

γ 行列 (Dirac 表示) γ^μ ($\mu=0,1,2,3$) ($i=1,2,3$)

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & 0 \\ 0 & \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad 4 \times 4 \text{行列}$$

$$\text{パウリ行列: } \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad \text{反交換子 } \{A, B\} = AB + BA$$

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} : \text{Minkowski計量}$$

$$\gamma^{0\dagger} = \gamma^0 \quad \gamma^{i\dagger} = -\gamma^i \quad \gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0 \quad \sigma^{\mu\nu} \equiv \frac{i}{2} [\gamma^\mu, \gamma^\nu] = \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$$

Dirac方程式

$$(\gamma^\mu i\partial_\mu - m)\psi(x) = 0 \quad \mathcal{L} = \bar{\psi} (\gamma^\mu i\partial_\mu - m)\psi \quad \bar{\psi} \equiv \psi^\dagger \gamma^0$$

Dirac spinor (4成分スピノール)

$$\text{粒子 } u_s(p) = N \begin{pmatrix} \phi_s \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} \phi_s \end{pmatrix} \quad \text{反粒子 } v_s(p) = N \begin{pmatrix} \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} \chi_s \\ \chi_s \end{pmatrix} \quad N = \sqrt{E+m}$$

$$\phi_s, \chi_s \text{ は2成分spinorの3軸方向のスピン固有状態} \quad \boldsymbol{\sigma} \cdot \mathbf{p} \equiv p_1 \sigma_1 + p_2 \sigma_2 + p_3 \sigma_3$$

$$\phi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \phi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \chi_\uparrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \chi_\downarrow = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$u_s^\dagger u_r = v_s^\dagger v_r = 2E \delta_{sr} \quad \bar{u}_s u_r = -\bar{v}_s v_r = 2m \delta_{sr} \quad \bar{u}_s v_r = 0$$

$$\sum_{s=\uparrow\downarrow} u_s(p) \bar{u}_s(p) = \not{p} + m \quad \sum_{s=\uparrow\downarrow} v_s(p) \bar{v}_s(p) = \not{p} - m$$

$$(\not{p} - m)u_s = 0 \quad (\not{p} + m)v_s = 0 \quad \bar{u}_s(\not{p} - m) = 0 \quad \bar{v}_s(\not{p} + m) = 0$$

$$\not{p} \equiv p_\mu \gamma^\mu : \text{Feynman slash 表記}$$

カイラル演算子

$$\gamma^5 \equiv i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (\gamma^5)^2 = 1 \quad \{\gamma^5, \gamma^\mu\} = 0 \quad \gamma^{5\dagger} = \gamma^5$$

$$(1 \pm \gamma^5)^2 = 2(1 \pm \gamma^5) \quad 1 - \gamma^5 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$P_L \equiv \frac{1}{2}(1 - \gamma^5) \quad P_R \equiv \frac{1}{2}(1 + \gamma^5) \quad P_L + P_R = 1 \quad P_L^2 = P_L \quad P_R^2 = P_R$$

$$\gamma^5 P_L = -P_L \quad \gamma^5 P_R = P_R \quad \gamma^\mu P_L = P_R \gamma^\mu \quad \gamma^\mu P_R = P_L \gamma^\mu$$

$$\psi_L \equiv P_L \psi \quad \bar{\psi}_L = (P_L \psi)^\dagger \gamma^0 = \psi^\dagger P_L \gamma^0 = \psi^\dagger \gamma^0 P_R = \bar{\psi} P_R$$

Charge conjugation 荷電共役

$$C \equiv i\gamma^2 \gamma^0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \quad C = C^* = -C^{-1} = -C^T = -C^\dagger$$

$$\psi^C \equiv C \bar{\psi}^T = i\gamma^2 \psi^* \quad \bar{\psi}^C = -\psi^T C^{-1} = -i\bar{\psi}^* \gamma^2$$

Matrix Element 計算 任意の列行列 $X = (x_i)$, $Y = (y_i) \implies X^T Y = \text{Tr}[Y X^T]$

$$|\bar{\psi}_f(\Gamma) \psi_i|^2 = \bar{\psi}_f(\Gamma) \psi_i \bar{\psi}_i (\gamma^0 \Gamma^\dagger \gamma^0) \psi_f = \text{Tr}[(\Gamma) \psi_i \bar{\psi}_i (\gamma^0 \Gamma^\dagger \gamma^0) \psi_f \bar{\psi}_f]$$

$\psi_{f,i}$: Dirac spinor (4成分) Γ : γ 行列 (4×4行列)

$$\gamma^0(1 - \gamma^5)^\dagger \gamma^0 = 1 + \gamma^5 \quad \gamma^0[\gamma^\mu(1 - \gamma^5)]^\dagger \gamma^0 = \gamma^\mu(1 - \gamma^5)$$

