Lorentz vector

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & \\ & \mathbf{-1} \end{pmatrix}$$

$$x^\mu = (c\,t\,, \vec{x}\,) \qquad \partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \qquad p^\mu = \left(\frac{E}{c}, \vec{p}\right)$$

$$k^\mu = \left(\frac{\omega}{c}, \vec{k}\right) \qquad j^\mu = \left(c\rho\,, \vec{j}\,\right) \qquad A^\mu = \left(\frac{\phi}{c}, \vec{A}\right)$$

Lorentz scalar: $d^4x = d\,x^0 d\,x^1 d\,x^2 d\,x^3 \qquad \dfrac{d^3p}{E/c} \qquad \dfrac{d^3k}{\omega/c}$

$$p^\mu = i\partial^\mu \qquad E = i\frac{\partial}{\partial t} \qquad \vec{p} = -i\nabla$$

Electro-Magnetic Field

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

$$-\frac{1}{4\,\mu_0}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2\mu_0}\left(\frac{\vec{E}^{\,2}}{c^2} - \vec{B}^{\,2}\right) = \frac{1}{2}(\varepsilon_0\vec{E}^{\,2} - \mu_0\vec{H}^{\,2}) \qquad \frac{1}{\mu_0\varepsilon_0} = c^2$$

$$\mathcal{L} = -\frac{1}{4\,\mu_0}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$$

$$\partial_\mu F^{\mu\nu} = \Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = \mu_0 j^\mu \qquad \partial_\mu j^\mu = 0$$

$$\mathcal{H} = \pi^\mu(\partial_0 A_\mu) - \mathcal{L} = \frac{1}{2}(\varepsilon_0\vec{E}^{\,2} + \mu_0\vec{H}^{\,2}) + \varepsilon_0\vec{E}\cdot\nabla\phi$$

$$\pi^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_0 A_\mu)} = -\frac{1}{\mu_0}F^{0\mu} = -\frac{1}{\mu_0}\left(0, \varepsilon_0 c\vec{E}\,\right) \qquad \partial_t A_\mu = \left(-\nabla\cdot\vec{A}, \frac{1}{c}\left(\vec{E} + \nabla\phi\right)\right)$$

Gauge transformation

$$A^\mu \to A'^\mu = A^\mu - \partial^\mu\Lambda$$

$$\partial_\mu A^\mu = 0 \qquad \Longrightarrow \qquad \Box\Lambda = \partial_\mu A^\mu \qquad : \text{Lorenz gauge}$$

$$\nabla\cdot\vec{A} = 0 \qquad \Longrightarrow \qquad \Delta\Lambda = -\nabla\cdot\vec{A} \qquad : \text{Coulomb gauge}$$

Heaviside-Lorentz unit: $\varepsilon_0 = \mu_0 = 1$ Natural unit: $\hbar = c = 1$ $\dfrac{e^2}{4\pi} = \dfrac{1}{137}$

Euler Equation

$$S = \int\mathcal{L}(\phi, \partial_\mu\phi)d^4x = \int\mathcal{L}\,d^3x\,dt = \int L dt \qquad \text{action}$$

$$\delta S = 0 \qquad \Longrightarrow \qquad \frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = 0 \qquad \text{Least action principle}$$

Proca Equation

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu - j^\mu A_\mu$$

$$\frac{\partial\mathcal{L}}{\partial A_\mu} - \partial_\nu\frac{\partial\mathcal{L}}{\partial(\partial_\nu A_\mu)} = 0 \Rightarrow \Box A^\mu - \partial^\mu(\partial_\nu A^\nu) + m^2 A^\mu = j^\mu \Rightarrow m^2\partial_\mu A^\mu = \partial_\mu j^\mu = 0$$

$$\Rightarrow (\Box + m^2)A^\mu = j^\mu$$

Fock Space

$\hat{a}\,,\hat{a}^{\dagger}$: rising and lowering operator $\qquad [\hat{a}\,,\hat{a}^{\dagger}]=1$

$\hat{N}=\hat{a}^{\dagger}\hat{a}$: number operator $\qquad \hat{N}\,|n\rangle = n|n\rangle$

$\hat{a}^{\dagger}|n\rangle=\sqrt{n+1}|n+1\rangle \qquad \hat{a}\,|n\rangle=\sqrt{n}|n-1\rangle \qquad \hat{a}\,|0\rangle=0$

$$|n\rangle=\frac{1}{\sqrt{n!}}(\hat{a}^{\dagger})^n|0\rangle$$

e.g. $[\hat{x}\,,\hat{p}]=i \qquad \hat{p}\,,\hat{x}$: Hermitian

$$\hat{a}=\frac{1}{\sqrt{2m\omega}}(m\omega\hat{x}+i\hat{p}) \qquad \hat{a}^{\dagger}=\frac{1}{\sqrt{2m\omega}}(m\omega\hat{x}-i\hat{p}) \qquad [\hat{a}\,,\hat{a}^{\dagger}]=1$$

$$\hat{H}=\frac{1}{2m}\hat{p}^2+\frac{1}{2}m\omega^2\hat{x}^2=\omega\left(\hat{a}^{\dagger}\hat{a}+\frac{1}{2}\right)$$

Invariant Delta Function

$$\begin{aligned}\Delta(x;m^2) &\equiv \frac{1}{i(2\pi)^3}\int d^4p\,\epsilon(p_0)\delta(p^2-m^2)e^{-ipx}\\ &=\frac{1}{i(2\pi)^3}\int d^3p\frac{1}{2\sqrt{m^2+\boldsymbol{p}^2}}[e^{-ipx}-e^{ipx}]\\ &=-\frac{1}{2\pi}\,\epsilon(x_0)\left[\delta(x^2)-\frac{m^2}{2}\theta(x^2)\frac{J_1\left(m\sqrt{x^2}\right)}{m\sqrt{x^2}}\right]\end{aligned} \qquad \begin{cases}\epsilon(u)\equiv\operatorname{sgn}u\equiv u/|u|\\ \epsilon(0)=0\end{cases}$$

