Spin flip probability of Top Quark by Gluon Radiation

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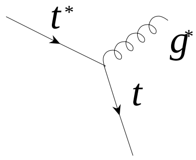
Assume off-shell gluon radiation via tree diagram from off-shell top quark $t^* \to t + g^*$



$$\mathfrak{M} \propto \varepsilon_\mu^* \, \bar{u}_t \gamma^\mu u_{t^*}$$

In the t^* rest frame, assume t flight direction to be z direction.

p: magnitude of momenta of t and g^*

m_g: virtual mass of g^*

We suppose a matrix element with index(i,j, and k) for spin states of t, t^*, and g^*

$$\mathfrak{M}_{ijk} \propto \varepsilon_\mu^*(k) \bar{u}_t(i) \gamma^\mu u_{t^*}(j)$$

$$u_t(i) = \sqrt{E_t + m_t} \begin{pmatrix} \phi_i \\ \dfrac{\sigma_3 p}{E_t + m_t} \phi_i \end{pmatrix} \qquad E_t^2 = m_t^2 + p^2$$

$$u_{t^*}(j) = \sqrt{m_{t^*}} \begin{pmatrix} \psi_j \\ 0 \end{pmatrix} \qquad m_{t^*} = \sqrt{m_t^2 + p^2} + \sqrt{m_{g^*}^2 + p^2}$$

$$\begin{aligned} \mathfrak{M}_{ijk} &\propto \varepsilon_\mu^*(k) \bar{u}_t(i) \gamma^\mu u_{t^*}(j) \\ &\propto \varepsilon_\mu^*(k) \begin{pmatrix} \phi_i^\dagger & -\phi_i^\dagger \dfrac{\sigma_3 p}{E_t + m_t} \end{pmatrix} \gamma^\mu \begin{pmatrix} \psi_j \\ 0 \end{pmatrix} \\ &\propto \varepsilon_\mu^*(k) \left\{ \phi_i^\dagger \psi_j ; \phi_i^\dagger \frac{\sigma_3 p}{E_t + m_t} \sigma_{[123]} \psi_j \right\} \\ &\propto \varepsilon_\mu^*(k) \left\{ \phi_i^\dagger \psi_j ; \frac{p}{E_t + m_t} \phi_i^\dagger \left[\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}, \begin{pmatrix} & -i \\ -i & \end{pmatrix}, 1 \right] \psi_j \right\} \end{aligned}$$

Breakdown as for spin states of t and t^*

$$\mathfrak{M}_{\uparrow\uparrow k} \propto \varepsilon_\mu^*(k) \left\{ 1 ; \frac{p}{E_t + m_t} [0, 0, 1] \right\}$$

$$\mathfrak{M}_{\uparrow\downarrow k} \propto \varepsilon_\mu^*(k) \left\{ 0 ; \frac{p}{E_t + m_t} [1, -i, 0] \right\}$$

$$\mathfrak{M}_{\downarrow\uparrow k} \propto \varepsilon_\mu^*(k) \left\{ 0 ; \frac{p}{E_t + m_t} [-1, -i, 0] \right\}$$

$$\mathfrak{M}_{\downarrow\downarrow k} \propto \varepsilon_\mu^*(k) \left\{ 1 ; \frac{p}{E_t + m_t} [0, 0, 1] \right\}$$

Remind polarized vectors (λ: helicity) for spin-1 particles

$$\varepsilon^\mu(p, \lambda = \pm) = (0; \mp 1/\sqrt{2}, -i/\sqrt{2}, 0)$$

$$\varepsilon^\mu(p, \lambda = 0) = (p; 0, 0, E_{g^*}) / m_{g^*}$$

Therefore

$$\varepsilon_\mu^*(-p, \lambda = \pm) = (0; \pm 1/\sqrt{2}, -i/\sqrt{2}, 0)$$

$$\varepsilon_\mu^*(-p, \lambda = 0) = (-p; 0, 0, -E_{g^*}) / m_{g^*}$$

Finally we obtain the complete set of matrix elements for each spin state.

$$\mathfrak{M}_{\uparrow\uparrow\pm} = \mathfrak{M}_{\downarrow\downarrow\pm} = 0 \qquad \mathfrak{M}_{\uparrow\uparrow 0} = \mathfrak{M}_{\downarrow\downarrow 0} = -\left(1 + \frac{E_{g^*}}{E_t + m_t}\right) p/m_{g^*}$$

$$\mathfrak{M}_{\uparrow\downarrow\pm} = \frac{p}{E_t + m_t}\left(\pm 1/\sqrt{2} + 1/\sqrt{2}\right) \qquad \mathfrak{M}_{\uparrow\downarrow 0} = 0$$

$$\mathfrak{M}_{\downarrow\uparrow\pm} = \frac{p}{E_t + m_t}\left(\mp 1/\sqrt{2} + 1/\sqrt{2}\right) \qquad \mathfrak{M}_{\downarrow\uparrow 0} = 0$$

Only the followings suvive among above:

$$\begin{aligned}\mathfrak{M}_{\uparrow\uparrow 0} = \mathfrak{M}_{\downarrow\downarrow 0} &= -\left(1 + \frac{E_{g^*}}{E_t + m_t}\right) p/m_{g^*} = -\left(\frac{E_t + E_{g^*} + m_t}{E_t + m_t}\right) p/m_{g^*} \\ &= -\frac{p}{E_t + m_t} \cdot \frac{m_{t^*} + m_t}{m_{g^*}}\end{aligned}$$

$$\mathfrak{M}_{\uparrow\downarrow +} = \mathfrak{M}_{\downarrow\uparrow -} = \frac{\sqrt{2}p}{E_t + m_t}$$

The spin flip probability of top quark by gluon radiation is given as

$$\begin{aligned}P_{\text{flip}} &= \frac{|\mathfrak{M}_{\uparrow\downarrow +}|^2 + |\mathfrak{M}_{\downarrow\uparrow -}|^2}{|\mathfrak{M}_{\uparrow\downarrow +}|^2 + |\mathfrak{M}_{\downarrow\uparrow -}|^2 + |\mathfrak{M}_{\uparrow\uparrow 0}|^2 + |\mathfrak{M}_{\downarrow\downarrow 0}|^2} \\ &= \frac{2}{2 + \left(\frac{m_{t^*} + m_t}{m_{g^*}}\right)^2} \\ &= \frac{2m_{g^*}^2}{2m_{g^*}^2 + (m_{t^*} + m_t)^2}\end{aligned}$$

[EOF]