Weinberg-Salam理論

Fermion (Dirac場)

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad j,k = 1 \sim 3$$

	T	T^3	Y	$Q = T^3 + \frac{Y}{2}$
ν_L	$1/2$	$1/2$	-1	0
ℓ_L		$-1/2$		-1
ν_R	0	0	0	0
ℓ_R	0	0	-2	-1

	T	T^3	Y	Q
u_L	$1/2$	$1/2$	$1/3$	$2/3$
d_L		$-1/2$		$-1/3$
u_R	0	0	$4/3$	$2/3$
d_R	0	0	$-2/3$	$-1/3$

Scalar場

$$\Phi(x) \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = e^{i\frac{\tau^l}{2}\chi^l(x)} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \phi(x) \end{pmatrix}$$

	T	T^3	Y	Q
ϕ^+	$1/2$	$1/2$	1	1
ϕ^0		$-1/2$		0

$$\tilde{\Phi}(x) \equiv i\tau^2 \Phi^*(x)$$

$$\left[\frac{\tau^l}{2}, \frac{\tau^m}{2}\right] = i\epsilon_{lmn}\frac{\tau^n}{2} \qquad lmn = 1 \sim 3$$

$$\mathcal{L} = \mathcal{L}_{\texttt{gauge}} + \mathcal{L}_{\texttt{Higgs}} + \mathcal{L}^{\texttt{kin}}_{\texttt{lepton}} + \mathcal{L}^{\texttt{mass}}_{\texttt{lepton}} + \mathcal{L}^{\texttt{kin}}_{\texttt{quark}} + \mathcal{L}^{\texttt{mass}}_{\texttt{quark}}$$

$$\mathcal{L}_{\texttt{gauge}} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu W^l_\nu - \partial_\nu W^l_\mu - g_2 \epsilon_{lmn} W^m_\mu W^n_\nu)^2$$

$$\mathcal{L}_{\texttt{Higgs}} = \left| \left(i\partial_\mu - g_1 \frac{1}{2} Y_H B_\mu - g_2 \frac{\tau^l}{2} W^l_\mu \right) \Phi \right|^2 - V(\Phi) \qquad V(\Phi) \equiv -\mu^2 \Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2$$

$$\mathcal{L}^{\texttt{kin}}_{\texttt{lepton}} = \bar{L}^j_L \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{\ell L}}{2} B_\mu - g_2 \frac{\tau^l}{2} W^l_\mu \right] L^j_L + \bar{\ell}^j_R \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{\ell R}}{2} B_\mu \right] \ell^j_R + \bar{\nu}^j_R \gamma^\mu i\partial_\mu \nu^j_R$$

$$\mathcal{L}^{\texttt{mass}}_{\texttt{lepton}} = -y^j_\ell \big(\bar{L}^j_L \Phi\big) \ell^j_R - y^j_\nu \big[(\bar{L}_L U_\nu)^j \tilde{\Phi} \big] \big(U^\dagger_\nu \nu_R\big)^j + h.c. \qquad U_\nu\text{: PMNS matrix}$$

$$\begin{aligned} \mathcal{L}^{\texttt{kin}}_{\texttt{quark}} = \ & \bar{Q}^j_L \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{QL}}{2} B_\mu - g_2 \frac{\tau^l}{2} W^l_\mu \right] Q^j_L \\ & + \bar{u}^j_R \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{uR}}{2} B_\mu \right] u^j_R + \bar{d}^j_R \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{dR}}{2} B_\mu \right] d^j_R \end{aligned}$$

$$\mathcal{L}^{\texttt{mass}}_{\texttt{quark}} = -y^j_d \big[\big(\bar{Q}^j_L U_d\big)^j \Phi \big] \big(U^\dagger_d d_R\big)^j - y^j_u \big(\bar{Q}^j_L \tilde{\Phi}\big) u^j_R + h.c. \qquad U_d\text{: CKM matrix}$$

