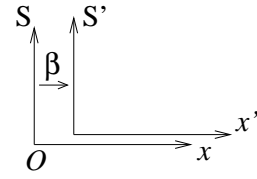


Lorentz Transformation

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \cosh \alpha & -\sinh \alpha \\ -\sinh \alpha & \cosh \alpha \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh \alpha$$

$$\gamma^2\beta^2 = \gamma^2 - 1 = \sinh^2 \alpha \quad \gamma\beta = \sinh \alpha \quad (\alpha > 0)$$



β/γ of Particle

$$\vec{\beta} = \frac{\vec{p}}{E} \quad \gamma = \frac{E}{m} \quad \vec{\beta}\gamma = \frac{\vec{p}}{m}$$

General Lorentz transformation

$$a^\mu = (a^0; \vec{a}) \text{ in } S \quad a'^\mu = (a'^0; \vec{a}') \text{ in } S' \quad p^\mu = (p^0; \vec{p}) : \text{4-mom. for } S \rightarrow S'$$

$$a'^0 = (p^0 a^0 - \vec{p} \cdot \vec{a})/m = (a^0 - \vec{\beta} \cdot \vec{a})\gamma$$

$$\vec{a}' = \vec{a} + \frac{1}{m} \left(\frac{\vec{p} \cdot \vec{a}}{m + p^0} - a^0 \right) \vec{p} = \vec{a} + \left(\frac{\gamma}{1 + \gamma} \vec{\beta} \cdot \vec{a} - a^0 \right) \vec{\beta}\gamma \quad \text{where } m = \sqrt{p^2}$$

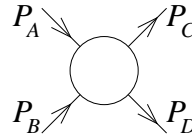
Boost

$$a^0 = (p^0 a'^0 + \vec{p} \cdot \vec{a}')/m = (a'^0 + \vec{\beta} \cdot \vec{a}')\gamma$$

$$\vec{a} = \vec{a}' + \frac{1}{m} \left(\frac{\vec{p} \cdot \vec{a}'}{m + p^0} + a'^0 \right) \vec{p} = \vec{a}' + \left(\frac{\gamma}{1 + \gamma} \vec{\beta} \cdot \vec{a}' + a'^0 \right) \vec{\beta}\gamma$$

Invariant Variables (Mandelstam variables)

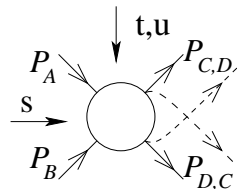
$$\begin{aligned} s &= (P_A + P_B)^2 = (P_C + P_D)^2 \\ t &= (P_A - P_C)^2 = (P_B - P_D)^2 \\ u &= (P_A - P_D)^2 = (P_B - P_C)^2 \\ s + t + u &= m_A^2 + m_B^2 + m_C^2 + m_D^2 \end{aligned}$$



$$\begin{array}{c} m \quad p \quad m \\ \circ \longrightarrow \circ \end{array} \quad s = 2m(m + E)$$

$$\begin{array}{c} m \quad \quad \quad m \\ \circ \longrightarrow \quad \quad \quad \circ \\ \quad \quad \quad \nearrow \theta \quad \nwarrow \end{array} \quad \begin{aligned} s &= 4(p^2 + m^2) \\ t &= -2p^2(1 - \cos \theta) \\ u &= -2p^2(1 + \cos \theta) \end{aligned}$$

Crossing Symmetry



	<i>s</i> -channel	<i>t</i> -channel	<i>u</i> -channel
<i>s</i>	$A + B \rightarrow C + D$ $(P_A + P_B)^2$	$A + \bar{C} \rightarrow \bar{B} + D$ $(P_A - P_C)^2$	$A + \bar{D} \rightarrow \bar{B} + C$ $(P_A - P_D)^2$
<i>t</i>	$(P_A - P_C)^2$	$(P_A + P_B)^2$	$(P_A + P_B)^2$
<i>u</i>	$(P_A - P_D)^2$	$(P_A - P_D)^2$	$(P_A - P_C)^2$

