

**Lorentz vector**

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & \\ & -\mathbf{1} \end{pmatrix}$$

$$x^\mu = (ct, \vec{x}) \quad \partial_\mu = \left( \frac{1}{c} \frac{\partial}{\partial t}, \nabla \right) \quad p^\mu = \left( \frac{E}{c}, \vec{p} \right)$$

$$k^\mu = \left( \frac{\omega}{c}, \vec{k} \right) \quad j^\mu = (c\rho, \vec{j}) \quad A^\mu = \left( \frac{\phi}{c}, \vec{A} \right)$$

$$d^4x = dx^0 dx^1 dx^2 dx^3 \quad \frac{d^3p}{E/c} \quad \frac{d^3k}{\omega/c} \quad \text{Lorentz invariant}$$

$$p^\mu = i\partial^\mu \quad E = i\frac{\partial}{\partial t} \quad \vec{p} = -i\nabla$$

**Electro-Magnetic Field**

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

$$-\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2\mu_0} \left( \frac{\vec{E}^2}{c^2} - \vec{B}^2 \right) = \frac{1}{2} (\varepsilon_0 \vec{E}^2 - \mu_0 \vec{H}^2) \quad \frac{1}{\mu_0 \varepsilon_0} = c^2$$

$$\mathcal{L} = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

$$\partial_\mu F^{\mu\nu} = \square A^\nu - \partial^\nu (\partial_\mu A^\mu) = \mu_0 j^\nu \quad \partial_\mu j^\mu = 0$$

$$\mathcal{H} = \pi^\mu (\partial_t A_\mu) - \mathcal{L} = \frac{1}{2} (\varepsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2) + \varepsilon_0 \vec{E} \cdot \nabla \phi$$

$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_t A_\mu)} = -\frac{1}{\mu_0} F^{0\mu} = -\frac{1}{\mu_0} (0, \varepsilon_0 c \vec{E}) \quad \partial_t A_\mu = \left( -\nabla \cdot \vec{A}, \frac{1}{c} (\vec{E} + \nabla \phi) \right)$$

**Gauge transformation**

$$A^\mu \rightarrow A'^\mu = A^\mu - \partial^\mu \Lambda$$

$$\partial_\mu A^\mu = 0 \quad \Rightarrow \quad \square \Lambda = \partial_\mu A^\mu \quad : \text{ Lorenz gauge}$$

$$\nabla \cdot \vec{A} = 0 \quad \Rightarrow \quad \Delta \Lambda = -\nabla \cdot \vec{A} \quad : \text{ Coulomb gauge}$$

$$\text{Heaviside-Lorentz unit: } \varepsilon_0 = \mu_0 = 1 \quad \text{Natural unit: } \hbar = c = 1 \quad \frac{e^2}{4\pi} = \frac{1}{137}$$

**Euler Equation**

$$S = \int \mathcal{L}(\phi, \partial_\mu \phi) d^4x = \int \mathcal{L} d^3x dt = \int L dt \quad \text{action}$$

$$\delta S = 0 \quad \Rightarrow \quad \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} = 0 \quad \text{Least action principle}$$

**Proca Equation**

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu - j^\mu A_\mu$$

$$\frac{\partial \mathcal{L}}{\partial A_\mu} - \partial_\nu \frac{\partial \mathcal{L}}{\partial (\partial_\nu A_\mu)} = 0 \Rightarrow \square A^\mu - \partial^\mu (\partial_\nu A^\nu) + m^2 A^\mu = j^\mu \Rightarrow m^2 \partial_\mu A^\mu = \partial_\mu j^\mu = 0$$

$$\Rightarrow \partial_\mu A^\mu = 0 \quad (\square + m^2) A^\mu = j^\mu$$

## Fock Space

$$\hat{a}, \hat{a}^\dagger : \text{rising and lowering operator} \quad [\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{N} = \hat{a}^\dagger \hat{a} : \text{number operator} \quad \hat{N}|n\rangle = n|n\rangle$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \quad \hat{a}|n\rangle = \sqrt{n}|n-1\rangle \quad \hat{a}|0\rangle = 0$$

$$|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^\dagger)^n|0\rangle$$

e.g.  $[\hat{x}, \hat{p}] = i$        $\hat{p}, \hat{x} : \text{Hermitian}$

$$\hat{a} = \frac{1}{\sqrt{2m\omega}}(m\omega\hat{x} + i\hat{p}) \quad \hat{a}^\dagger = \frac{1}{\sqrt{2m\omega}}(m\omega\hat{x} - i\hat{p}) \quad [\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2 = \omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

## Invariant Delta Function

$$\begin{aligned} \Delta(x; m^2) &\equiv \frac{1}{i(2\pi)^3} \int d^4p \epsilon(p_0) \delta(p^2 - m^2) e^{-ipx} \\ &= \frac{1}{i(2\pi)^3} \int d^3p \frac{1}{2\sqrt{m^2 + \vec{p}^2}} [e^{-ipx} - e^{ipx}] \\ &= -\frac{1}{2\pi} \epsilon(x_0) \left[ \delta(x^2) - \frac{m^2}{2} \theta(x^2) \frac{J_1(m\sqrt{x^2})}{m\sqrt{x^2}} \right] \end{aligned} \quad \begin{cases} \epsilon(u) \equiv \text{sgn } u \equiv u/|u| \\ \epsilon(0) = 0 \end{cases}$$

$$\Delta(-x; m^2) = -\Delta(x; m^2) \quad x^2 > 0: \text{timelike} \quad x^2 < 0: \text{spacelike}$$