Electro-Weak Symmetry Breaking (EWSB)

$$\Phi(x)\to\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\phi(x)\end{pmatrix}\qquad\tilde{\Phi}(x)\to\frac{1}{\sqrt{2}}\begin{pmatrix}v+\phi(x)\\ 0\end{pmatrix}\qquad\frac{v}{\sqrt{2}}=\frac{\mu}{\sqrt{\lambda}}$$

$$\begin{aligned}\mathcal{L}_{\texttt{gauge}}= &-\frac{1}{4}F^{A}_{\mu\nu}F^{A\mu\nu}-\frac{1}{4}F^{Z}_{\mu\nu}F^{Z\mu\nu}-\frac{1}{2}(\mathcal{D}_\mu W_\nu-\mathcal{D}_\nu W_\mu)^\dagger(\mathcal{D}^\mu W^\nu-\mathcal{D}^\nu W^\mu)\\ &+i(e\,F^{A}_{\mu\nu}+g_2\cos\theta_W F^{Z}_{\mu\nu})W^{\dagger\mu}W^\nu+\frac{g_2^2}{2}(|W_\mu W^\mu|^2-|W_\mu W^\nu|^2)\end{aligned}$$

$$W_\mu\equiv\frac{1}{\sqrt{2}}(W^1_\mu-i\,W^2_\mu)\qquad\cos\theta_W\equiv\frac{g_2}{\sqrt{g_1^2+g_2^2}}\qquad\sin\theta_W\equiv\frac{g_1}{\sqrt{g_1^2+g_2^2}}$$

$$\begin{pmatrix}A_\mu\\ Z_\mu\end{pmatrix}\equiv\begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}B_\mu\\ W^3_\mu\end{pmatrix}$$

$$F^{A}_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu\qquad F^{Z}_{\mu\nu}\equiv\partial_\mu Z_\nu-\partial_\nu Z_\mu$$

$$e\equiv\frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}=g_2\sin\theta_W=g_1\cos\theta_W\qquad\sin^2\theta_W\approx 0.23$$

$$\mathcal{D}_\mu W_\nu\equiv(\partial_\mu+i\,g_2W^3_\mu)W_\nu=(\partial_\mu+i\,e\,A_\mu+i\,g_2\cos\theta_W Z_\mu)W_\nu$$

$$\begin{aligned}\mathcal{L}_{\texttt{Higgs}}= &\;M_W^2W^\dagger_\mu W^\mu+\frac{1}{2}M_Z^2Z_\mu^2+\left(g\,M_W\phi+\frac{g^2}{4}\phi^2\right)\left(W^\dagger_\mu W^\mu+\frac{1}{2\cos^2\theta_W}Z_\mu^2\right)\\ &+\frac{1}{2}[(\partial_\mu\phi)^2-m^2\phi^2]-\frac{m\sqrt{\lambda}}{2}\phi^3-\frac{\lambda}{8}\phi^4-V\left(\frac{v}{\sqrt{2}}\right)\end{aligned}$$

$$M_W\equiv\frac{g\,v}{2}\qquad M_Z\equiv\frac{1}{2}\sqrt{g^2+g'^2}\,v=\frac{M_W}{\cos\theta_W}\qquad m\equiv\sqrt{2}\mu$$

$$\mathcal{L}_{\texttt{lepton}}=\bar{\ell}^{\,j}i\partial\!\!\!/\ell^j+\bar{\nu}^{\,j}i\partial\!\!\!/\nu^j-\left(m^j_\ell+\frac{y^j_\ell}{\sqrt{2}}\phi\right)\bar{\ell}^{\,j}\ell^j-\left(m^j_\nu+\frac{y^j_\nu}{\sqrt{2}}\phi\right)\bar{\nu}^{\,\prime j}\nu^{\prime j}+\mathcal{L}^{(\text{lepton})}_{\text{EW int}}$$

$$\mathcal{L}_{\texttt{quark}}=\bar{u}^{\,j}i\partial\!\!\!/u^j+\bar{d}^{\,j}i\partial\!\!\!/d^j-\left(m^j_d+\frac{y^j_d}{\sqrt{2}}\phi\right)\bar{d}^{\,\prime j}d^{\prime j}-\left(m^j_u+\frac{y^j_u}{\sqrt{2}}\phi\right)\bar{u}^{\,j}u^j+\mathcal{L}^{(\text{quark})}_{\text{EW int}}$$

$$m^j_f\equiv\frac{y^j_f v}{\sqrt{2}}\qquad y^j_f=\sqrt{2}\,\frac{m^j_f}{v}=g_2\frac{m^j_f}{\sqrt{2}M_W}\qquad\texttt{where}\quad f=\ell,\nu,u,d$$

$$\nu^j\equiv(U_\nu)_{jk}\nu'^k\qquad d^j\equiv(U_d)_{jk}d'^k$$

$$\mathcal{L}_{\text{EW int}}=-\frac{g}{\sqrt{2}}\left(J^{\mu\dagger}W_\mu+J^\mu W^\dagger_\mu\right)-e\,J^\mu_{\text{EM}}A_\mu-\frac{g}{\cos\theta_W}J^\mu_Z Z_\mu$$

