素粒子物理学授業 公式集 2019/7/22

γ行列（Dirac 表示）γ^μ $(\mu=0,1,2,3)$ $(i=1,2,3)$

$$\gamma^0=\begin{pmatrix}1&0\\0&-1\end{pmatrix}=\begin{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}&0\\0&\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\end{pmatrix}\qquad \gamma^i=\begin{pmatrix}0&\sigma_i\\-\sigma_i&0\end{pmatrix}\qquad 4\times4\text{行列}$$

$$\text{パウリ行列：}\ \sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}\qquad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\qquad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\{\gamma^\mu,\gamma^\nu\}=\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}\qquad \text{反交換子}\ \{A,B\}=AB+BA$$

$$g^{\mu\nu}=\begin{pmatrix}1&&&\\&-1&&\\&&-1&\\&&&-1\end{pmatrix}\text{：Minkowski計量}$$

$$\gamma^{0\dagger}=\gamma^0\qquad \gamma^{i\dagger}=-\gamma^i\qquad \gamma^{\mu\dagger}=\gamma^0\gamma^\mu\gamma^0\qquad \sigma^{\mu\nu}\equiv\frac{i}{2}[\gamma^\mu,\gamma^\nu]=\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$$

Dirac方程式

$$(\gamma^\mu i\partial_\mu-m)\psi(x)=0\qquad \mathcal{L}=\bar\psi\,(\gamma^\mu i\partial_\mu-m)\psi\qquad \bar\psi\equiv\psi^\dagger\gamma^0$$

Dirac spinor（4成分スピノール）

$$\text{粒子}\ \ u_s(p)=N\begin{pmatrix}\phi_s\\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\phi_s\end{pmatrix}\qquad \text{反粒子}\ \ v_s(p)=N\begin{pmatrix}\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi_s\\ \chi_s\end{pmatrix}\qquad N=\sqrt{E+m}$$

ϕ_s，χ_s は2成分spinorの3軸方向のスピン固有状態　$\boldsymbol{\sigma}\cdot\boldsymbol{p}\equiv p_1\sigma_1+p_2\sigma_2+p_3\sigma_3$

$$\phi_\uparrow=\begin{pmatrix}1\\0\end{pmatrix}\qquad \phi_\downarrow=\begin{pmatrix}0\\1\end{pmatrix}\qquad \chi_\uparrow=\begin{pmatrix}0\\1\end{pmatrix}\qquad \chi_\downarrow=\begin{pmatrix}-1\\0\end{pmatrix}$$

$$u_s^\dagger u_r=v_s^\dagger v_r=2E\,\delta_{sr}\qquad \bar u_s u_r=-\bar v_s v_r=2m\,\delta_{sr}\qquad \bar u_s v_r=0$$

$$\sum_{s=\uparrow\downarrow}u_s(p)\bar u_s(p)=\not p+m\qquad \sum_{s=\uparrow\downarrow}v_s(p)\bar v_s(p)=\not p-m$$

$$(\not p-m)u_s=0\qquad (\not p+m)v_s=0\qquad \bar u_s(\not p-m)=0\qquad \bar v_s(\not p+m)=0$$

$\not p\equiv p_\mu\gamma^\mu$ ：Feynman slash 表記

カイラル演算子

$$\gamma^5\equiv i\,\gamma^0\gamma^1\gamma^2\gamma^3=\begin{pmatrix}0&1\\1&0\end{pmatrix}\qquad (\gamma^5)^2=1\qquad \{\gamma^5,\gamma^\mu\}=0\qquad \gamma^{5\dagger}=\gamma^5$$

$$(1\pm\gamma^5)^2=2\,(1\pm\gamma^5)\qquad 1-\gamma^5=\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$$

$$P_L\equiv\frac{1}{2}(1-\gamma^5)\qquad P_R\equiv\frac{1}{2}(1+\gamma^5)\qquad P_L+P_R=1\qquad P_L^2=P_L\qquad P_R^2=P_R$$

$$\gamma^5P_L=-P_L\qquad \gamma^5P_R=P_R\qquad \gamma^\mu P_L=P_R\gamma^\mu\qquad \gamma^\mu P_R=P_L\gamma^\mu$$

$$\psi_L\equiv P_L\psi\qquad \bar\psi_L=(P_L\psi)^\dagger\gamma^0=\psi^\dagger P_L\gamma^0=\psi^\dagger\gamma^0P_R=\bar\psi P_R$$

