Lorentz vector

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & \\ & -\mathbf{1} \end{pmatrix}$$

$$x^\mu = (c\,t\,,\vec{x}) \qquad \partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \qquad p^\mu = \left(\frac{E}{c}, \vec{p}\right)$$

$$k^\mu = \left(\frac{\omega}{c}, \vec{k}\right) \qquad j^\mu = (c\rho\,,\vec{j}) \qquad A^\mu = \left(\frac{\phi}{c}, \vec{A}\right)$$

$$d^4x = d\,x^0 d\,x^1 d\,x^2 d\,x^3 \qquad \frac{d^3p}{E/c} \qquad \frac{d^3k}{\omega/c} \qquad \text{Lorentz invariant}$$

$$p^\mu = i\partial^\mu \qquad E = i\frac{\partial}{\partial t} \qquad \vec{p} = -i\nabla$$

Electro-Magnetic Field

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

$$-\frac{1}{4\,\mu_0}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2\mu_0}\left(\frac{\vec{E}^2}{c^2} - \vec{B}^2\right) = \frac{1}{2}(\varepsilon_0\vec{E}^2 - \mu_0\vec{H}^2) \qquad \frac{1}{\mu_0\varepsilon_0} = c^2$$

$$\mathcal{L} = -\frac{1}{4\,\mu_0}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$$

$$\partial_\mu F^{\mu\nu} = \Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = \mu_0 j^\mu \qquad \partial_\mu j^\mu = 0$$

$$\mathcal{H} = \pi^\mu(\partial_t A_\mu) - \mathcal{L} = \frac{1}{2}(\varepsilon_0\vec{E}^2 + \mu_0\vec{H}^2) + \varepsilon_0\vec{E}\cdot\nabla\phi$$

$$\pi^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_t A_\mu)} = -\frac{1}{\mu_0}F^{0\mu} = -\frac{1}{\mu_0}(0, \varepsilon_0 c\vec{E}) \qquad \partial_t A_\mu = \left(-\nabla\cdot\vec{A}, \frac{1}{c}(\vec{E} + \nabla\phi)\right)$$

Gauge transformation

$$A^\mu \to A'^\mu = A^\mu - \partial^\mu\Lambda$$

$$\partial_\mu A^\mu = 0 \qquad \Longrightarrow \qquad \Box\Lambda = \partial_\mu A^\mu \qquad : \text{Lorenz gauge}$$

$$\nabla\cdot\vec{A} = 0 \qquad \Longrightarrow \qquad \Delta\Lambda = -\nabla\cdot\vec{A} \qquad : \text{Coulomb gauge}$$

Heaviside-Lorentz unit: $\varepsilon_0 = \mu_0 = 1$ Natural unit: $\hbar = c = 1$ $\frac{e^2}{4\pi} = \frac{1}{137}$

Euler Equation

$$S = \int \mathcal{L}(\phi, \partial_\mu\phi)d^4x = \int \mathcal{L}\,d^3x\,dt = \int L dt \qquad \text{action}$$

$$\delta S = 0 \qquad \Longrightarrow \qquad \frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = 0 \qquad \text{Least action principle}$$

Proca Equation

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu - j^\mu A_\mu$$

$$\frac{\partial\mathcal{L}}{\partial A_\mu} - \partial_\nu\frac{\partial\mathcal{L}}{\partial(\partial_\nu A_\mu)} = 0 \Rightarrow \Box A^\mu - \partial^\mu(\partial_\nu A^\nu) + m^2 A^\mu = j^\mu \Rightarrow m^2\partial_\mu A^\mu = \partial_\mu j^\mu = 0$$

$$\Rightarrow \partial_\mu A^\mu = 0 \qquad (\Box + m^2)A^\mu = j^\mu$$

Fock Space

$\hat{a}, \hat{a}^\dagger$: rising and lowering operator $\qquad [\hat{a}, \hat{a}^\dagger] = 1$

$\hat{N} = \hat{a}^\dagger \hat{a}$: number operator $\qquad \hat{N}|n\rangle = n|n\rangle$

$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \qquad \hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \hat{a}|0\rangle = 0$

$$|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^\dagger)^n|0\rangle$$

e.g. $[\hat{x}, \hat{p}] = i \qquad \hat{p}, \hat{x}$: Hermitian

$$\hat{a} = \frac{1}{\sqrt{2m\omega}}(m\omega\hat{x} + i\hat{p}) \qquad \hat{a}^\dagger = \frac{1}{\sqrt{2m\omega}}(m\omega\hat{x} - i\hat{p}) \qquad [\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2 = \omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