$$J^\mu\equiv\bar{\ell}^{\,j}_L\gamma^\mu\nu^j_L+\bar{d}^{\,j}_L\gamma^\mu u^j_L\qquad J^\mu_{\text{EM}}\equiv(-)\bar{\ell}^{\,j}\gamma^\mu\ell^j+\left(\frac{2}{3}\right)\bar{u}^{\,j}\gamma^\mu u^j+\left(-\frac{1}{3}\right)\bar{d}^{\,j}\gamma^\mu d^j$$

$$J^{3\mu}\equiv\bar{L}^j_\ell\gamma^\mu\frac{\tau^3}{2}L^j_\ell+\bar{Q}^j_\ell\gamma^\mu\frac{\tau^3}{2}Q^j_\ell=\frac{1}{2}\bar{\nu}^j_L\gamma^\mu\nu^j_L-\frac{1}{2}\bar{\ell}^{\,j}_L\gamma^\mu\ell^j_L+\frac{1}{2}\bar{u}^j_L\gamma^\mu u^j_L-\frac{1}{2}\bar{d}^j_L\gamma^\mu d^j_L$$

$$J^\mu_Z\equiv J^{3\mu}-\sin^2\theta_W J^\mu_{\text{EM}}$$

$$\left(\frac{g_2}{2\sqrt{2}}\right)^2J^\dagger_\mu\frac{g^{\mu\nu}}{p^2-M_W^2}J_\nu\quad\xrightarrow{p^2\ll M_W^2}-\frac{G}{\sqrt{2}}J^\dagger_\mu J^\mu\qquad\frac{G}{\sqrt{2}}=\frac{g_2^2}{8M_W^2}=\frac{1}{2v^2}$$