## Canonical Quantization

$$\pi_i = \frac{\partial \mathcal{L}(\phi_i, \partial\phi_i)}{\partial[\partial_t\phi_i]} \quad \mathcal{H} = \pi_i \partial_t \phi_i - \mathcal{L} \quad H = \int d^3x \mathcal{H}$$

$$i\partial_t \phi = [\phi, H] \quad i\partial_t \pi(x) = [\pi, H]$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial[\partial_\mu\phi_i]} \partial^\nu \phi_i - g^{\mu\nu} \mathcal{L}: \text{Energy-momentum tensor}$$

$$T^{00} = \mathcal{H} \quad p_i = \int d^3x T^{0i} \quad \partial_\mu T^{\mu\nu} = 0$$

## Neutral Scalar Field

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \quad (\square + m^2)\phi(x) = 0$$

$$\pi = \frac{\partial \mathcal{L}}{\partial[\partial_t\phi]} = \partial_t \phi$$

$$\mathcal{H} = \pi \partial_t \phi - \mathcal{L} = \frac{1}{2} [(\partial_t \phi)^2 + (\nabla \phi)^2 + m^2 \phi^2]$$

$$[\phi(t, \vec{x}), \pi(t, \vec{x}')] = i\delta(\vec{x} - \vec{x}') \quad \text{canonical quantization}$$

$$[\phi(t, \vec{x}), \phi(t, \vec{x}')] = [\pi(t, \vec{x}), \pi(t, \vec{x}')] = 0$$

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3 2E} [a(\vec{p}) e^{-ipx} + a^\dagger(\vec{p}) e^{ipx}]$$

$$a(\vec{p}) = \int d^3x e^{-i\vec{p}\cdot\vec{x}} [E\phi(0, \vec{x}) + i\pi(0, \vec{x})]$$

$$H = \int \frac{d^3p}{2(2\pi)^3} \frac{1}{2} [a^\dagger(\vec{p}) a(\vec{p}) + a(\vec{p}) a^\dagger(\vec{p})] = \int \frac{d^3p}{2(2\pi)^3} a^\dagger(\vec{p}) a(\vec{p}) + \frac{1}{2} \int \frac{d^3p d^3x}{(2\pi)^3} E$$

$$[a(\vec{p}), a^\dagger(\vec{p}')] = (2\pi)^3 2E \delta(\vec{p} - \vec{p}') \quad [a(\vec{p}), a(\vec{p}')] = 0$$

#### 4-dimensional commutation relation

$$[\phi(x), \phi(y)] = i\Delta(x - y; m^2) \quad \text{invariant delta function}$$

#### Complex Scalar Field

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi$$

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \quad \phi^\dagger = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \quad \phi_1, \phi_2 : \text{Hermitian}$$

$$\pi = \frac{\partial \mathcal{L}}{\partial[\partial_t \phi]} = \partial_t \phi^\dagger \quad \pi^\dagger = \frac{\partial \mathcal{L}}{\partial[\partial_t \phi^\dagger]} = \partial_t \phi$$

$$\mathcal{H} = \pi \partial_t \phi + (\partial_t \phi^\dagger) \pi^\dagger - \mathcal{L} = \partial_t \phi^\dagger \partial_t \phi + \nabla \phi^\dagger \cdot \nabla \phi + m^2 \phi^\dagger \phi$$

$$\phi(x) = \int \frac{d^3 p}{(2\pi)^3 2E} [a(\vec{p}) e^{-ipx} + b^\dagger(\vec{p}) e^{ipx}] \quad \pi(x) = i \int \frac{d^3 p}{(2\pi)^3 2} [a^\dagger(\vec{p}) e^{ipx} - b(\vec{p}) e^{-ipx}]$$

$$a(\vec{p}) = \int d^3 x e^{-i\vec{p}\cdot\vec{x}} [E\phi(0, \vec{x}) + i\pi^\dagger(0, \vec{x})] \quad b(\vec{p}) = \int d^3 x e^{-i\vec{p}\cdot\vec{x}} [E\phi^\dagger(0, \vec{x}) + i\pi(0, \vec{x})]$$

$$[\phi(t, \vec{x}), \pi(t, \vec{x}')] = i\delta(\vec{x} - \vec{x}') \quad [\phi(x), \phi^\dagger(y)] = i\Delta(x - y; m^2)$$

$$[\phi(t, \vec{x}), \phi(t, \vec{x}')] = [\phi, \phi^\dagger] = [\pi, \pi] = [\pi, \pi^\dagger] = [\phi, \pi^\dagger] = 0$$

$$[a(\vec{p}), a^\dagger(\vec{p}')] = [b(\vec{p}), b^\dagger(\vec{p}')] = (2\pi)^3 2E \delta(\vec{p} - \vec{p}')$$

$$[a, a] = [a, b] = [a, b^\dagger] = [b, b] = 0$$

$$H = \int \frac{d^3 p}{2(2\pi)^3} [a^\dagger(\vec{p}) a(\vec{p}) + b(\vec{p}) b^\dagger(\vec{p})] = \int \frac{d^3 p}{2(2\pi)^3} [a^\dagger(\vec{p}) a(\vec{p}) + b^\dagger(\vec{p}) b(\vec{p})] + \int \frac{d^3 p d^3 x}{(2\pi)^3} E$$