$\Delta(-x;m^2)=-\Delta(x;m^2) \qquad x^2>0$: timelike $\qquad x^2<0$: spacelike

Canonical Quantization

$$\pi_i=\frac{\partial\mathcal{L}(\phi_i,\partial\phi_i)}{\partial[\partial_t\phi_i]} \qquad \mathcal{H}=\pi_i\partial_t\phi_i-\mathcal{L} \qquad H=\int d^3x\mathcal{H}$$

$$i\partial_t\phi=[\phi,H] \qquad i\partial_t\pi(x)=[\pi,H]$$

$T^{\mu\nu}=\dfrac{\partial\mathcal{L}}{\partial[\partial_\mu\phi_i]}\partial^\nu\phi_i-g^{\mu\nu}\mathcal{L}$: Energy-momentum tensor

$$T^{00}=\mathcal{H} \qquad p_i=\int d^3xT^{0i} \qquad \partial_\mu T^{\mu\nu}=0$$

Neutral Scalar Field

$$\mathcal{L}=\frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi-\frac{1}{2}\,m^2\phi^2 \qquad (\Box+m^2)\phi(x)=0$$

$$\pi=\frac{\partial\mathcal{L}}{\partial[\partial_t\phi]}=\partial_t\phi$$

$$\mathcal{H}=\pi\partial_t\phi-\mathcal{L}=\frac{1}{2}[(\partial_t\phi)^2+(\nabla\phi)^2+m^2\phi^2]$$

$[\phi(t,\vec{x}\,),\pi(t,\vec{x}\,')]=i\delta(\vec{x}-\vec{x}\,')$ $\qquad$ canonical quantization

$$[\phi(t,\vec{x}\,),\phi(t,\vec{x}\,')]=[\pi(t,\vec{x}\,),\pi(t,\vec{x}\,')]=0$$

$$\phi(x)=\int\frac{d^3p}{(2\pi)^3\,2E}[a(\vec{p}\,)e^{-ipx}+a^{\dagger}(\vec{p}\,)e^{ipx}]$$

$$a(\vec{p}\,)=\int d^3xe^{-i\vec{p}\cdot\vec{x}}[E\phi(0,\vec{x}\,)+i\pi(0,\vec{x}\,)]$$

$$H=\int\frac{d^3p}{2(2\pi)^3}\frac{1}{2}[a^{\dagger}(\vec{p}\,)a(\vec{p}\,)+a(\vec{p}\,)a^{\dagger}(\vec{p}\,)]=\int\frac{d^3p}{2(2\pi)^3}\,a^{\dagger}(\vec{p}\,)a(\vec{p}\,)+\frac{1}{2}\delta^3(0)\int d^3pE$$

$$[a(\vec{p}\,),a^{\dagger}(\vec{p}\,')]=(2\pi)^3\,2E\,\delta(\vec{p}-\vec{p}\,') \qquad [a(\vec{p}\,),a(\vec{p}\,')]=0$$

4-dimensional commutation relation

$$[\phi(x),\phi(y)] = i\Delta(x-y;m^2) \qquad \text{invariant delta function}$$

Complex Scalar Field

$$\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi$$

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \qquad \phi^\dagger = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \qquad \phi_1, \phi_2 : \text{Hermitian}$$

$$\pi = \frac{\partial\mathcal{L}}{\partial[\partial_t\phi]} = \partial_t\phi^\dagger \qquad \pi^\dagger = \frac{\partial\mathcal{L}}{\partial[\partial_t\phi^\dagger]} = \partial_t\phi$$

$$\mathcal{H} = \pi\partial_t\phi + (\partial_t\phi^\dagger)\pi^\dagger - \mathcal{L} = \partial_t\phi^\dagger\partial_t\phi + \nabla\phi^\dagger\cdot\nabla\phi + m^2\phi^\dagger\phi$$

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3\,2E}\,[a(\vec{p}\,)e^{-ipx} + b^\dagger(\vec{p}\,)e^{ipx}] \qquad \pi(x) = i\int \frac{d^3p}{(2\pi)^3\,2}\,[a^\dagger(\vec{p}\,)e^{ipx} - b(\vec{p}\,)e^{-ipx}]$$

$$a(\vec{p}\,) = \int d^3x\, e^{-i\vec{p}\cdot\vec{x}}[E\phi(0,\vec{x}\,) + i\pi^\dagger(0,\vec{x}\,)] \qquad b(\vec{p}\,) = \int d^3x\, e^{-i\vec{p}\cdot\vec{x}}[E\phi^\dagger(0,\vec{x}\,) + i\pi(0,\vec{x}\,)]$$

$$[\phi(t,\vec{x}\,),\pi(t,\vec{x}\,')] = i\delta(\vec{x}-\vec{x}\,') \qquad [\phi(x),\phi^\dagger(y)] = i\Delta(x-y;m^2)$$

$$[\phi(t,\vec{x}\,),\phi(t,\vec{x}\,')] = [\phi,\phi^\dagger] = [\pi,\pi] = [\pi,\pi^\dagger] = [\phi,\pi^\dagger] = 0$$

$$[a(\vec{p}\,),a^\dagger(\vec{p}\,')] = [b(\vec{p}\,),b^\dagger(\vec{p}\,')] = (2\pi)^3\,2E\,\delta(\vec{p}-\vec{p}\,')$$

$$[a,a] = [a,b] = [a,b^\dagger] = [b,b] = 0$$

$$H = \int \frac{d^3p}{2(2\pi)^3}[a^\dagger(\vec{p}\,)a(\vec{p}\,) + b(\vec{p}\,)b^\dagger(\vec{p}\,)] = \int \frac{d^3p}{2(2\pi)^3}[a^\dagger(\vec{p}\,)a(\vec{p}\,) + b^\dagger(\vec{p}\,)b(\vec{p}\,)] + \delta^3(0)\int d^3pE$$