途中計算式

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} =& -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu W_\nu^l - \partial_\nu W_\mu^l - g_2\epsilon_{lmn}W_\mu^m W_\nu^n)^2 \\
=& -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu W_\nu^l - \partial_\nu W_\mu^l)^2 \\
& \quad + \frac{1}{2}g(\partial_\mu W_\nu^l - \partial_\nu W_\mu^l)\epsilon_{lmn}W^{m\mu}W^{n\nu} - \frac{1}{4}(g_2\epsilon_{lmn}W_\mu^m W_\nu^n)^2 \\
=& -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& \quad + \frac{1}{2}g_2(\partial_\mu A_\nu^1 - \partial_\nu A_\mu^1)(W^{2\mu}W^{3\nu} - W^{3\mu}W^{2\nu}) \\
& \quad + \frac{1}{2}g_2(\partial_\mu A_\nu^2 - \partial_\nu A_\mu^2)(W^{3\mu}W^{1\nu} - W^{1\mu}W^{3\nu}) \\
& \quad + \frac{1}{2}g_2(\partial_\mu A_\nu^l - \partial_\nu A_\mu^l)(W^{1\mu}W^{2\nu} - W^{2\mu}W^{1\nu}) \\
& \quad - \frac{g_2^2}{4}(W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2)^2 - \frac{g_2^2}{4}(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3)^2 - \frac{g_2^2}{4}(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1)^2 \\
=& -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& \quad + \frac{i}{4}g_2\{\partial_\mu(W_\nu + W_\nu^\dagger) - \partial_\nu(W_\mu + W_\mu^\dagger)\}\{(W^\mu - W^{\mu\dagger})W^{3\nu} - W^{3\mu}(W^\nu - W^{\nu\dagger})\} \\
& \quad + \frac{i}{4}g_2\{\partial_\mu(W_\nu - W_\nu^\dagger) - \partial_\nu(W_\mu - W_\mu^\dagger)\}\{W^{3\mu}(W^\nu + W^{\nu\dagger}) - (W^\mu + W^{\mu\dagger})W^{3\nu}\} \\
& \quad + \frac{i}{4}g_2(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3)\{(W^\mu + W^{\mu\dagger})(W^\nu - W^{\nu\dagger}) - (W^\mu - W^{\mu\dagger})(W^\nu + W^{\nu\dagger})\} \\
& \quad + \frac{g_2^2}{8}\{(W_\mu - W_\mu^\dagger)W_\nu^3 - W_\mu^3(W_\nu - W_\nu^\dagger)\}^2 - \frac{g_2^2}{8}\{W_\mu^3(W_\nu + W_\nu^\dagger) - (W_\mu + W_\mu^\dagger)W_\nu^3\}^2 \\
& \quad + \frac{g_2^2}{16}((W_\mu + W_\mu^\dagger)(W_\nu - W_\nu^\dagger) - (W_\mu - W_\mu^\dagger)(W_\nu + W_\nu^\dagger))^2 \\
=& -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& \quad + \frac{i}{4}g_2(\partial_\mu W_\nu - \partial_\nu W_\mu + \partial_\mu W_\nu^\dagger - \partial_\nu W_\mu^\dagger)(W^\mu W^{3\nu} - W^{3\mu}W^\nu - W^{\mu\dagger}W^{3\nu} + W^{3\mu}W^{\nu\dagger}) \\
& \quad + \frac{i}{4}g_2(\partial_\mu W_\nu - \partial_\nu W_\mu - \partial_\mu W_\nu^\dagger + \partial_\nu W_\mu^\dagger)(-W^\mu W^{3\nu} + W^{3\mu}W^\nu - W^{\mu\dagger}W^{3\nu} + W^{3\mu}W^{\nu\dagger}) \\
& \quad + \frac{i}{2}g_2(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3)(W^{\mu\dagger}W^\nu - W^{\nu\dagger}W^\mu) \\
& \quad + \frac{g_2^2}{8}(W_\mu W_\nu^3 - W_\mu^\dagger W_\nu^3 - W_\mu^3 W_\nu + W_\mu^3 W_\nu^\dagger)^2 \\
& \quad - \frac{g_2^2}{8}(W_\mu^3 W_\nu + W_\mu^3 W_\nu^\dagger - W_\mu W_\nu^3 - W_\mu^\dagger W_\nu^3)^2 \\
& \quad + \frac{g_2^2}{4}(W_\mu^\dagger W_\nu - W_\mu W_\nu^\dagger)^2 \\
=& -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& \quad - \frac{i}{2}g_2\{(\partial_\mu W_\nu - \partial_\nu W_\mu)(W^{\mu\dagger}W^{3\nu} - W^{3\mu}W^{\nu\dagger}) \\
& \qquad - (\partial_\mu W_\nu^\dagger - \partial_\nu W_\mu^\dagger)(W^\mu W^{3\nu} - W^{3\mu}W^\nu)\}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{i}{2}g_2(\partial_\mu W^3_\nu-\partial_\nu W^3_\mu)(W^{\mu\dagger}W^\nu-W^{\nu\dagger}W^\mu)\\
&-\frac{g_2^2}{2}(W_\mu W^3_\nu-W^3_\mu W_\nu)(W^{\mu\dagger}W^{3\nu}-W^{3\mu}W^{\nu\dagger})\\
&+\frac{g_2^2}{2}\big(W^\dagger_\mu W^{\dagger\mu}W_\nu W^\nu-W_\mu W^\nu W^{\dagger\mu}W^\dagger_\nu\big)\\
=&-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu}-\frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu}\\
&-\frac{1}{2}\{\partial_\mu W_\nu-\partial_\nu W_\mu-i\,g_2(W_\mu W^3_\nu-W^3_\mu W_\nu)\}^\dagger\times\\
&\qquad\{\partial^\mu W^\nu-\partial^\nu W^\mu-i\,g_2(W^\mu W^{3\nu}-W^{3\mu}W^\nu)\}\\
&+i\,g_2(\partial_\mu W^3_\nu-\partial_\nu W^3_\mu)W^{\mu\dagger}W^\nu+\frac{g_2^2}{2}(|W_\mu W^\mu|^2-|W_\mu W^\nu|^2)\\
=&-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu}-\frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu}-\frac{1}{2}(\mathcal{D}_\mu W_\nu-\mathcal{D}_\nu W_\mu)^\dagger(\mathcal{D}^\mu W^\nu-\mathcal{D}^\nu W^\mu)\\
&+i\big(eF^A_{\mu\nu}+g_2\cos\theta_W F^Z_{\mu\nu}\big)W^{\mu\dagger}W^\nu+\frac{g_2^2}{2}(|W_\mu W^\mu|^2-|W_\mu W^\nu|^2)
\end{aligned}
$$

但し $W_\mu\equiv\frac{1}{\sqrt{2}}(W^1_\mu-i\,W^2_\mu)\qquad W^1_\mu=\frac{1}{\sqrt{2}}\big(W_\mu+W^\dagger_\mu\big)\qquad W^2_\mu=\frac{i}{\sqrt{2}}\big(W_\mu-W^\dagger_\mu\big)$

$$\cos\theta_W\equiv\frac{g_2}{\sqrt{g_1^2+g_2^2}}\qquad \sin\theta_W\equiv\frac{g_1}{\sqrt{g_1^2+g_2^2}}$$