#### Noether's theorem

$$\delta S = \int d^4 x \sum_i \partial_\mu \left[ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta \phi_i \right]$$

$$J^\mu = \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta \phi_i \quad \Longrightarrow \quad \partial_\mu J^\mu = 0$$

$$Q \equiv \int d^3 x J^0 \quad \frac{dQ}{dt} = 0$$

e.g.  $\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi \quad \phi \rightarrow \phi' = e^{i\alpha} \phi \rightarrow (1 + i\alpha) \phi \quad \delta \phi = i\alpha \phi$

$$J^\mu = i[\phi^\dagger(\partial_\mu \phi) - (\partial_\mu \phi^\dagger)\phi]$$

$$Q = i \int d^3 x [\phi^\dagger(\partial_t \phi) - (\partial_t \phi^\dagger)\phi] = \int \frac{d^3 p}{(2\pi)^3 2E} [a^\dagger(\vec{p}) a(\vec{p}) - b(\vec{p}) b^\dagger(\vec{p})]$$

$$= \int \frac{d^3 p}{(2\pi)^3 2E} [a^\dagger(\vec{p}) a(\vec{p}) - b^\dagger(\vec{p}) b(\vec{p})] - \int \frac{d^3 p d^3 x}{(2\pi)^3}$$

#### Quantization of Electro-magnetic Field ( Lorenz gauge $\partial_\mu A^\mu = 0$ )

$$A^\mu(x) = \int \frac{d^3 k}{(2\pi)^3 2\omega} \sum_\lambda [a_\lambda(\vec{k}) \varepsilon_\lambda^\mu(\vec{k}) e^{-ikx} + a_\lambda^\dagger(\vec{k}) \varepsilon_\lambda^{\mu*}(\vec{k}) e^{ikx}]$$

$$k^2 = 0 \quad k_\mu \varepsilon^\mu = k_\mu \varepsilon^{\mu*} = 0 \quad \lambda = \pm \text{ for real photon}$$

$$[a_\lambda(\vec{k}), a_\rho^\dagger(\vec{k}')] = (2\pi)^3 2\omega (-g^{\lambda\rho}) \delta(\vec{k} - \vec{k}') \quad [a, a] = [a^\dagger, a^\dagger] = 0$$

$$H = \int \frac{d^3 k}{2(2\pi)^3} \sum_{\lambda=\pm} a_\lambda^\dagger(\vec{k}) a_\lambda(\vec{k}) + \sum_{\lambda=\pm} \int \frac{d^3 k d^3 x \omega}{(2\pi)^3 2}$$

#### Polarization Vector of Spin 1 Particle

$$\varepsilon^\mu(\lambda = +) = \frac{1}{\sqrt{2}}(0, -1, -i, 0) \quad \lambda : \text{helicity on traveling along the } z\text{-axis}$$

$$\varepsilon^\mu(\lambda = -) = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$$

$$\varepsilon^\mu(\lambda = 0) = (0, 0, 0, 1) \quad \varepsilon^\mu(\lambda = s) = (1, 0, 0, 0)$$

$$\varepsilon_\mu(\lambda)\varepsilon^{\mu*}(\rho) = g_{\lambda\rho} \quad (\lambda, \rho = s, +, -, 0)$$

$$p^\mu = (E, 0, 0, p) \Rightarrow \varepsilon^\mu(p; \lambda = 0) = (p, 0, 0, E)/M \quad M^2 = E^2 - p^2$$

$$\varepsilon^\mu(p; \lambda = s) = (E, 0, 0, p)/M = p^\mu/M$$

$$\varepsilon^\mu(p; \lambda = \pm) = \varepsilon^\mu(\lambda = \pm)$$

$$\sum_{\lambda=\pm,0} \varepsilon^\mu(p; \lambda)\varepsilon^{\nu*}(p; \lambda) = -g^{\mu\nu} + p^\mu p^\nu/M^2$$