Neother's theorem

$$\delta S = \int d^4x \sum_i \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i\right]$$

$$J^\mu = \sum_i \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i \qquad \Longrightarrow \qquad \partial_\mu J^\mu = 0$$

$$Q \equiv \int d^3x\, J^0 \qquad \frac{dQ}{dt} = 0$$

e.g. $\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi \qquad \phi \to \phi' = e^{i\alpha}\phi \to (1+i\alpha)\phi \qquad \delta\phi = i\alpha\phi$

$$J^\mu = i[\phi^\dagger(\partial_\mu\phi) - (\partial_\mu\phi^\dagger)\phi]$$

$$\begin{aligned} Q &= i\int d^3x\,[\phi^\dagger(\partial_t\phi) - (\partial_t\phi^\dagger)\phi] = \int \frac{d^3p}{(2\pi)^3\,2E}\,[a^\dagger(\vec{p}\,)a(\vec{p}\,) - b(\vec{p}\,)b^\dagger(\vec{p}\,)] \\ &= \int \frac{d^3p}{(2\pi)^3\,2E}\,[a^\dagger(\vec{p}\,)a(\vec{p}\,) - b^\dagger(\vec{p}\,)b(\vec{p}\,)] - \delta^3(0)\int d^3p \end{aligned}$$

Quantization of Electro-magnetic Field (Lorentz gauge $\partial_\mu A^\mu = 0$)

$$A^\mu(x) = \int \frac{d^3k}{(2\pi)^3\,2\omega}\sum_\lambda \left[a_\lambda(\vec{k}\,)\varepsilon_\lambda^\mu(\vec{k}\,)e^{-ikx} + a_\lambda^\dagger(\vec{k}\,)\varepsilon_\lambda^{\mu*}(\vec{k}\,)e^{ikx}\right]$$

$$k^2 = 0 \qquad k_\mu\varepsilon^\mu = k_\mu\varepsilon^{\mu*} = 0 \qquad \lambda = \pm \text{ for real photon}$$

$$[a_\lambda(\vec{k}\,),a_\rho^\dagger(\vec{k}\,')] = (2\pi)^3\,2\omega\delta_{\lambda\rho}\delta(\vec{k}-\vec{k}\,') \qquad [a,a] = [a^\dagger,a^\dagger] = 0$$

$$H = \int \frac{d^3k}{2(2\pi)^3}\sum_\lambda a_\lambda^\dagger(\vec{k}\,)a_\lambda(\vec{k}\,) + \sum_\lambda \delta^3(0)\int d^3k\frac{\omega}{2}$$

Polarization Vector of Spin 1 Particle

$$\varepsilon^{\mu}(\lambda=+)=\frac{1}{\sqrt{2}}(0,-1,-i,0) \qquad \lambda : \text{helicity on traveling along the } z\text{-axis}$$

$$\varepsilon^{\mu}(\lambda=-)=\frac{1}{\sqrt{2}}(0,1,-i,0)$$

$$\varepsilon^{\mu}(\lambda=0)=(0,0,0,1) \qquad \varepsilon^{\mu}(\lambda=s)=(1,0,0,0)$$

$$\varepsilon_{\mu}(\lambda)\varepsilon^{\mu*}(\rho)=g^{\lambda\rho} \qquad (\lambda,\rho=s,+,-,0)$$

$$p^{\mu}=(E;0,0,p) \Rightarrow \varepsilon^{\mu}(p;\lambda=0)=(p,0,0,E)/M \qquad M^2=E^2-p^2$$

$$\varepsilon^{\mu}(p;\lambda=s)=(E,0,0,p)/M=p^{\mu}/M$$

$$\varepsilon^{\mu}(p;\lambda=\pm)=\varepsilon^{\mu}(\lambda=\pm)$$

$$\sum_{\lambda=\pm,0}\varepsilon^{\mu}(p;\lambda)\varepsilon^{\nu*}(p;\lambda)=-g^{\mu\nu}+p^{\mu}p^{\nu}/M^2$$

γ matrixes

$$\gamma^0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \qquad \gamma^i=\begin{pmatrix}0 & \sigma_i\\ -\sigma_i & 0\end{pmatrix} \qquad \gamma^5=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad 1-\gamma^5=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$\{\gamma^{\mu},\gamma^{\nu}\}=2\,g^{\mu\nu} \qquad g_{\mu\nu}g^{\mu\nu}=4$$