$$\begin{pmatrix}A_\mu\\ Z_\mu\end{pmatrix}\equiv\begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}B_\mu\\ W^3_\mu\end{pmatrix}\qquad \begin{pmatrix}B_\mu\\ W^3_\mu\end{pmatrix}=\begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}A_\mu\\ Z_\mu\end{pmatrix}$$

$$F^A_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu\qquad F^Z_{\mu\nu}\equiv\partial_\mu Z_\nu-\partial_\nu Z_\mu\qquad e\equiv\frac{g_1g_2}{\sqrt{g_1^2+g_2^2}}=g_2\sin\theta_W=g_1\cos\theta_W$$

$$g_2W^3_\mu=e\,A_\mu+g_2\cos\theta_W Z_\mu\qquad g_1B_\mu=e\,A_\mu-\frac{g_2}{\cos\theta_W}\sin^2\theta_W Z_\mu$$

$$\mathcal{D}_\mu W_\nu\equiv(\partial_\mu+i\,g_2W^3_\mu)W_\nu=(\partial_\mu+i\,e\,A_\mu+i\,g_2\cos\theta_W Z_\mu)W_\nu$$

$$
\begin{aligned}
\mathcal{L}_{\texttt{Higgs}}=&\left|\left(i\partial_\mu-g_1\frac{1}{2}Y_HB_\mu-g_2\frac{\tau^l}{2}W^l_\mu\right)\Phi\right|^2+\mu^2\Phi^\dagger\Phi-\frac{\lambda}{2}(\Phi^\dagger\Phi)^2\\
=&\frac{1}{2}\left|\begin{pmatrix} i\partial_\mu-g_1\frac{1}{2}B_\mu-\frac{g_2}{2}W^3_\mu & -\frac{g_2}{\sqrt{2}}W_\mu\\ -\frac{g_2}{\sqrt{2}}W^\dagger_\mu & i\partial_\mu-g_1\frac{1}{2}B_\mu+\frac{g_2}{2}W^3_\mu\end{pmatrix}\begin{pmatrix}0\\ v+\phi\end{pmatrix}\right|^2+\mu^2\Phi^\dagger\Phi-\frac{\lambda}{2}(\Phi^\dagger\Phi)^2\\
=&\frac{1}{2}\left|\begin{pmatrix} -\frac{g_2}{\sqrt{2}}W_\mu(v+\phi)\\ \big(i\partial_\mu-\frac{g_1}{2}B_\mu+\frac{g_2}{2}W^3_\mu\big)(v+\phi)\end{pmatrix}\right|^2+\frac{1}{2}\mu^2(v+\phi)^2-\frac{\lambda}{8}(v+\phi)^4\\
=&\frac{g_2^2}{4}W^\dagger_\mu W^\mu(v+\phi)^2+\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{8}|-g_1B_\mu+g_2W^3_\mu|^2(v+\phi)^2\\
&+\mu^2\left(\frac{v}{\sqrt{2}}\right)^2-\frac{\lambda}{2}\left(\frac{v}{\sqrt{2}}\right)^4+v\lambda\left(\frac{\mu^2}{\lambda}-\frac{v^2}{2}\right)\phi+\left(\frac{\mu^2}{2}-\frac{3\lambda v^2}{4}\right)\phi^2-\frac{\lambda v}{2}\phi^3-\frac{\lambda}{8}\phi^4\\
=&\frac{g_2^2}{4}W^\dagger_\mu W^\mu(v+\phi)^2+\frac{1}{8}\frac{g_2^2}{\cos^2\theta_W}Z^2_\mu(v+\phi)^2\\
&+\frac{1}{2}(\partial_\mu\phi)^2-\frac{\lambda v^2}{2}\phi^2-\frac{\lambda v}{2}\phi^3-\frac{\lambda}{8}\phi^4-V\left(\frac{v}{\sqrt{2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{g_2^2 v^2}{4} W_\mu^\dagger W^\mu + \frac{g_2^2 v^2}{8\cos^2\theta_W} Z_\mu^2 + \frac{g_2^2(2v\phi+\phi^2)}{4}\left(W_\mu^\dagger W^\mu + \frac{1}{2\cos^2\theta_W} Z_\mu^2 \right) \\
&\qquad + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda v^2}{2}\phi^2 - \frac{\lambda v}{2}\phi^3 - \frac{\lambda}{8}\phi^4 - V\left(\frac{v}{\sqrt{2}}\right) \\
&= M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu^2 + \left(g_2 M_W \phi + \frac{g_2^2}{4}\phi^2 \right)\left(W_\mu^\dagger W^\mu + \frac{1}{2\cos^2\theta_W} Z_\mu^2 \right) \\
&\qquad + \frac{1}{2}[(\partial_\mu\phi)^2 - m^2\phi^2] - \frac{m\sqrt{\lambda}}{2}\phi^3 - \frac{\lambda}{8}\phi^4 - V\left(\frac{v}{\sqrt{2}}\right)
\end{aligned}
$$