### $\gamma$ matrixes

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad 1 - \gamma^5 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad g_{\mu\nu}g^{\mu\nu} = 4$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (\gamma^5)^2 = 1 \quad \{\gamma^5, \gamma^\mu\} = 0 \quad (1 \pm \gamma^5)^2 = 2(1 \pm \gamma^5)$$

$$\gamma^{0\dagger} = \gamma^0 \quad \gamma^{i\dagger} = -\gamma^i \quad \gamma^{5\dagger} = \gamma^5 \quad \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$$

$$\gamma_\mu\gamma^\mu = 4 \quad \gamma_\mu\not{a}\gamma^\mu = -2\not{a} \quad \gamma_\mu\not{a}\not{b}\gamma^\mu = 4a \cdot b \quad \gamma_\mu\not{a}\not{b}\not{c}\gamma^\mu = -2\not{c}\not{a}\not{b}$$

$$\text{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu} \quad \text{Tr}[ABC] = \text{Tr}[BCA] \quad \text{Tr}[S^{-1}AS] = \text{Tr}A \quad \text{Tr}\mathbf{1} = 4$$

$$\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4[g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}]$$

$$\text{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\varepsilon^{\mu\nu\rho\sigma} \quad (\varepsilon^{0123} = -\varepsilon_{0123} = 1 \text{ convention})$$

$$\text{Tr}[\gamma^1\gamma^2 \dots \gamma^{2n+1}] = 0$$

$$\text{Tr}[\gamma^5] = \text{Tr}[\gamma^5\gamma^\mu] = \text{Tr}[\gamma^5\gamma^\mu\gamma^\nu] = \text{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho] = 0$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu\gamma\delta} = -2(g^{\rho\gamma}g^{\sigma\delta} - g^{\rho\delta}g^{\sigma\gamma})$$

$$\gamma^\mu\gamma^\nu\gamma^\rho = g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu - i\gamma^5\varepsilon^{\mu\nu\rho\sigma}\gamma_\sigma$$

### Dirac Field

$$\mathcal{L} = \bar{\psi}(\gamma^\mu i\overleftrightarrow{\partial}_\mu - m)\psi = \frac{1}{2}[\bar{\psi}\gamma^\mu(i\partial_\mu\psi) - (i\partial_\mu\bar{\psi})\gamma^\mu\psi] - m\bar{\psi}\psi \quad \bar{\psi} \equiv \psi^\dagger\gamma^0$$

$$(\gamma^\mu i\partial_\mu - m)\psi(x) = 0$$

$$\mathcal{L}(\psi, \partial_\mu\psi) = \bar{\psi}(x)(\gamma^\mu i\partial_\mu - m)\psi(x) : \text{Use this hereafter}$$

$$\pi = \frac{\partial\mathcal{L}}{\partial[\partial_t\psi]} = i\psi^\dagger$$

$$\mathcal{H} = \pi\partial_t\psi - \mathcal{L} = -\bar{\psi}\gamma^i i\partial_i\psi + m\bar{\psi}\psi = \psi^\dagger i\partial_i\psi$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{s=1,2} [c_s(\vec{p})u_s(\vec{p})e^{-ipx} + d_s^\dagger(\vec{p})v_s(\vec{p})e^{ipx}]$$

$$\bar{\psi}(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{s=1,2} [c_s^\dagger(\vec{p})\bar{u}_s(\vec{p})e^{ipx} + d_s(\vec{p})\bar{v}_s(\vec{p})e^{-ipx}]$$

$$\begin{aligned}
u_s(\vec{p}) &= N \begin{pmatrix} \phi_s \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \phi_s \end{pmatrix} & v_s(\vec{p}) &= N \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi_s \\ \chi_s \end{pmatrix} & N &= \sqrt{E+m} \\
c_s(\vec{p}) &= \int d^3x e^{-i\vec{p} \cdot \vec{x}} u_s^\dagger(\vec{p}) \psi(0, \vec{x}) & d_s(\vec{p}) &= \int d^3x e^{-i\vec{p} \cdot \vec{x}} \psi^\dagger(0, \vec{x}) v_s(\vec{p}) \\
\{\psi_\alpha(t, \vec{x}), \psi_\beta^\dagger(t, \vec{x}')\} &= \delta_{\alpha\beta} \delta(\vec{x} - \vec{x}') & \alpha, \beta &= 1 \sim 4: \text{Dirac spinor suffix} \\
\{\psi, \psi\} &= \{\psi^\dagger, \psi^\dagger\} = 0 \\
\{c_s(\vec{p}), c_r^\dagger(\vec{p}')\} &= \{d_s(\vec{p}), d_r^\dagger(\vec{p}')\} = (2\pi)^3 2E \delta_{sr} \delta(\vec{p} - \vec{p}') & s, r &= 1, 2 \\
\{c, c\} &= \{c, d\} = \{c, d^\dagger\} = \{d, d\} = 0 \\
H &= \int \frac{d^3p}{2(2\pi)^3} \sum_s [c_s^\dagger(\vec{p}) c_s(\vec{p}) - d_s(\vec{p}) d_s^\dagger(\vec{p})] \\
&= \int \frac{d^3p}{2(2\pi)^3} \sum_s [c_s^\dagger(\vec{p}) c_s(\vec{p}) + d_s^\dagger(\vec{p}) d_s(\vec{p})] - \sum_s \int \frac{d^3p d^3x}{(2\pi)^3} E \\
J^\mu &= q \bar{\psi} \gamma^\mu \psi & \partial_\mu J^\mu &= 0 \\
Q &= \int d^3x J^0 = q \int d^3x \psi^\dagger \psi = q \int \frac{d^3p}{(2\pi)^3 2E} \sum_s [c_s^\dagger(\vec{p}) c_s(\vec{p}) + d_s(\vec{p}) d_s^\dagger(\vec{p})] \\
&= q \int \frac{d^3p}{(2\pi)^3 2E} \sum_s [c_s^\dagger(\vec{p}) c_s(\vec{p}) - d_s^\dagger(\vec{p}) d_s(\vec{p})] + q \sum_s \int \frac{d^3p d^3x}{(2\pi)^3}
\end{aligned}$$