Invariant Delta Function

$$\begin{aligned}\Delta(x; m^2) &\equiv \frac{1}{i(2\pi)^3}\int d^4p\, \epsilon(p_0)\delta(p^2 - m^2)e^{-ipx} \\ &= \frac{1}{i(2\pi)^3}\int d^3p \frac{1}{2\sqrt{m^2 + \vec{p}^2}}[e^{-ipx} - e^{ipx}] \\ &= -\frac{1}{2\pi}\epsilon(x_0)\left[\delta(x^2) - \frac{m^2}{2}\theta(x^2)\frac{J_1\left(m\sqrt{x^2}\right)}{m\sqrt{x^2}}\right]\end{aligned} \qquad \begin{cases}\epsilon(u) \equiv \operatorname{sgn} u \equiv u/|u| \\ \epsilon(0) = 0\end{cases}$$

$\Delta(-x; m^2) = -\Delta(x; m^2) \qquad x^2 > 0$: timelike $\qquad x^2 < 0$: spacelike

Canonical Quantization

$$\pi_i = \frac{\partial\mathcal{L}(\phi_i, \partial\phi_i)}{\partial[\partial_t\phi_i]} \qquad \mathcal{H} = \pi_i\partial_t\phi_i - \mathcal{L} \qquad H = \int d^3x\mathcal{H}$$

$i\partial_t\phi = [\phi, H] \qquad i\partial_t\pi(x) = [\pi, H]$

$T^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial[\partial_\mu\phi_i]}\partial^\nu\phi_i - g^{\mu\nu}\mathcal{L}$: Energy-momentum tensor

$$T^{00} = \mathcal{H} \qquad p_i = \int d^3x T^{0i} \qquad \partial_\mu T^{\mu\nu} = 0$$

Neutral Scalar Field

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \qquad (\Box + m^2)\phi(x) = 0$$

$$\pi = \frac{\partial\mathcal{L}}{\partial[\partial_t\phi]} = \partial_t\phi$$

$$\mathcal{H} = \pi\partial_t\phi - \mathcal{L} = \frac{1}{2}[(\partial_t\phi)^2 + (\nabla\phi)^2 + m^2\phi^2]$$

$[\phi(t, \vec{x}), \pi(t, \vec{x}')] = i\delta(\vec{x} - \vec{x}') \qquad$ canonical quantization

$$[\phi(t, \vec{x}), \phi(t, \vec{x}')] = [\pi(t, \vec{x}), \pi(t, \vec{x}')] = 0$$

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3\,2E}[a(\vec{p})e^{-ipx} + a^\dagger(\vec{p})e^{ipx}]$$

$$a(\vec{p}) = \int d^3x e^{-i\vec{p}\cdot\vec{x}}[E\phi(0, \vec{x}) + i\pi(0, \vec{x})]$$

$$H = \int \frac{d^3p}{2(2\pi)^3}\frac{1}{2}[a^\dagger(\vec{p})a(\vec{p}) + a(\vec{p})a^\dagger(\vec{p})] = \int \frac{d^3p}{2(2\pi)^3}a^\dagger(\vec{p})a(\vec{p}) + \frac{1}{2}\int \frac{d^3p d^3x}{(2\pi)^3}E$$

$$[a(\vec{p}), a^\dagger(\vec{p}')] = (2\pi)^3\,2E\,\delta(\vec{p} - \vec{p}') \qquad [a(\vec{p}), a(\vec{p}')] = 0$$

4-dimensional commutation relation

$$[\phi(x),\phi(y)] = i\Delta(x-y;m^2) \qquad \text{invariant delta function}$$

Complex Scalar Field

$$\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi$$

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \qquad \phi^\dagger = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \qquad \phi_1,\phi_2 : \text{Hermitian}$$

$$\pi = \frac{\partial\mathcal{L}}{\partial[\partial_t\phi]} = \partial_t\phi^\dagger \qquad \pi^\dagger = \frac{\partial\mathcal{L}}{\partial[\partial_t\phi^\dagger]} = \partial_t\phi$$

$$\mathcal{H} = \pi\partial_t\phi + (\partial_t\phi^\dagger)\pi^\dagger - \mathcal{L} = \partial_t\phi^\dagger\partial_t\phi + \nabla\phi^\dagger\cdot\nabla\phi + m^2\phi^\dagger\phi$$