但し　$\Phi(x) \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\phi(x) \end{pmatrix}$　　$\tilde{\Phi} \equiv i\tau^2\Phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v+\phi(x) \\ 0 \end{pmatrix}$　　$\frac{v}{\sqrt{2}} \equiv \frac{\mu}{\sqrt{\lambda}}$

$V(\Phi) \equiv -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2$　　$\tau^l W_\mu^l = \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu \\ \sqrt{2}W_\mu^\dagger & -W_\mu^3 \end{pmatrix}$

$M_W \equiv \frac{g\,v}{2}$　　$M_Z \equiv \frac{1}{2}\sqrt{g_1^2+g_2^2}\,v = \frac{M_W}{\cos\theta_W}$　　$m \equiv \sqrt{2}\mu = v\sqrt{\lambda}$

$$
\begin{aligned}
\mathcal{L}_{\mathtt{lepton}}^{\mathtt{kin}} &= \bar{L}_L^j \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{\ell L}}{2} B_\mu - g_2 \frac{\tau^l}{2} W_\mu^l \right] L_L^j + \bar{\ell}_R^{\,j}\gamma^\mu\left[i\partial_\mu - g_1\frac{Y_{\ell R}}{2} B_\mu \right]\ell_R^j + \bar{\nu}_R^j\gamma^\mu i\partial_\mu \nu_R^j \\
&= \bar{\ell}^{\,j} i\partial\!\!\!/ \ell^j + \bar{\nu}^{\,j} i\partial\!\!\!/ \nu^j + \frac{1}{2}\bar{L}_L^j\gamma^\mu \begin{pmatrix} g_1 B_\mu - g_2 W_\mu^3 & -\sqrt{2} g_2 W_\mu \\ -\sqrt{2} g_2 W_\mu^\dagger & g_1 B_\mu + g_2 W_\mu^3 \end{pmatrix} L_L^j + g_1 B_\mu \bar{\ell}_R^{\,j}\gamma^\mu \ell_R^j \\
&= \bar{\ell}^{\,j} i\partial\!\!\!/ \ell^j + \bar{\nu}^{\,j} i\partial\!\!\!/ \nu^j + g_1 B_\mu \bar{\ell}_R^{\,j}\gamma^\mu\ell_R^j \\
&\qquad + \frac{1}{2}\bar{L}_L^j\gamma^\mu \begin{pmatrix} -\frac{g_2}{\cos\theta_W} Z_\mu & -\sqrt{2} g_2 W_\mu \\ -\sqrt{2} g_2 W_\mu^\dagger & 2e\,A_\mu + \frac{g_2}{\cos\theta_W}(1-2\sin^2\theta_W) Z_\mu \end{pmatrix} L_L^j \\
&= \bar{\ell}^{\,j} i\partial\!\!\!/ \ell^j + \bar{\nu}^{\,j} i\partial\!\!\!/ \nu^j + \left(e A_\mu - \frac{g_2}{\cos\theta_W}\sin^2\theta_W Z_\mu \right)\bar{\ell}_R^{\,j}\gamma^\mu\ell_R^j - \frac{g_2}{\sqrt{2}}\left(\bar{\nu}_L^j\gamma^\mu\ell_L^j W_\mu + \bar{\ell}_L^{\,j}\gamma^\mu\nu_L^j W_\mu^\dagger \right) \\
&\qquad - \frac{g_2}{2\cos\theta_W}\bar{\nu}_L^j\gamma^\mu\nu_L^j Z_\mu + \left\{ e\,A_\mu + \frac{g_2}{2\cos\theta_W}(1-2\sin^2\theta_W) Z_\mu \right\}\bar{\ell}_L^{\,j}\gamma^\mu\ell_L^j \\
&= \bar{\ell}^{\,j} i\partial\!\!\!/ \ell^j + \bar{\nu}^{\,j} i\partial\!\!\!/ \nu^j - \frac{g_2}{\sqrt{2}}\left\{ \left(\bar{\ell}_L^{\,j}\gamma^\mu\nu_L^j \right)^\dagger W_\mu + \bar{\ell}_L^{\,j}\gamma^\mu\nu_L^j W_\mu^\dagger \right\} \\
&\qquad + e\bar{\ell}^{\,j}\gamma^\mu\ell^j A_\mu - \frac{g_2}{\cos\theta_W}\left(\frac{1}{2}\bar{\nu}_L^j\gamma^\mu\nu_L^j - \frac{1}{2}\bar{\ell}_L^{\,j}\gamma^\mu\ell_L^j + \sin^2\theta_W \bar{\ell}^{\,j}\gamma^\mu\ell^j \right) Z_\mu \\
&= \bar{\ell}^{\,j} i\partial\!\!\!/ \ell^j + \bar{\nu}^{\,j} i\partial\!\!\!/ \nu^j - \frac{g_2}{\sqrt{2}}\left(J_\ell^{\mu\dagger} W_\mu + J_\ell^\mu W_\mu^\dagger \right) - e J_{\ell,\mathrm{EM}}^\mu A_\mu - \frac{g_2}{\cos\theta_W} J_{\ell,Z}^\mu Z_\mu
\end{aligned}
$$