Two Body Decay

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

$$E_2 = \frac{M^2 + m_2^2 - m_1^2}{2M}$$



$$T_1 = \Delta M \left(1 - \frac{m_1}{M} - \frac{\Delta M}{2M} \right) \quad \Delta M = M - m_1 - m_2$$

$$P = \frac{\lambda^{1/2}(M^2, m_1^2, m_2^2)}{2M} \quad \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$$

$$\lambda(\alpha^2, \beta^2, \gamma^2) = \{\alpha^2 - (\beta + \gamma)^2\} \{\alpha^2 - (\beta - \gamma)^2\}$$

Two Body Reaction

$$s = m_1^2 + m_2^2 + 2m_2 E_1$$

$$E_1^* = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}$$

$$E_2^* = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}} = \frac{m_2(m_2 + E_1)}{\sqrt{s}}$$

$$P^* = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}} = \frac{m_2 P_1}{\sqrt{s}}$$

$m_1 \longrightarrow m_2$
 $\circ \longrightarrow \circ$

CM
 $m_1 \longrightarrow m_2$
 $\circ \longrightarrow \circ$

In Lab system
 $p_1^\mu = (E_1; P_1)$
 $p_2^\mu = (m_2; 0)$

Three Body Decay $M(\text{rest}) \rightarrow m_1 + m_2 + m_3$

$$m_{12}^2 \equiv (P_1 + P_2)^2 = M^2 - 2ME_3 + m_3^2 \quad E_3 = \frac{M^2 - m_{12}^2 + m_3^2}{2M}$$

$$P_3 = \frac{\lambda^{1/2}(M^2, m_{12}^2, m_3^2)}{2M} \quad \cos\theta_{12} = \frac{P_3^2 - P_1^2 - P_2^2}{2P_1 P_2}$$

Transformation between Lab and CM-Systems

$$\gamma = \frac{E_1 + m_2}{\sqrt{s}} \quad \beta = \frac{P_1}{E_1 + m_2}$$

$m_1 \quad p_1 \quad E_1 \quad m_2$
 $\circ \longrightarrow \circ$

Energy: $E_1 + m_2$

Momentum: P_1

Mass: \sqrt{s}

$$\begin{pmatrix} E^* \\ P_{\parallel}^* \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E \\ P_{\parallel} \end{pmatrix} \quad \begin{pmatrix} E \\ P_{\parallel} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P_{\parallel}^* \end{pmatrix} \quad *:\text{CM-system}$$



$$\tan\theta = \frac{\sin\theta^*}{\gamma(\alpha + \cos\theta^*)} \quad \alpha \equiv \frac{\beta}{v_{\text{particle}}^*/c}$$

$$\cos\theta^* = \frac{\pm\sqrt{D} - \alpha\gamma^2 \tan^2\theta}{1 + \gamma^2 \tan^2\theta} \quad \text{if } \alpha < 1, \text{ positive sign only}$$

$$D \equiv 1 + \gamma^2(1 - \alpha^2)\tan^2\theta$$

Lorentz Invariant Phase Space

$$d^4 P \delta(M^2 - P^2) \theta(P^0) = \frac{d^3 P}{2E} = \frac{P}{2} dE d\Omega \quad E = \sqrt{\vec{P}^2 + M^2}$$

$$d^3 P = P^2 dP d\Omega = PE dE d\Omega$$

Phase Spce of n-bodies (normalization $\int_{\text{unit } V} \rho dV = 2E$)

Phase space volume of single particle: $h^3 = (2\pi\hbar)^3 = (2\pi)^3$

$$d\Phi_n(P; P_1, \dots, P_n) = (2\pi)^4 \delta^4\left(P - \sum_{i=1}^n P_i\right) \prod_{i=1}^n \frac{d^3 P_i}{(2\pi)^3 2E_i}$$