### Dirac Spinor

$$\begin{aligned}
\phi_\uparrow(z) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \phi_\downarrow(z) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \chi_\uparrow(z) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \chi_\downarrow(z) &= \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\
\phi_\uparrow(\theta, \varphi) &= \begin{pmatrix} e^{-i\varphi/2} \cos \frac{\theta}{2} \\ e^{i\varphi/2} \sin \frac{\theta}{2} \end{pmatrix} & \phi_\downarrow(\theta, \varphi) &= \begin{pmatrix} -e^{-i\varphi/2} \sin \frac{\theta}{2} \\ e^{i\varphi/2} \cos \frac{\theta}{2} \end{pmatrix} & & & & \text{: particle} \\
\chi_\uparrow(\theta, \varphi) &= \begin{pmatrix} -e^{-i\varphi/2} \sin \frac{\theta}{2} \\ e^{i\varphi/2} \cos \frac{\theta}{2} \end{pmatrix} & \chi_\downarrow(\theta, \varphi) &= \begin{pmatrix} -e^{-i\varphi/2} \cos \frac{\theta}{2} \\ -e^{i\varphi/2} \sin \frac{\theta}{2} \end{pmatrix} & & & & \text{: anti-particle} \\
\sigma(\theta, \varphi) \phi_\uparrow(\theta, \varphi) &= \phi_\uparrow(\theta, \varphi) & \sigma(\theta, \varphi) \phi_\downarrow(\theta, \varphi) &= -\phi_\downarrow(\theta, \varphi) \\
\sigma(\theta, \varphi) &\equiv \sin\theta \cos\varphi \sigma_1 + \sin\theta \sin\varphi \sigma_2 + \cos\theta \sigma_3 = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix} \\
u_s^\dagger(\vec{p}) u_r(\vec{p}) &= v_s^\dagger(\vec{p}) v_r(\vec{p}) = 2E \delta_{sr} & u_s^\dagger(\vec{p}) v_r(-\vec{p}) &= v_s^\dagger(\vec{p}) u_r(-\vec{p}) = 0 \\
\bar{u}_s(\vec{p}) u_r(\vec{p}) &= -\bar{v}_s(\vec{p}) v_r(\vec{p}) = 2m \delta_{sr} & \bar{u}_s(\vec{p}) v_r(\vec{p}) &= \bar{v}_s(\vec{p}) u_r(\vec{p}) = 0 \\
\sum_{s=1,2} u_s(p) \bar{u}_s(p) &= \not{p} + m & \sum_{s=1,2} v_s(p) \bar{v}_s(p) &= \not{p} - m \\
(\not{p} - m)u &= 0 & (\not{p} + m)v &= 0 & \bar{u}(\not{p} - m) &= 0 & \bar{v}(\not{p} + m) &= 0
\end{aligned}$$

### Rotation of 2 component spinor

$$\begin{aligned}
R_i(\theta) &= e^{-i\sigma_i \theta/2} & (\text{Rotation of coordinates: } \tilde{R}_i(\theta) &= e^{i\sigma_i \theta/2}) \\
R_1 &= \begin{pmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} & R_2 &= \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} & R_3 &= \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}
\end{aligned}$$

$$R(\theta, \varphi) = R_3(\varphi)R_2(\theta) = \begin{pmatrix} e^{-i\varphi/2}\cos\frac{\theta}{2} & -e^{-i\varphi/2}\sin\frac{\theta}{2} \\ e^{i\varphi/2}\sin\frac{\theta}{2} & e^{i\varphi/2}\cos\frac{\theta}{2} \end{pmatrix}$$

Magnetic moment of dirac particle (gyromagnetic ratio)

$$qe\bar{\psi}\gamma^\mu\psi A_\mu \rightarrow qe\bar{u}_s(p)u_s(p)\frac{p^\mu}{m}A_\mu(x) - \frac{qe}{2m}\bar{u}_s(p)\sigma^{\mu\nu}u_s(p)\partial_\nu A_\mu(x)$$

on conditions of  $|\vec{p}| \ll E \sim m$ ,  $u_s \sim N \begin{pmatrix} \phi_s \\ \vec{\sigma} \cdot \vec{v}\phi_s/2 \end{pmatrix}$ ,  $N = 1$

$$\rightarrow qe(\phi - \vec{v} \cdot \vec{A}) - \frac{qe}{m}\vec{s} \cdot (\vec{B} - \vec{v} \times \vec{E}), \text{ where } \vec{s} \equiv \frac{1}{2}\phi_s^\dagger \vec{\sigma} \phi_s$$

$$\vec{\mu} \equiv g\frac{qe}{2m}\vec{s} = gq\mu_B\vec{s} \quad g = 2 \quad \mu_B \equiv \frac{e}{2m}: \text{ Bohr magneton}$$