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\gamma^5=i\,\gamma^0\gamma^1\gamma^2\gamma^3 \qquad (\gamma^5)^2=1 \qquad \{\gamma^5,\gamma^{\mu}\}=0 \qquad (1\pm\gamma^5)^2=2\,(1\pm\gamma^5)$$

$$\gamma^{0\dagger}=\gamma^0 \qquad \gamma^{i\dagger}=-\gamma^i \qquad \gamma^{5\dagger}=\gamma^5 \qquad \gamma^{\mu\dagger}=\gamma^0\gamma^{\mu}\gamma^0$$

$$\gamma_{\mu}\gamma^{\mu}=4 \qquad \gamma_{\mu}\not{a}\gamma^{\mu}=-2\not{a} \qquad \gamma_{\mu}\not{a}\not{b}\gamma^{\mu}=4a\cdot b \qquad \gamma_{\mu}\not{a}\not{b}\not{c}\gamma^{\mu}=-2\not{c}\not{b}\not{a}$$

$$\mathrm{Tr}[\gamma^{\mu}\gamma^{\nu}]=4g^{\mu\nu} \qquad \mathrm{Tr}[A\,B\,C]=\mathrm{Tr}[B\,C\,A] \qquad \mathrm{Tr}[S^{-1}A\,S]=\mathrm{Tr}\,A \qquad \mathrm{Tr}\mathbf{1}=4$$

$$\mathrm{Tr}[\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}]=4[g^{\mu\nu}g^{\rho\sigma}+g^{\mu\sigma}g^{\nu\rho}-g^{\mu\rho}g^{\nu\sigma}]$$

$$\mathrm{Tr}[\gamma^5\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}]=-4i\,\varepsilon^{\mu\nu\rho\sigma} \qquad (\varepsilon^{0123}=-\varepsilon_{0123}=1\text{ convention})$$

$$\mathrm{Tr}[\gamma^1\gamma^2\cdots\gamma^{2n+1}]=0$$

$$\mathrm{Tr}[\gamma^5]=\mathrm{Tr}[\gamma^5\gamma^{\mu}]=\mathrm{Tr}[\gamma^5\gamma^{\mu}\gamma^{\nu}]=\mathrm{Tr}[\gamma^5\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}]=0$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu}{}^{\gamma\delta}=-2(g^{\rho\gamma}g^{\sigma\delta}-g^{\rho\delta}g^{\sigma\gamma})$$

$$\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}=g^{\mu\nu}\gamma^{\rho}-g^{\mu\rho}\gamma^{\nu}+g^{\nu\rho}\gamma^{\mu}-i\gamma^5\varepsilon^{\mu\nu\rho\sigma}\gamma_{\sigma}$$

Dirac Field

$$\mathcal{L}=\bar{\psi}\,(\gamma^{\mu}i\overleftrightarrow{\partial}_{\mu}-m)\psi=\frac{1}{2}[\bar{\psi}\gamma^{\mu}(i\partial_{\mu}\psi)-(i\partial_{\mu}\bar{\psi})\gamma^{\mu}\psi]-m\bar{\psi}\psi \qquad \bar{\psi}\equiv\psi^{\dagger}\gamma^0$$

$$(\gamma^{\mu}i\partial_{\mu}-m)\psi(x)=0$$

$$\mathcal{L}(\psi,\partial_{\mu}\psi)=\bar{\psi}\,(x)(\gamma^{\mu}i\partial_{\mu}-m)\psi(x) : \text{Use this hereafter}$$

$$\pi=\frac{\partial\mathcal{L}}{\partial[\partial_t\psi]}=i\psi^{\dagger}$$

$$\mathcal{H}=\pi\partial_t\psi-\mathcal{L}=-\bar{\psi}\gamma^i i\partial_i\psi+m\bar{\psi}\psi=\psi^{\dagger}i\partial_t\psi$$

$$\psi(x)=\int\frac{d^3p}{(2\pi)^3\,2E}\sum_{s=1,2}[c_s(\vec{p})u_s(\vec{p})e^{-ipx}+d_s^{\dagger}(\vec{p})v_s(\vec{p})e^{ipx}]$$

$$\bar{\psi}(x) = \int \frac{d^3p}{(2\pi)^3\,2E} \sum_{s=1,2} [c_s^\dagger(\vec{p})\bar{u}_s(\vec{p})e^{ipx} + d_s(\vec{p})\bar{v}_s(\vec{p})e^{-ipx}]$$

$$u_s(\vec{p}) = N\begin{pmatrix} \phi_s \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\phi_s \end{pmatrix} \qquad v_s(\vec{p}) = N\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s \\ \chi_s \end{pmatrix} \qquad N = \sqrt{E+m}$$

$$c_s(\vec{p}) = \int d^3x\, e^{-i\vec{p}\cdot\vec{x}} u_s^\dagger(\vec{p})\psi(0,\vec{x}) \qquad d_s(\vec{p}) = \int d^3x\, e^{-i\vec{p}\cdot\vec{x}} \psi^\dagger(0,\vec{x}) v_s(\vec{p})$$

$$\{\psi_\alpha(t,\vec{x}), \psi_\beta^\dagger(t,\vec{x}\,')\} = \delta_{\alpha\beta}\delta(\vec{x}-\vec{x}\,') \qquad \alpha,\beta = 1\sim 4\text{: Dirac spinor suffix}$$

$$\{\psi,\psi\} = \{\psi^\dagger,\psi^\dagger\} = 0$$

$$\{c_s(\vec{p}), c_r^\dagger(\vec{p}\,')\} = \{d_s(\vec{p}), d_r^\dagger(\vec{p}\,')\} = (2\pi)^3\,2E\,\delta_{sr}\delta(\vec{p}-\vec{p}\,') \qquad s,r = 1,2$$

$$\{c,c\} = \{c,d\} = \{c,d^\dagger\} = \{d,d\} = 0$$

$$\begin{aligned} H &= \int \frac{d^3p}{2(2\pi)^3}\sum_s [c_s^\dagger(\vec{p})c_s(\vec{p}) - d_s(\vec{p})d_s^\dagger(\vec{p})] \\ &= \int \frac{d^3p}{2(2\pi)^3}\sum_s [c_s^\dagger(\vec{p})c_s(\vec{p}) + d_s^\dagger(\vec{p})d_s(\vec{p})] - \sum_s \delta^3(0)\int d^3pE \end{aligned}$$