Charge conjugation 荷電共役

$$C\equiv i\gamma^2\gamma^0=\begin{pmatrix}0&-i\sigma_2\\-i\sigma_2&0\end{pmatrix}\qquad C=C^*=-C^{-1}=-C^T=-C^\dagger$$

$$\psi^C\equiv C\bar\psi^{\,T}=i\gamma^2\psi^*\qquad \bar\psi^{\,C}=-\psi^TC^{-1}=-i\bar\psi^{\,*}\gamma^2$$

ファインマン則

外線

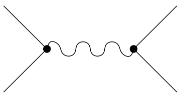
spin 1/2　　incoming: $u(p,s)$ or $v(p,s)$　　outgoing: $\bar{u}(p,s)$ or $\bar{v}(p,s)$

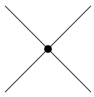
spin 1　　incoming: $\varepsilon_\mu(p,\lambda)$　　outgoing: $\varepsilon^*_\mu(p,\lambda)$

内線

Gauge field: $i\tilde{D}_F(p) = \dfrac{-ig^{\mu\nu}}{p^2 - m^2 + i\varepsilon}$

Dirac field: $i\tilde{S}_F(p) = \dfrac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$





$$J^\mu = \frac{1}{2}\bar{\ell}^{\,j}\gamma^\mu(1-\gamma^5)\nu^j + \frac{1}{2}\bar{d}^{\,j}\gamma^\mu(1-\gamma^5)u^j$$

$$\left(\frac{g}{\sqrt{2}}\right)^2 J^\dagger_\mu \frac{g^{\mu\nu}}{p^2 - M_W^2} J_\nu \xrightarrow{p^2 \ll M_W^2} -\frac{4G}{\sqrt{2}} J^\dagger_\mu J^\mu \qquad \frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$

Matrix Element 計算　　任意の列行列$X = (x_i)$, $Y = (y_i)$ $\Longrightarrow X^T Y = \mathrm{Tr}[Y X^T]$

$$|\bar{\psi}_f(\Gamma)\psi_i|^2 = \bar{\psi}_f(\Gamma)\psi_i\bar{\psi}_i(\gamma^0\Gamma^\dagger\gamma^0)\psi_f = \mathrm{Tr}[(\Gamma)\psi_i\bar{\psi}_i(\gamma^0\Gamma^\dagger\gamma^0)\psi_f\bar{\psi}_f]$$

$\psi_{f,i}$: Dirac spinor（4成分）　　Γ : γ行列（4×4行列）

$$\gamma^0(1-\gamma^5)^\dagger\gamma^0 = 1 + \gamma^5 \qquad \gamma^0[\gamma^\mu(1-\gamma^5)]^\dagger\gamma^0 = \gamma^\mu(1-\gamma^5)$$

γ行列のトレース計算公式

$$\gamma_\mu\gamma^\mu = 4 \qquad \gamma_\mu\not{a}\gamma^\mu = -2\not{a} \qquad \gamma_\mu\not{a}\not{b}\gamma^\mu = 4a\cdot b \qquad \gamma_\mu\not{a}\not{b}\not{c}\gamma^\mu = -2\not{c}\not{b}\not{a}$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu} \qquad \mathrm{Tr}[ABC] = \mathrm{Tr}[BCA] \qquad \mathrm{Tr}[S^{-1}AS] = \mathrm{Tr}A \qquad \mathrm{Tr}\mathbf{1} = 4$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4[g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}]$$