**Covariance of Dirac spinor**

$$x^\mu \rightarrow x'^\mu = a^\mu{}_\nu x^\nu$$

$$\psi(x) \rightarrow \psi'(x') = S(a)\psi(x) \quad \bar{\psi}(x) \rightarrow \bar{\psi}(x)S(a)^{-1}$$

$$S^{-1}\gamma^\mu S = a^\mu{}_\nu \gamma^\nu \quad S^{-1} = \gamma^0 S^\dagger \gamma^0$$

boost with 4 momentum  $p^\mu = (E, \vec{p})$ ,  $m = \sqrt{p^2}$

$$S(\vec{p}) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 & \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} & 1 \end{pmatrix} \quad \vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_3 & p_1 - ip_2 \\ p_1 + ip_2 & -p_3 \end{pmatrix}$$

boost with  $\hat{n}$ ,  $\cosh\alpha = \gamma$

$$S(\alpha) = \begin{pmatrix} \cosh\frac{\alpha}{2} & \hat{n} \cdot \vec{\sigma} \sinh\frac{\alpha}{2} \\ \hat{n} \cdot \vec{\sigma} \sinh\frac{\alpha}{2} & \cosh\frac{\alpha}{2} \end{pmatrix} = \sqrt{\frac{\gamma+1}{2}} \begin{pmatrix} 1 & \hat{n} \cdot \vec{\sigma} \frac{\gamma\beta}{\gamma+1} \\ \hat{n} \cdot \vec{\sigma} \frac{\gamma\beta}{\gamma+1} & 1 \end{pmatrix}$$

charge conjugation of Dirac spinor

$$C = i\gamma^2\gamma^0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}$$

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu \quad C\{\gamma^\mu(1-\gamma^5)\}^T C^{-1} = -\gamma^\mu(1+\gamma^5)$$

$$C = C^* = -C^{-1} = -C^T = -C^\dagger$$

$$\psi^C = C\bar{\psi}^T = i\gamma^2\psi^* \quad \bar{\psi}^C = -\psi^T C^{-1} = -i\bar{\psi}^*\gamma^2$$

$$u = \begin{pmatrix} \phi_s \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m}\phi_s \end{pmatrix} \rightarrow u^C = i\gamma^2 u^* = \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E+m}\chi_s \\ \chi_s \end{pmatrix}, \text{ where } \chi_s = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \phi_s^*$$

$$v = \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E+m}\chi_s \\ \chi_s \end{pmatrix} \rightarrow v^C = i\gamma^2 v^* = \begin{pmatrix} \phi_s \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m}\phi_s \end{pmatrix}, \text{ where } \phi_s = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \chi_s^*$$

parity transformation of Dirac spinor

$$P = \eta_P \gamma^0 = \eta_P \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad P^2 = 1 \quad \eta_P^2 = 1 \quad P^{-1} = \eta_P \gamma^0$$

$$P^{-1}\gamma^\mu P = (\gamma^0, -\gamma^i) = \gamma_\mu$$

$$Pu_s(\vec{p}) = u_s(-\vec{p}) \quad Pv_s(\vec{p}) = -v_s(-\vec{p}) \quad \eta_P \equiv 1$$

$$P_L = \frac{1 - \gamma^5}{2} \quad P_R = \frac{1 + \gamma^5}{2}$$

$$P_L + P_R = 1 \quad P_{L,R}^2 = P_{L,R} \quad P_L P_R = P_R P_L = 0 \quad P^{-1} P_{L,R} P = P_{R,L}$$

Formula

$$\begin{pmatrix} u \uparrow \sigma_1 v \uparrow & u \uparrow \sigma_2 v \uparrow & u \uparrow \sigma_3 v \uparrow \\ u \uparrow \sigma_1 v \downarrow & u \uparrow \sigma_2 v \downarrow & u \uparrow \sigma_3 v \downarrow \\ u \downarrow \sigma_1 v \uparrow & u \downarrow \sigma_2 v \uparrow & u \downarrow \sigma_3 v \uparrow \\ u \downarrow \sigma_1 v \downarrow & u \downarrow \sigma_2 v \downarrow & u \downarrow \sigma_3 v \downarrow \end{pmatrix} = \begin{pmatrix} \cos\theta \cos\varphi + i \sin\varphi & \cos\theta \sin\varphi - i \cos\varphi & -\sin\theta \\ -\sin\theta \cos\varphi & -\sin\theta \sin\varphi & -\cos\theta \\ -\sin\theta \cos\varphi & -\sin\theta \sin\varphi & -\cos\theta \\ -\cos\theta \cos\varphi + i \sin\varphi & -\cos\theta \sin\varphi - i \cos\varphi & \sin\theta \end{pmatrix}$$

$$\text{where } u \uparrow(\theta, \varphi) = -v \downarrow = \begin{pmatrix} e^{-i\varphi/2} \cos \frac{\theta}{2} \\ e^{i\varphi/2} \sin \frac{\theta}{2} \end{pmatrix}, \quad u \downarrow = v \uparrow = \begin{pmatrix} -e^{-i\varphi/2} \sin \frac{\theta}{2} \\ e^{i\varphi/2} \cos \frac{\theta}{2} \end{pmatrix}$$