$$J^\mu = q\bar{\psi}\gamma^\mu\psi \qquad \partial_\mu J^\mu = 0$$

$$\begin{aligned} Q &= \int d^3x\, J^0 = q\int d^3x\, \psi^\dagger\psi = q\int \frac{d^3p}{(2\pi)^3\,2E}\sum_s [c_s^\dagger(\vec{p})c_s(\vec{p}) + d_s(\vec{p})d_s^\dagger(\vec{p})] \\ &= q\int \frac{d^3p}{(2\pi)^3\,2E}\sum_s [c_s^\dagger(\vec{p})c_s(\vec{p}) - d_s^\dagger(\vec{p})d_s(\vec{p})] + q\sum_s \delta^3(0)\int d^3p \end{aligned}$$

Dirac Spinor

$$\phi_\uparrow(z) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \phi_\downarrow(z) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \chi_\uparrow(z) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \chi_\downarrow(z) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\phi_\uparrow(\theta,\varphi) = \begin{pmatrix} e^{-i\varphi/2}\cos\dfrac{\theta}{2} \\ e^{i\varphi/2}\sin\dfrac{\theta}{2} \end{pmatrix} \quad \phi_\downarrow(\theta,\varphi) = \begin{pmatrix} -e^{-i\varphi/2}\sin\dfrac{\theta}{2} \\ e^{i\varphi/2}\cos\dfrac{\theta}{2} \end{pmatrix}\text{: particle}$$

$$\chi_\uparrow(\theta,\varphi) = \begin{pmatrix} -e^{-i\varphi/2}\sin\dfrac{\theta}{2} \\ e^{i\varphi/2}\cos\dfrac{\theta}{2} \end{pmatrix} \quad \chi_\downarrow(\theta,\varphi) = \begin{pmatrix} -e^{-i\varphi/2}\cos\dfrac{\theta}{2} \\ -e^{i\varphi/2}\sin\dfrac{\theta}{2} \end{pmatrix}\text{: anti-particle}$$

$$\sigma(\theta,\varphi)\phi_\uparrow(\theta,\varphi) = \phi_\uparrow(\theta,\varphi) \qquad \sigma(\theta,\varphi)\phi_\downarrow(\theta,\varphi) = -\phi_\downarrow(\theta,\varphi)$$

$$\sigma(\theta,\varphi) \equiv \sin\theta\cos\varphi\sigma_1 + \sin\theta\sin\varphi\sigma_2 + \cos\theta\sigma_3 = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$$

$$u_s^\dagger(\vec{p})u_r(\vec{p}) = v_s^\dagger(\vec{p})v_r(\vec{p}) = 2E\delta_{sr} \qquad u_s^\dagger(\vec{p})v_r(-\vec{p}) = v_s^\dagger(\vec{p})u_r(-\vec{p}) = 0$$

$$\bar{u}_s(\vec{p})u_r(\vec{p}) = -\bar{v}_s(\vec{p})v_r(\vec{p}) = 2m\delta_{sr} \qquad \bar{u}_s(\vec{p})v_r(\vec{p}) = \bar{v}_s(\vec{p})u_r(\vec{p}) = 0$$

$$\sum_{s=1,2} u_s(p)\bar{u}_s(p) = \not{p} + m \qquad \sum_{s=1,2} v_s(p)\bar{v}_s(p) = \not{p} - m$$

$$(\not{p}-m)u = 0 \qquad (\not{p}+m)v = 0 \qquad \bar{u}\,(\not{p}-m) = 0 \qquad \bar{v}\,(\not{p}+m) = 0$$

Rotation of 2 component spinor

$$R_i(\theta) = e^{-i\sigma_i\theta/2} \qquad (\text{Rotation of coordinates: } \tilde{R}_i(\theta) = e^{i\sigma_i\theta/2})$$

$$R_1 = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \qquad R_2 = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \qquad R_3 = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$R(\theta,\varphi) = R_3(\varphi)R_2(\theta) = \begin{pmatrix} e^{-i\varphi/2}\cos\frac{\theta}{2} & -e^{-i\varphi/2}\sin\frac{\theta}{2} \\ e^{i\varphi/2}\sin\frac{\theta}{2} & e^{i\varphi/2}\cos\frac{\theta}{2} \end{pmatrix}$$

Magnetic moment of dirac particle (gyromagnetic ratio)

$$qe\bar{\psi}\gamma^\mu\psi A_\mu \to qe\bar{u}_s(p)u_s(p)\frac{p^\mu}{m}A_\mu(x) + \frac{qe}{2m}\bar{u}_s(p)\sigma^{\mu\nu}u_s(p)\partial_\nu A_\mu(x)$$

on conditions of $|\vec{p}\,| \ll E \sim m$, $u_s \sim N\begin{pmatrix} \phi_s \\ \vec{\sigma}\cdot\vec{v}\phi_s/2 \end{pmatrix}$, $N=1$

$$\to qe\big(\phi - \vec{v}\cdot\vec{A}\,\big) - \frac{qe}{m}\vec{s}\cdot\big(\vec{B} - \vec{v}\times\vec{E}\,\big),\ \text{where}\ \vec{s} \equiv \frac{1}{2}\phi_s^\dagger\vec{\sigma}\phi_s$$

$$\vec{\mu} \equiv g\frac{qe}{2m}\vec{s} = gq\mu_B\vec{s} \qquad g=2 \qquad \mu_B \equiv \frac{e}{2m}\text{: Bohr magneton}$$