$M(\text{rest}) \rightarrow m_1 + m_2$

$$d\Phi_2(M; P_1, P_2) = \frac{1}{16\pi^2} \frac{|\vec{P}_1|}{M} d\Omega_1$$

$M(\text{rest}) \rightarrow m_1 + m_2 + m_3$ (In case no special direction in the initial state)

$$d\Phi_3(M; P_1, P_2, P_3) = \frac{1}{4(2\pi)^3} dE_1 dE_2 = \frac{1}{16(2\pi)^3 M^2} dm_{12}^2 dm_{23}^2$$

$$d\Gamma = \frac{|\mathfrak{M}|^2}{2M} d\Phi_n(P; P_1, \dots, P_n)$$

$$\text{2-bodies decay} \quad d\Gamma = \frac{1}{32\pi^2} |\mathfrak{M}|^2 \frac{|\vec{P}_1|}{M^2} d\Omega_1$$

$$\text{3-bodies decay} \quad d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8M} |\mathfrak{M}|^2 dE_1 dE_2$$

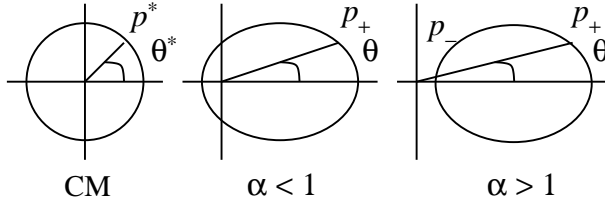
$$d\sigma = \frac{|\mathfrak{M}|^2}{4\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2}} d\Phi_n(P_1 + P_2; P_3, \dots, P_{n+2})$$

$$\begin{array}{c} m_1 \quad \mathbf{p}_1 \quad m_2 \\ \circ \longrightarrow \circ \end{array} \quad \sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2} = m_2 P_1$$

$$\begin{array}{c} m_1 \quad \mathbf{p}_1 \quad -\mathbf{p}_1 \quad m_2 \\ \circ \longrightarrow \longleftarrow \circ \end{array} \quad \sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2} = P_1 \sqrt{s}$$

$$d\Phi_2(P_1 + P_2; P_3, P_4) = \frac{1}{16\pi^2} \frac{|\vec{P}_3|}{\sqrt{s}} d\Omega_3 \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{P}_3|}{|\vec{P}_1|} |\mathfrak{M}|^2$$

Jacobian of $\frac{d\sigma}{d\Omega_{\text{CM}}} \Leftrightarrow \frac{d\sigma}{d\Omega_{\text{Lab}}}$



$$\alpha \equiv \frac{v_{\text{Target}}^*}{v_{\text{Scattered}}^*}$$

$$\frac{d\Omega}{d\Omega_{\pm}^*} = \pm \sqrt{D} \cos\theta \left(\frac{P^*}{P_{\pm}} \right)^2 \quad D \equiv 1 + \gamma^2 (1 - \alpha^2) \tan^2\theta$$

$$\left(\frac{P^*}{P_{\pm}} \right)^2 = \frac{\cos^2\theta}{\gamma^2} \left(\frac{1 + \gamma^2 \tan^2\theta}{\alpha \pm \sqrt{D}} \right)^2$$

Rapidity

$$y \equiv \frac{1}{2} \ln \left(\frac{1 + \beta}{1 - \beta} \right) \quad \beta = \tanh y \quad \gamma = \cosh y$$

$$y = y_0 + y' \quad \frac{dN}{dy} = \frac{dN}{dy'} \quad \beta = \frac{\beta_0 + \beta'}{1 + \beta_0 \beta'}$$

$$y \equiv \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$$

$$\eta = -\ln \left(\tan \frac{\theta}{2} \right): \text{pseudo rapidity} \quad \cos\theta = \tanh\eta$$