**Chirality(Weyl) Representation**

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\psi_L \equiv P_L \psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \quad \psi_R \equiv P_R \psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \quad P_L \gamma^\mu = \gamma^\mu P_R$$

$$\bar{\psi}_L = \bar{\psi} P_R \quad \bar{\psi}_L \psi_L = \bar{\psi}_R \psi_R = 0 \quad \bar{\psi}_L \gamma^\mu \psi_R = \bar{\psi}_R \gamma^\mu \psi_L = 0$$

$$\psi_D = T \psi_W \quad \gamma_D = T \gamma_W T^{-1} \quad T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad T^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$S_W(\vec{p}) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 - \frac{\vec{\sigma} \cdot \vec{p}}{E+m} & 0 \\ 0 & 1 + \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \end{pmatrix}$$

**Bilinear Expression**

$\bar{\psi} \psi$	$\bar{\psi} \gamma^5 \psi$	$\bar{\psi} \gamma^\mu \psi$	$\bar{\psi} \gamma^\mu \gamma^5 \psi$	$\bar{\psi} \sigma^{\mu\nu} \psi$
(S)	(P)	(V)	(A)	(T)
1	1	4	4	6

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] = i [\gamma^\mu \gamma^\nu - g^{\mu\nu}]$$

$$\bar{u}(p') \gamma^\mu u(p) = \frac{1}{m+m'} \bar{u}(p') [(p+p')^\mu - i \sigma^{\mu\nu} q_\nu] u(p) \quad q \equiv p - p'$$

**Propagator**

$$i \Delta_F(x' - x) = \langle 0 | \mathcal{T}(\phi(x') \phi^\dagger(x)) | 0 \rangle$$

$$= \theta(t' - t) \langle 0 | \phi(x') \phi^\dagger(x) | 0 \rangle \pm \theta(t - t') \langle 0 | \phi^\dagger(x) \phi(x') | 0 \rangle$$

boson  
fermion

$$\theta(t) = \frac{-1}{2\pi i} \int dk^0 \frac{e^{-ik^0 t}}{k^0 + i\varepsilon}$$

$$i \Delta_F(x' - x) = i \int \frac{d^4 k}{(2\pi)^4} i \tilde{\Delta}_F(k) e^{-ik(x' - x)}$$

$$\text{Klein-Gordon: } i \tilde{\Delta}_F(p) = \frac{i}{p^2 - m^2 + i\varepsilon}$$

$$\text{Gauge field: } i\tilde{D}_F(p) = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\varepsilon}$$

$$\text{Dirac field: } i\tilde{S}_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

### Local Gauge Transformation: scalar field with U(1)

$$D_\mu = \partial_\mu + iqA_\mu \quad (p^\mu \rightarrow p^\mu - qA^\mu)$$

$$\begin{aligned} \mathcal{L} &= (D^\mu\phi)^\dagger(D_\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &= (\partial^\mu\phi)^\dagger(\partial_\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j_\mu A^\mu \end{aligned} \quad j_\mu = qJ_\mu = iq[\phi^\dagger\partial_\mu\phi - (\partial_\mu\phi)^\dagger\phi]$$

### Gauge Field

$$U(1): e^{ig_1\beta(x)\cdot Y/2} \quad B_\mu : \text{Abelian}$$

$$SU(2): e^{ig_2\vec{\alpha}(x)\cdot\vec{\tau}/2} \quad W_\mu : \text{non-Abelian} \quad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = if_{abc}\frac{\tau_c}{2}$$

$$D_\mu = \partial_\mu + ig_1\frac{Y}{2}B_\mu + ig_2\frac{\tau_a}{2}W_{a\mu}$$

$$B_\mu \rightarrow B'_\mu = B_\mu - \partial_\mu\beta \quad W_{a\mu} \rightarrow W'_{a\mu} = W_{a\mu} - \partial_\mu\alpha_a + g_2f_{abc}W_{b\mu}\alpha_c$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \quad W_{a\mu\nu} \equiv \partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} - g_2f_{abc}W_{b\mu}W_{c\nu}$$

### SU(3)

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if_{abc}\frac{\lambda_c}{2} \quad \text{Tr}(\lambda_a) = 0 \quad \text{Tr}(\lambda_a\lambda_b) = 2\delta_{ab} \quad f_{abc} = \frac{1}{4i}\text{Tr}([\lambda_a, \lambda_b]\lambda_c)$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$f_{123} = 1$$

$$f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

$$\text{Jacobi identity } f_{abc}f_{ckl} + f_{bkc}f_{cal} + f_{kac}f_{cbl} = 0$$

### S-Matrix

$$H_0|n, t\rangle = i\frac{\partial}{\partial t}|n, t\rangle \quad |n, t\rangle = U_0(t)|n\rangle$$

$$(H_0 + V)|\psi, t\rangle = i\frac{\partial}{\partial t}|\psi, t\rangle \quad |\psi, t\rangle = U(t)|\psi\rangle \quad V(t) = -\int d^3x \mathcal{L}_{\text{int}}(x)$$