Covariance of Dirac spinor

$$x^\mu \to x'^\mu = a^\mu{}_\nu x^\nu$$

$$\psi(x) \to \psi'(x') = S(a)\psi(x) \qquad \bar{\psi}\,(x) \to \bar{\psi}\,(x)S(a)^{-1}$$

$$S^{-1}\gamma^\mu S = a^\mu{}_\nu\gamma^\nu \qquad S^{-1} = \gamma^0 S^\dagger\gamma^0$$

boost with 4 momentum $p^\mu = (E,\vec{p}\,)$, $m = \sqrt{p^2}$

$$S(\vec{p}\,) = \sqrt{\frac{E+m}{2\,m}}\begin{pmatrix} 1 & \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} & 1 \end{pmatrix} \qquad \vec{\sigma}\cdot\vec{p} = \begin{pmatrix} p_3 & p_1 - i\,p_2 \\ p_1 + i\,p_2 & -p_3 \end{pmatrix}$$

boost with $\hat{n}$, $\cosh\alpha = \gamma$

$$S(\alpha) = \begin{pmatrix} \cosh\frac{\alpha}{2} & \hat{n}\cdot\vec{\sigma}\sinh\frac{\alpha}{2} \\ \hat{n}\cdot\vec{\sigma}\sinh\frac{\alpha}{2} & \cosh\frac{\alpha}{2} \end{pmatrix} = \sqrt{\frac{\gamma+1}{2}}\begin{pmatrix} 1 & \hat{n}\cdot\vec{\sigma}\frac{\gamma\beta}{\gamma+1} \\ \hat{n}\cdot\vec{\sigma}\frac{\gamma\beta}{\gamma+1} & 1 \end{pmatrix}$$

charge conjugation of Dirac spinor

$$C = i\gamma^2\gamma^0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}$$

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu \qquad C\{\gamma^\mu(1-\gamma^5)\}^T C^{-1} = -\gamma^\mu(1+\gamma^5)$$

$$C = C^* = -C^{-1} = -C^T = -C^\dagger$$

$$\psi^C = C\bar{\psi}^{\,T} = i\gamma^2\psi^* \qquad \bar{\psi}^{\,C} = -\psi^T C^{-1} = -i\bar{\psi}^{\,*}\gamma^2$$

$$u = \begin{pmatrix} \phi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\phi_s \end{pmatrix} \to u^C = i\gamma^2 u^* = \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s \\ \chi_s \end{pmatrix},\ \text{where}\ \chi_s = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\phi_s^*$$

$$v = \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s \\ \chi_s \end{pmatrix} \to v^C = i\gamma^2 v^* = \begin{pmatrix} \phi_\alpha \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\phi_s \end{pmatrix},\ \text{where}\ \phi_s = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\chi_s^*$$

parity transformation of Dirac spinor

$$P=\eta_P\gamma^0=\eta_P\begin{pmatrix}1&0\\0&-1\end{pmatrix}\qquad P^2=1\qquad \eta_P^2=1\qquad P^{-1}=\eta_P\gamma^0$$

$$P^{-1}\gamma^\mu P=(\gamma^0;-\gamma^i)=\gamma_\mu$$

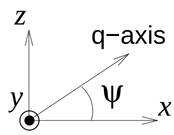
$$Pu_s(\vec{p}\,)=u_s(-\vec{p}\,)\qquad Pv_s(\vec{p}\,)=-v_s(-\vec{p}\,)\qquad \eta_P\equiv 1$$

$$P_L=\frac{1-\gamma^5}{2}\qquad P_R=\frac{1+\gamma^5}{2}$$

$$P_L+P_R=1\qquad P_{L,R}^2=P_{L,R}\qquad P_LP_R=P_RP_L=0\qquad P^{-1}P_{L,R}P=P_{R,L}$$

Formula

$$\begin{pmatrix}u_\uparrow^\dagger\sigma_1v_\uparrow & u_\uparrow^\dagger\sigma_2v_\uparrow & u_\uparrow^\dagger\sigma_3v_\uparrow\\ u_\uparrow^\dagger\sigma_1v_\downarrow & u_\uparrow^\dagger\sigma_2v_\downarrow & u_\uparrow^\dagger\sigma_3v_\downarrow\\ u_\downarrow^\dagger\sigma_1v_\uparrow & u_\downarrow^\dagger\sigma_2v_\uparrow & u_\downarrow^\dagger\sigma_3v_\uparrow\\ u_\downarrow^\dagger\sigma_1v_\downarrow & u_\downarrow^\dagger\sigma_2v_\downarrow & u_\downarrow^\dagger\sigma_3v_\downarrow\end{pmatrix}=\begin{pmatrix}\cos\psi & -i & -\sin\psi\\ -\sin\psi & 0 & -\cos\psi\\ -\sin\psi & 0 & -\cos\psi\\ -\cos\psi & -i & \sin\psi\end{pmatrix}$$



$$\text{where } u_\uparrow(\psi)=-v_\downarrow=\begin{pmatrix}\cos\frac{\psi}{2}\\ \sin\frac{\psi}{2}\end{pmatrix},\ u_\downarrow=v_\uparrow=\begin{pmatrix}-\sin\frac{\psi}{2}\\ \cos\frac{\psi}{2}\end{pmatrix}$$

$$=\begin{pmatrix}\cos\theta\cos\varphi+i\sin\varphi & \cos\theta\sin\varphi-i\cos\varphi & -\sin\theta\\ -\sin\theta\cos\varphi & -\sin\theta\sin\varphi & -\cos\theta\\ -\sin\theta\cos\varphi & -\sin\theta\sin\varphi & -\cos\theta\\ -\cos\theta\cos\varphi+i\sin\varphi & -\cos\theta\sin\varphi-i\cos\varphi & \sin\theta\end{pmatrix}$$

$$\text{where } u_\uparrow(\theta,\varphi)=-v_\downarrow=\begin{pmatrix}e^{-i\varphi/2}\cos\frac{\theta}{2}\\ e^{i\varphi/2}\sin\frac{\theta}{2}\end{pmatrix},\ u_\downarrow=v_\uparrow=\begin{pmatrix}-e^{-i\varphi/2}\sin\frac{\theta}{2}\\ e^{i\varphi/2}\cos\frac{\theta}{2}\end{pmatrix}$$