$\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4i\varepsilon^{\mu\nu\rho\sigma}$　（$\varepsilon^{\mu\nu\rho\sigma}$: レヴィチヴィタの反対称テンソル）

$\mathrm{Tr}[\gamma^1\gamma^2\cdots\gamma^{2n+1}] = 0$　（任意の奇数個のγ行列の積のトレースは0）

$$\mathrm{Tr}[\gamma^5] = \mathrm{Tr}[\gamma^5\gamma^\mu] = \mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu] = \mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho] = 0$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu}{}^{\gamma\delta} = -2(g^{\rho\gamma}g^{\sigma\delta} - g^{\rho\delta}g^{\sigma\gamma})$$

$$\gamma^\mu\gamma^\nu\gamma^\rho = g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu + i\gamma^5\varepsilon^{\mu\nu\rho\sigma}\gamma_\sigma$$

ローレンツ不変な位相体積 Lorentz Invariant Phase Space (LIPS)

$$d^4P\delta(M^2 - P^2)\theta(P^0) = \frac{d^3P}{2E} = \frac{P}{2}dEd\Omega \qquad E = \sqrt{\vec{P}^2 + M^2}$$

$$d^3P = P^2 dP d\Omega = PEdEd\Omega$$

$$d\Phi_n(P; P_1, \cdots, P_n) = (2\pi)^4\delta^4\left(P - \sum_{i=1}^n P_i\right)\prod_{i=1}^n \frac{d^3P_i}{(2\pi)^3 2E_i}$$

n体の位相体積（normalization $\int \rho dV = 2E$）

$M(\mathrm{rest}) \to m_1 + m_2$ ： 静止系での二体崩壊

$$d\Phi_2(M; P_1, P_2) = \frac{1}{16\pi^2}\frac{|\vec{P}_1|}{M}\, d\Omega_1$$

$M(\mathrm{rest}) \to m_1 + m_2 + m_3$ ：静止系での三体崩壊（親粒子に特別な方向が無い場合）

$$d\Phi_3(M; P_1, P_2, P_3) = \frac{1}{4(2\pi)^3}\, dE_1 dE_2 = \frac{1}{16(2\pi)^3 M^2}\, dm_{12}^2 dm_{23}^2$$

微分崩壊幅：$d\Gamma = \dfrac{|\mathfrak{M}|^2}{2M}\, d\Phi_n(P; P_1, \cdots, P_n)$

2-body decay $\qquad d\Gamma = \dfrac{1}{32\pi^2}|\mathfrak{M}|^2\dfrac{|\vec{P}_1|}{M^2}d\Omega_1$

3-body decay $\qquad d\Gamma = \dfrac{1}{(2\pi)^3}\dfrac{1}{8M}|\mathfrak{M}|^2 dE_1 dE_2$

微分断面積：$d\sigma = \dfrac{|\mathfrak{M}|^2}{4\sqrt{(P_1\cdot P_2)^2 - m_1^2 m_2^2}}\, d\Phi_n(P_1 + P_2; P_3, \cdots, P_{n+2})$

$$\overset{m_1}{\bigcirc}\xrightarrow{\ \boldsymbol{p}_1\ }\overset{m_2}{\bigcirc} \qquad \sqrt{(P_1\cdot P_2)^2 - m_1^2 m_2^2} = m_2 P_1$$

$$\overset{m_1}{\bigcirc}\xrightarrow{\boldsymbol{p}_1}\ \xleftarrow{-\boldsymbol{p}_1}\overset{m_2}{\bigcirc} \qquad \sqrt{(P_1\cdot P_2)^2 - m_1^2 m_2^2} = P_1\sqrt{s} \qquad s \equiv (P_1 + P_2)^2$$

$$d\Phi_2(P_1 + P_2; P_3, P_4) = \frac{1}{16\pi^2}\frac{|\vec{P}_3|}{\sqrt{s}}d\Omega_3 \qquad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{P}_3|}{|\vec{P}_1|}|\mathfrak{M}|^2$$