{(2\pi)^5}\delta^4(p - p_1 - p_2 - p_3)\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2} d^4p_3\theta(E_3)\delta(p_3^2 - m_3^2)$$

ここで，d^4p_3 の積分を実行すると更に

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{(2\pi)^5}\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\theta(m - E_1 - E_2)\delta((p - p_1 - p_2)^2 - m_3^2)$$

と書ける．更に前問により，

$$\frac{d^3p_1}{2E_1} = \frac{p_1}{2} dE_1 d\Omega_1 \qquad \frac{d^3p_2}{2E_2} = \frac{p_2}{2} dE_2 d(\cos\theta_{12}) d\varphi_2$$

但しθ_{12}は，$\vec{p}_1$と$\vec{p}_2$のなす角

親粒子が特定の方向を持たないとして，$d\Omega_1$, $d(\cos\theta_{12})$, $d\varphi_2$での積分を実行せよ．

3. μ粒子の崩壊幅(自然単位系)

$$\Gamma = \frac{G_F^2 m^5}{192\pi^3} \qquad G_F = 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2},\ m = 0.1057\,\mathrm{GeV}$$

から寿命を計算せよ．

自然単位系におけるエネルギーから時間への変換は，$\hbar = 6.58 \times 10^{-25}\,\mathrm{GeV} \cdot s$ を使う

1. Prove the following equations:

$$\frac{d^3p}{2E} = d^4p\delta(p^2 - m^2)\theta(E) \qquad \text{where } p = (E; \vec{p}\,),\ E = \sqrt{m^2 + \vec{p}^{\,2}}$$

and also

$$\frac{d^3p}{2E} = \frac{|p|}{2} dE d\Omega$$

hint: by using

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|} \qquad \text{where } f(x_i) = 0$$

show

$$\delta(m^2 - p^2)\theta(E) = \frac{\delta\Big(E - \sqrt{m^2 + \vec{p}^{\,2}}\Big)}{2E}$$

and perform the integral on dE.

2. Prove that the dLISP for a three-body decay of a particle with no special direction can be described as follows:

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{4(2\pi)^3} dE_1 dE_2$$

hint: Using the equation given in the previous question, the dLIPS for a three-body decay is to be

$$\begin{aligned} d\Phi_3(m; p_1, p_2, p_3) &= (2\pi)^4 \delta^4(p - p_1 - p_2 - p_3) \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3} \\ &= \frac{1}{(2\pi)^5} \delta^4(p - p_1 - p_2 - p_3) \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} d^4p_3 \theta(E_3) \delta(p_3^2 - m_3^2) \end{aligned}$$

Then the integral on d^4p_3 yields

$$d\Phi_3(m; p_1, p_2, p_3) = \frac{1}{(2\pi)^5} \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \theta(m - E_1 - E_2) \delta((p - p_1 - p_2)^2 - m_3^2)$$

Again, use the following equations which are given in the previous question.

$$\frac{d^3p_1}{2E_1} = \frac{p_1}{2} dE_1 d\Omega_1 \qquad \frac{d^3p_2}{2E_2} = \frac{p_2}{2} dE_2 d(\cos\theta_{12}) d\varphi_2$$

where θ_{12} is the angle between $\vec{p}_1$ and $\vec{p}_2$

and perform the integral on $d\Omega_1$, $d(\cos\theta_{12})$, $d\varphi_2$ on the assumption of a particle with no special direction.

3. Obtain the muon lifetime using the muon decay width:

$$\Gamma = \frac{G_F^2 m^5}{192\pi^3} \qquad G_F = 1.17 \times 10^{-5}\,\text{GeV}^{-2},\ m = 0.1057\,\text{GeV} \qquad \text{(natural unit)}$$

Use the constant $\hbar = 6.58 \times 10^{-25}\,\text{GeV} \cdot s$ to convert the unit from energy to time.