Chirality(Weyl) Representation

$$\psi=\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}$$

$$\gamma^0=\begin{pmatrix}0&1\\1&0\end{pmatrix}\qquad \gamma^i=\begin{pmatrix}0&\sigma_i\\-\sigma_i&0\end{pmatrix}\qquad \gamma^5=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$$

$$\psi_L\equiv P_L\psi=\begin{pmatrix}\psi_L\\0\end{pmatrix}\qquad \psi_R\equiv P_R\psi=\begin{pmatrix}0\\ \psi_R\end{pmatrix}\qquad P_L\gamma^\mu=\gamma^\mu P_R$$

$$\bar{\psi}_L=\bar{\psi}P_R\qquad \bar{\psi}_L\psi_L=\bar{\psi}_R\psi_R=0\qquad \bar{\psi}_L\gamma^\mu\psi_R=\bar{\psi}_R\gamma^\mu\psi_L=0$$

$$\psi_D=T\psi_W\qquad \gamma_D=T\gamma_WT^{-1}\qquad T=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\-1&1\end{pmatrix}\qquad T^{-1}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}$$

$$S_W(\vec{p}\,)=\sqrt{\frac{E+m}{2m}}\begin{pmatrix}1-\frac{\vec{\sigma}\cdot\vec{p}}{E+m} & 0\\ 0 & 1+\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\end{pmatrix}$$

Bilinear Expression

$\bar{\psi}\psi$	$\bar{\psi}\gamma^5\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma^5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$
(S)	(P)	(V)	(A)	(T)
1	1	4	4	6

$$\sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu]=i[\gamma^\mu\gamma^\nu-g^{\mu\nu}]$$

$$\bar{u}(p')\gamma^\mu u(p) = \frac{1}{m+m'}\bar{u}(p')[(p+p')^\mu - i\sigma^{\mu\nu}q_\nu]u(p) \qquad q \equiv p - p'$$

Propagator

$$\begin{aligned} i\,\Delta_F(x'-x) &= \langle 0|\mathcal{T}(\phi(x')\phi^\dagger(x))|0\rangle \\ &= \theta(t'-t)\langle 0|\phi(x')\phi^\dagger(x)|0\rangle \underset{\substack{\text{boson}\\ \text{fermion}}}{\pm} \theta(t-t')\langle 0|\phi^\dagger(x)\phi(x')|0\rangle \end{aligned}$$

$$\theta(t) = \frac{-1}{2\pi\, i}\int dk^0 \frac{e^{-ik^0 t}}{k^0 + i\varepsilon}$$

$$i\Delta_F(x'-x) = i\int \frac{d^4k}{(2\pi)^4}\, i\tilde{\Delta}_F(k)\, e^{-ik(x'-x)}$$

Klein-Gordon: $i\tilde{\Delta}_F(p) = \dfrac{i}{p^2 - m^2 + i\,\varepsilon}$

Gauge field: $i\tilde{D}_F(p) = \dfrac{-ig^{\mu\nu}}{p^2 - m^2 + i\,\varepsilon}$

Dirac field: $i\tilde{S}_F(p) = \dfrac{i(\not{p} + m)}{p^2 - m^2 + i\,\varepsilon}$

Local Gauge Transformation: scalar field with U(1)

$$D_\mu = \partial_\mu + iqA_\mu \qquad (p^\mu \to p^\mu - qA^\mu)$$

$$\begin{aligned} \mathcal{L} &= (D^\mu\phi)^\dagger(D_\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &= (\partial^\mu\phi)^\dagger(\partial_\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j_\mu A^\mu \end{aligned} \qquad j_\mu = qJ_\mu = iq[\phi^\dagger\partial_\mu\phi - (\partial_\mu\phi)^\dagger\phi]$$

Gauge Field

U(1): $e^{ig_1\beta(x)\cdot Y/2}$ $\qquad B_\mu$: Abelian

SU(2): $e^{ig_2\vec{\alpha}(x)\cdot\vec{\tau}/2}$ $\qquad W_\mu$: non-Abelian $\qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = if_{abc}\frac{\tau_c}{2}$

$$D_\mu = \partial_\mu + i\,g_1\frac{Y}{2}B_\mu + i\,g_2\,\frac{\tau_a}{2}\,W_{a\mu}$$

$$B_\mu \to B'_\mu = B_\mu - \partial_\mu\beta \qquad W_{a\mu} \to W'_{a\mu} = W_{a\mu} - \partial_\mu\alpha_a + g_2 f_{abc}W_{b\mu}\alpha_c$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \qquad W_{a\mu\nu} \equiv \partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} - g_2 f_{abc}W_{b\mu}W_{c\nu}$$

SU(3)