γ 行列のトレース計算公式

$$\gamma_\mu \gamma^\mu = 4 \quad \gamma_\mu \not{a} \gamma^\mu = -2 \not{a} \quad \gamma_\mu \not{a} \not{b} \gamma^\mu = 4a \cdot b \quad \gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu = -2 \not{c} \not{b} \not{a}$$

$$\text{Tr}[\gamma^\mu \gamma^\nu] = 4g^{\mu\nu} \quad \text{Tr}[A B C] = \text{Tr}[B C A] \quad \text{Tr}[S^{-1} A S] = \text{Tr} A \quad \text{Tr} \mathbf{1} = 4$$

$$\text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4[g^{\mu\nu} g^{\rho\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\mu\rho} g^{\nu\sigma}]$$

$$\text{Tr}[\gamma^5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4i \varepsilon^{\mu\nu\rho\sigma} \quad (\varepsilon^{\mu\nu\rho\sigma}: \text{レヴイチヴイタの反対称テンソル})$$

$$\text{Tr}[\gamma^1 \gamma^2 \dots \gamma^{2n+1}] = 0 \quad (\text{任意の奇数個の}\gamma\text{行列の積のトレースは0})$$

$$\text{Tr}[\gamma^5] = \text{Tr}[\gamma^5 \gamma^\mu] = \text{Tr}[\gamma^5 \gamma^\mu \gamma^\nu] = \text{Tr}[\gamma^5 \gamma^\mu \gamma^\nu \gamma^\rho] = 0$$

$$\varepsilon^{\mu\nu\rho\sigma} \varepsilon_{\mu\nu} \gamma^\delta = -2(g^{\rho\gamma} g^{\sigma\delta} - g^{\rho\delta} g^{\sigma\gamma})$$

$$\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu} \gamma^\rho - g^{\mu\rho} \gamma^\nu + g^{\nu\rho} \gamma^\mu + i \gamma^5 \varepsilon^{\mu\nu\rho\sigma} \gamma_\sigma$$

ローレンツ不変な位相体積 Lorentz Invariant Phase Space (LIPS)

$$d^4 P \delta(M^2 - P^2) \theta(P^0) = \frac{d^3 P}{2E} = \frac{P}{2} dE d\Omega \quad E = \sqrt{\vec{P}^2 + M^2}$$

$$d^3 P = P^2 dP d\Omega = P E dE d\Omega$$

$$d\Phi_n(P; P_1, \dots, P_n) = (2\pi)^4 \delta^4\left(P - \sum_{i=1}^n P_i\right) \prod_{i=1}^n \frac{d^3 P_i}{(2\pi)^3 2E_i}$$

$$n\text{体の位相体積 (normalization } \int \rho dV = 2E)$$

$M(\text{rest}) \rightarrow m_1 + m_2$: 静止系での二体崩壊

$$d\Phi_2(M; P_1, P_2) = \frac{1}{16\pi^2} \frac{|\vec{P}_1|}{M} d\Omega_1$$

$M(\text{rest}) \rightarrow m_1 + m_2 + m_3$: 静止系での三体崩壊(親粒子に特別な方向が無い場合)

$$d\Phi_3(M; P_1, P_2, P_3) = \frac{1}{4(2\pi)^3} dE_1 dE_2 = \frac{1}{16(2\pi)^3 M^2} dm_{12}^2 dm_{23}^2$$

$$\text{微分崩壊幅: } d\Gamma = \frac{|\mathfrak{M}|^2}{2M} d\Phi_n(P; P_1, \dots, P_n)$$

$$2\text{-body decay} \quad d\Gamma = \frac{1}{32\pi^2} |\mathfrak{M}|^2 \frac{|\vec{P}_1|}{M^2} d\Omega_1$$

$$3\text{-body decay} \quad d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8M} |\mathfrak{M}|^2 dE_1 dE_2$$

$$\text{微分断面積: } d\sigma = \frac{|\mathfrak{M}|^2}{4\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2}} d\Phi_n(P_1 + P_2; P_3, \dots, P_{n+2})$$

$$\begin{array}{c} m_1 \quad \mathbf{p}_1 \\ \bigcirc \end{array} \rightarrow \begin{array}{c} m_2 \\ \bigcirc \end{array} \quad \sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2} = m_2 P_1$$

$$\begin{array}{c} m_1 \quad \mathbf{p}_1 \quad - \mathbf{p}_1 \quad m_2 \\ \bigcirc \rightarrow \leftarrow \bigcirc \end{array} \quad \sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2} = P_1 \sqrt{s} \quad s \equiv (P_1 + P_2)^2$$

$$d\Phi_2(P_1 + P_2; P_3, P_4) = \frac{1}{16\pi^2} \frac{|\vec{P}_3|}{\sqrt{s}} d\Omega_3 \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{P}_3|}{|\vec{P}_1|} |\mathfrak{M}|^2$$