但し　$J_\ell^\mu \equiv \bar{\ell}_L^{\,j}\gamma^\mu\nu_L^j$　　$J_{\ell,\mathrm{EM}}^\mu \equiv (-)\bar{\ell}^{\,j}\gamma^\mu\ell^j$

$J_\ell^{3\mu} \equiv \bar{L}_\ell^j\gamma^\mu\frac{\tau^3}{2}L_\ell^j = \frac{1}{2}\bar{\nu}_L^j\gamma^\mu\nu_L^j - \frac{1}{2}\bar{\ell}_L^{\,j}\gamma^\mu\ell_L^j$　　$J_{\ell,Z}^\mu \equiv J_\ell^{3\mu} - \sin^2\theta_W J_{\ell,\mathrm{EM}}^\mu$

$$
\begin{aligned}
\mathcal{L}_{\mathtt{lepton}}^{\mathtt{mass}} &= -y_\ell^j\left(\bar{L}_L^j\Phi\right)\ell_R^j - y_\nu^j\left[(\bar{L}_L U_\nu)^j\tilde{\Phi}\right]\left(U_\nu^\dagger\nu_R\right)^j + h.c. \qquad U_\nu\text{: PMNS matrix} \\
&= -\frac{1}{\sqrt{2}} y_\ell^j \bar{\ell}_L^{\,j}(v+\phi)\ell_R^j - \frac{1}{\sqrt{2}} y_\nu^j \bar{\nu}_L'^j (v+\phi)\nu'^j_R + h.c. \\
&= -\left(m_\ell^j + \frac{y_\ell^j}{\sqrt{2}}\phi \right)\bar{\ell}^{\,j}\ell^j - \left(m_\nu^j + \frac{y_\nu^j}{\sqrt{2}}\phi \right)\bar{\nu}'^j\nu'^j
\end{aligned}
$$

但し　$\nu^j \equiv U_\nu^{jk} \nu'^k \qquad m_\ell^j \equiv \frac{y_\ell^j v}{\sqrt{2}} \qquad m_\nu^j \equiv \frac{y_\nu^j v}{\sqrt{2}}$

$$\mathcal{L}_{\mathrm{quark}}^{\mathrm{kin}} = \bar{Q}_L^j \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{QL}}{2} B_\mu - g_2 \frac{\tau^l}{2} W_\mu^l \right] Q_L^j$$

$$+ \bar{u}_R^j \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{uR}}{2} B_\mu \right] u_R^j + \bar{d}_R^j \gamma^\mu \left[i\partial_\mu - g_1 \frac{Y_{dR}}{2} B_\mu \right] d_R^j$$

$$= \bar{u}^{\,j} i\partial\!\!\!/ u^j + \bar{d}^{\,j} i\partial\!\!\!/ d^j + \frac{1}{2} \bar{Q}_L^j \gamma^\mu \begin{pmatrix} -\frac{1}{3} g_1 B_\mu - g_2 W_\mu^3 & -\sqrt{2} g_2 W_\mu \\ -\sqrt{2} g_2 W_\mu^\dagger & -\frac{1}{3} g_1 B_\mu + g_2 W_\mu^3 \end{pmatrix} Q_L^j$$

$$-\frac{2}{3} g_1 B_\mu \bar{u}_R^j \gamma^\mu u_R^j + \frac{1}{3} g_1 B_\mu \bar{d}_R^j \gamma^\mu d_R^j$$