$$U(t) = U_0(t)\mathcal{T}\exp\left[-i\int_{t_0}^t dt' U_0^\dagger(t') V(t') U_0(t')\right] U_0^\dagger(t_0)$$

$$S \equiv U(\infty)U^\dagger(-\infty) = U_0(\infty)\mathcal{T}\exp\left[i\int d^4x' U_0^\dagger(t') \mathcal{L}_{\text{int}}(x') U_0(t')\right] U_0^\dagger(-\infty)$$



$$\begin{aligned}
T_{fi} &\equiv \langle f, \infty | S | i, -\infty \rangle = \left\langle f \left| \mathcal{T} \exp \left[ i \int d^4x' U_0^\dagger(t') \mathcal{L}_{\text{int}}(x') U_0(t') \right] \right| i \right\rangle \\
&= \delta_{fi} + i \int d^4x' \langle f, t' | \mathcal{L}_{\text{int}}(x') | i, t' \rangle \\
&\quad + \frac{i^2}{2!} \int d^4x' \int d^4x'' \sum_n \mathcal{T} [\langle f, t' | \mathcal{L}_{\text{int}}(x') | n, t' \rangle \langle n, t'' | \mathcal{L}_{\text{int}}(x'') | i, t'' \rangle] + \dots \\
&\equiv \delta_{fi} - i(2\pi)^4 \delta^4(p_f - p_i) \mathcal{M}_{fi}
\end{aligned}$$

### Matrix Element for $-i\mathcal{M}$

$$\begin{aligned}
|\bar{\psi}_f \Gamma \psi_i|^2 &= \bar{\psi}_f \Gamma \psi_i \bar{\psi}_i \gamma^0 \Gamma^\dagger \gamma^0 \psi_f = \text{Tr}[\Gamma \psi_i \bar{\psi}_i \gamma^0 \Gamma^\dagger \gamma^0 \psi_f \bar{\psi}_f] \\
X &= (x_i), Y = (y_i) \implies X^T Y = \text{Tr}[Y X^T] \\
\gamma^0 \gamma^{\mu\dagger} \gamma^0 &= \gamma^\mu \quad \gamma^0 (1 - \gamma^5)^\dagger \gamma^0 = 1 + \gamma^5 \quad \gamma^0 (\psi_i \bar{\psi}_j)^\dagger \gamma^0 = \psi_j \bar{\psi}_i
\end{aligned}$$

### External Lines

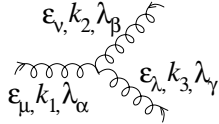
$$\begin{aligned}
\text{spin } 1/2 &\quad \text{incoming: } u(p, s) \text{ or } v(p, s) \quad \text{outgoing: } \bar{u}(p, s) \text{ or } \bar{v}(p, s) \\
\text{spin } 1 &\quad \text{incoming: } \varepsilon_\mu(p, \lambda) \quad \text{outgoing: } \varepsilon_\mu^*(p, \lambda)
\end{aligned}$$

### Internal Lines

$$\begin{aligned}
\text{spin } 0 &\quad \frac{i}{p^2 - m^2 + i\varepsilon} \\
\text{spin } 1/2 &\quad \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon} \\
\text{spin } 1 &\quad \frac{i}{k^2 + i\varepsilon} \left[ -g^{\mu\nu} + (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right] \quad \xi = 1 \text{ for Feynman gauge} \\
&\quad \frac{i}{p^2 - m^2 + i\varepsilon} \left( -g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2} \right) \quad \text{Unitary gauge} \\
&\quad \frac{i}{p^2 - m^2 + i\varepsilon} \left[ -g^{\mu\nu} + (1 - \xi) \frac{p^\mu p^\nu}{p^2 - \xi m^2} \right] \quad \text{R gauge}
\end{aligned}$$

### Vertexes

$$\begin{aligned}
\text{EM current for charge } +e &\quad -ie\gamma^\mu \\
\text{EW charged current} &\quad -i \frac{g}{\sqrt{2}} \gamma^\mu \frac{1 - \gamma^5}{2} \\
\text{3 gluons vertex} &\quad -g_s f_{\alpha\beta\gamma} [g^{\mu\nu}(k_1 - k_2)^\lambda + g^{\nu\lambda}(k_2 - k_3)^\mu + g^{\lambda\mu}(k_3 - k_1)^\nu]
\end{aligned}$$



$$\begin{aligned}
&\gamma(k_\gamma, \varepsilon_\mu) - W^+(k_+, \varepsilon_\nu) - W^-(k_-, \varepsilon_\lambda) \text{ vertex} \\
&\quad ie [g^{\nu\lambda}(k_+ - k_-)^\mu + g^{\lambda\mu}(k_- - k_\gamma)^\nu + g^{\mu\nu}(k_\gamma - k_+)^\lambda]
\end{aligned}$$

### Conservation law

Q num \ Int.	Strong	EM	Weak
Isospin $I$	0	X	X
$I_3$	0	0	X
Parity $P$	0	0	X
C-parity $C$	0	0	X
G-parity $G$	0	X	X