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if_{abc}\frac{\lambda_c}{2} \qquad \mathrm{Tr}(\lambda_a) = 0 \qquad \mathrm{Tr}(\lambda_a\lambda_b) = 2\delta_{ab} \qquad f_{abc} = \frac{1}{4i}\mathrm{Tr}([\lambda_a, \lambda_b]\lambda_c)$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$f_{123} = 1$$

$$f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}$$

$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$

Jacobi identity $f_{abc} f_{ck\ell} + f_{bkc} f_{ca\ell} + f_{kac} f_{cb\ell} = 0$

S-Matrix

$$H_0 |n,t\rangle = i\frac{\partial}{\partial t}|n,t\rangle \qquad |n,t\rangle = U_0(t)|n\rangle$$

$$(H_0 + V)|\psi,t\rangle = i\frac{\partial}{\partial t}|\psi,t\rangle \qquad |\psi,t\rangle = U(t)|\psi\rangle \qquad V(t) = -\int d^3x \mathcal{L}_{\rm int}(x)$$

$$U(t) = U_0(t)\mathcal{T}\exp\left[-i\int_{t_0}^{t} dt' U_0^\dagger(t')\, V(t') U_0(t')\right] U_0^\dagger(t_0)$$

$$S \equiv U(\infty)U^\dagger(-\infty) = U_0(\infty)\mathcal{T}\exp\left[i\int d^4x' U_0^\dagger(t')\mathcal{L}_{\rm int}(x')U_0(t')\right]U_0^\dagger(-\infty)$$

$$\begin{aligned} T_{fi} &\equiv \langle f,\infty|S|i,-\infty\rangle = \left\langle f\left|\mathcal{T}\exp\left[i\int d^4x' U_0^\dagger(t')\mathcal{L}_{\rm int}(x')U_0(t')\right]\right|i\right\rangle \\ &= \delta_{fi} + i\int d^4x' \langle f,t'|\mathcal{L}_{\rm int}(x')|i,t'\rangle \\ &\quad + \frac{i^2}{2!}\int d^4x' \int d^4x'' \sum_n \mathcal{T}[\langle f,t'|\mathcal{L}_{\rm int}(x')|n,t'\rangle\langle n,t''|\mathcal{L}_{\rm int}(x'')|i,t''\rangle] + \cdots \\ &\equiv \delta_{fi} - i(2\pi)^4\delta^4(p_f - p_i)\mathcal{M}_{fi} \end{aligned}$$

Matrix Element for $-i\mathcal{M}$

$$|\bar{\psi}_f\,\Gamma\,\psi_i|^2 = \bar{\psi}_f\,\Gamma\,\psi_i\,\bar{\psi}_i\,\gamma^0\,\Gamma^\dagger\,\gamma^0\,\psi_f = \mathrm{Tr}[\Gamma\,\psi_i\,\bar{\psi}_i\,\gamma^0\,\Gamma^\dagger\,\gamma^0\,\psi_f\bar{\psi}_f\,]$$

$$X = (x_i),\; Y = (y_i) \Longrightarrow X^T Y = \mathrm{Tr}[Y X^T]$$

$$\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu \qquad \gamma^0(1-\gamma^5)^\dagger\gamma^0 = 1+\gamma^5 \qquad \gamma^0(\psi_i\,\bar{\psi}_j)^\dagger\gamma^0 = \psi_j\,\bar{\psi}_i$$

External Lines

spin 1/2 incoming: $u(p,s)$ or $v(p,s)$ outgoing: $\bar{u}\,(p,s)$ or $\bar{v}\,(p,s)$

spin 1 incoming: $\varepsilon_\mu(p,\lambda)$ outgoing: $\varepsilon^*_\mu(p,\lambda)$

Internal Lines

spin 0 $\quad \dfrac{i}{p^2 - m^2 + i\,\varepsilon}$

spin 1/2 $\quad \dfrac{i(\not{p} + m)}{p^2 - m^2 + i\,\varepsilon}$

spin 1 $\quad \dfrac{i}{p^2 + i\,\varepsilon}\left[-g^{\mu\nu} + (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right]$ $\quad \xi = 1$ for Feynman gauge

$\dfrac{i}{p^2 - m^2 + i\,\varepsilon}\left(-g^{\mu\nu} + \dfrac{p^\mu p^\nu}{m^2}\right)$ Unitary gauge

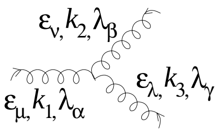
$\dfrac{i}{p^2 - m^2 + i\,\varepsilon}\left[-g^{\mu\nu} + (1-\xi)\dfrac{p^\mu p^\nu}{p^2 - \xi m^2}\right]$ R gauge

Vertexes

EM current for charge $+e$ $\quad -ie\gamma^\mu$

EW charged current $\quad -i\dfrac{g}{\sqrt{2}}\gamma^\mu\dfrac{1-\gamma^5}{2}$

3 gluons vertex $\quad -g_s f_{\alpha\beta\gamma}[g^{\mu\nu}(k_1 - k_2)^\lambda + g^{\nu\lambda}(k_2 - k_3)^\mu + g^{\lambda\mu}(k_3 - k_1)^\nu]$



$\gamma(k_\gamma, \varepsilon_\mu)$-$W^+(k_+, \varepsilon_\nu)$-$W^-(k_-, \varepsilon_\lambda)$ vertex

$$ie[g^{\nu\lambda}(k_+ - k_-)^\mu + g^{\lambda\mu}(k_- - k_\gamma)^\nu + g^{\mu\nu}(k_\gamma - k_+)^\lambda]$$

Conservation law

Q num\Int.	Strong	EM	Weak
Isospin I	O	X	X
I_3	O	O	X
Parity P	O	O	X
C-parity C	O	O	X
G-parity G	O	X	X