$$= \bar{u}^{\,j} i\partial\!\!\!/ u^j + \bar{d}^{\,j} i\partial\!\!\!/ d^j$$

$$+ \bar{Q}_L^j \gamma^\mu \begin{pmatrix} -\frac{2}{3} e A_\mu + \frac{g_2\left(\frac{4}{3}\sin^2\theta_W - 1\right)}{2\cos\theta_W} Z_\mu & -\frac{g_1}{\sqrt{2}} W_\mu \\ -\frac{g_2}{\sqrt{2}} W_\mu^\dagger & \frac{1}{3} e A_\mu + \frac{g_2\left(-\frac{2}{3}\sin^2\theta_W + 1\right)}{2\cos\theta_W} Z_\mu \end{pmatrix} Q_L^j$$

$$-\frac{2}{3}\left(e A_\mu - \frac{g_2}{\cos\theta_W} \sin^2\theta_W Z_\mu \right) \bar{u}_R^j \gamma^\mu u_R^j + \frac{1}{3}\left(e A_\mu - \frac{g_2}{\cos\theta_W} \sin^2\theta_W Z_\mu \right) \bar{d}_R^j \gamma^\mu d_R^j$$

$$= \bar{u}^{\,j} i\partial\!\!\!/ u^j + \bar{d}^{\,j} i\partial\!\!\!/ d^j - \frac{g_2}{\sqrt{2}} \bar{u}_L^j \gamma^\mu d_L^j W_\mu - \frac{g_2}{\sqrt{2}} \bar{d}_L^j \gamma^\mu u_L^j W_\mu^\dagger - \frac{2}{3} e \bar{u}^{\,j} \gamma^\mu u^j A_\mu + \frac{1}{3} e \bar{d}^{\,j} \gamma^\mu d^j A_\mu$$

$$-\frac{g_2}{\cos\theta_W} \left\{ \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \bar{u}_L^j \gamma^\mu u_L^j - \frac{2}{3} \sin^2\theta_W \bar{u}_R^j \gamma^\mu u_R^j \right\} Z_\mu$$

$$-\frac{g_2}{\cos\theta_W} \left\{ \left(\frac{1}{3}\sin^2\theta_W - \frac{1}{2} \right) \bar{d}_L^j \gamma^\mu d_L^j + \frac{1}{3} \sin^2\theta_W \bar{d}_R^j \gamma^\mu d_R^j \right\} Z_\mu$$

$$= \bar{u}^{\,j} i\partial\!\!\!/ u^j + \bar{d}^{\,j} i\partial\!\!\!/ d^j - \frac{g_2}{\sqrt{2}} \left(J_q^{\mu\dagger} W_\mu + J_q^\mu W_\mu^\dagger \right) - e J_{q,\mathrm{EM}}^\mu A_\mu - \frac{g_2}{\cos\theta_W} J_{q,Z}^\mu Z_\mu$$

但し　$J_q^\mu \equiv \bar{d}_L^j \gamma^\mu u_L^j \qquad J_{q,\mathrm{EM}}^\mu \equiv \left(\frac{2}{3} \right) \bar{u}^{\,j} \gamma^\mu u^j + \left(-\frac{1}{3} \right) \bar{d}^{\,j} \gamma^\mu d^j$

$$J_q^{3\mu} \equiv \bar{Q}_\ell^j \gamma^\mu \frac{\tau^3}{2} Q_\ell^j = \frac{1}{2} \bar{u}_L^j \gamma^\mu u_L^j - \frac{1}{2} \bar{d}_L^j \gamma^\mu d_L^j \qquad J_{q,Z}^\mu \equiv J_q^{3\mu} - \sin^2\theta_W J_{q,\mathrm{EM}}^\mu$$

$$\mathcal{L}_{\mathrm{quark}}^{\mathrm{mass}} = -y_d^j [(\bar{Q}_L U_d)^j \Phi] \left(U_d^\dagger d_R \right)^j - y_u^j \left(\bar{Q}_L^j \tilde{\Phi} \right) u_R^j + h.c. \qquad U_d\text{: CKM matrix}$$

$$= -\frac{1}{\sqrt{2}} y_d^j \bar{d}_L'^j (v+\phi) d'^j_R - \frac{1}{\sqrt{2}} y_u^j \bar{u}_L^j (v+\phi) u_R^j + h.c.$$

$$= -\left(m_d^j + \frac{y_d^j}{\sqrt{2}} \phi \right) \bar{d}^{\,\prime j} d'^j - \left(m_u^j + \frac{y_u^j}{\sqrt{2}} \phi \right) \bar{u}^{\,j} u^j$$

但し　$d^j \equiv U_d^{jk} d'^k \qquad m_d^j \equiv \frac{y_d^j v}{\sqrt{2}} \qquad m_u^j \equiv \frac{y_u^j v}{\sqrt{2}}$