$$\phi(x) = \int\frac{d^3p}{(2\pi)^3\,2E}\,[a(\vec{p})e^{-ipx} + b^\dagger(\vec{p})e^{ipx}] \qquad \pi(x) = i\int\frac{d^3p}{(2\pi)^3\,2}\,[a^\dagger(\vec{p})e^{ipx} - b(\vec{p})e^{-ipx}]$$

$$a(\vec{p}) = \int d^3x\, e^{-i\vec{p}\cdot\vec{x}}[E\phi(0,\vec{x}) + i\pi^\dagger(0,\vec{x})] \qquad b(\vec{p}) = \int d^3x\, e^{-i\vec{p}\cdot\vec{x}}[E\phi^\dagger(0,\vec{x}) + i\pi(0,\vec{x})]$$

$$[\phi(t,\vec{x}),\pi(t,\vec{x}')] = i\delta(\vec{x}-\vec{x}') \qquad [\phi(x),\phi^\dagger(y)] = i\Delta(x-y;m^2)$$

$$[\phi(t,\vec{x}),\phi(t,\vec{x}')] = [\phi,\phi^\dagger] = [\pi,\pi] = [\pi,\pi^\dagger] = [\phi,\pi^\dagger] = 0$$

$$[a(\vec{p}),a^\dagger(\vec{p}')] = [b(\vec{p}),b^\dagger(\vec{p}')] = (2\pi)^3\,2E\,\delta(\vec{p}-\vec{p}')$$

$$[a,a] = [a,b] = [a,b^\dagger] = [b,b] = 0$$

$$H = \int\frac{d^3p}{2(2\pi)^3}[a^\dagger(\vec{p})a(\vec{p}) + b(\vec{p})b^\dagger(\vec{p})] = \int\frac{d^3p}{2(2\pi)^3}[a^\dagger(\vec{p})a(\vec{p}) + b^\dagger(\vec{p})b(\vec{p})] + \int\frac{d^3pd^3x}{(2\pi)^3}E$$

Noether's theorem

$$\delta S = \int d^4x\sum_i\,\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i\right]$$

$$J^\mu = \sum_i\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i \qquad \Longrightarrow \qquad \partial_\mu J^\mu = 0$$

$$Q \equiv \int d^3x\,J^0 \qquad \frac{dQ}{dt} = 0$$

e.g. $\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi \qquad \phi\to\phi' = e^{i\alpha}\phi\to(1+i\alpha)\phi \qquad \delta\phi = i\alpha\phi$

$$J^\mu = i[\phi^\dagger(\partial_\mu\phi) - (\partial_\mu\phi^\dagger)\phi]$$

$$\begin{aligned} Q &= i\int d^3x\,[\phi^\dagger(\partial_t\phi) - (\partial_t\phi^\dagger)\phi] = \int\frac{d^3p}{(2\pi)^3\,2E}\,[a^\dagger(\vec{p})a(\vec{p}) - b(\vec{p})b^\dagger(\vec{p})] \\ &= \int\frac{d^3p}{(2\pi)^3\,2E}\,[a^\dagger(\vec{p})a(\vec{p}) - b^\dagger(\vec{p})b(\vec{p})] - \int\frac{d^3pd^3x}{(2\pi)^3} \end{aligned}$$

Quantization of Electro-magnetic Field (Lorenz gauge $\partial_\mu A^\mu = 0$)

$$A^\mu(x) = \int\frac{d^3k}{(2\pi)^3\,2\omega}\sum_\lambda\,[a_\lambda(\vec{k})\varepsilon_\lambda^\mu(\vec{k})e^{-ikx} + a_\lambda^\dagger(\vec{k})\varepsilon_\lambda^{\mu*}(\vec{k})e^{ikx}]$$

$$k^2 = 0 \qquad k_\mu\varepsilon^\mu = k_\mu\varepsilon^{\mu*} = 0 \qquad \lambda = \pm \text{ for real photon}$$

$$[a_\lambda(\vec{k}),a_\rho^\dagger(\vec{k}')] = (2\pi)^3\,2\omega(-g^{\lambda\rho})\delta(\vec{k}-\vec{k}') \qquad [a,a] = [a^\dagger,a^\dagger] = 0$$

$$H = \int\frac{d^3k}{2(2\pi)^3}\sum_{\lambda=\pm}\,a_\lambda^\dagger(\vec{k})a_\lambda(\vec{k}) + \sum_{\lambda=\pm}\,\int\frac{d^3kd^3x}{(2\pi)^3}\frac{\omega}{2}$$

Polarization Vector of Spin 1 Particle

$$\varepsilon^\mu(\lambda=+)=\frac{1}{\sqrt{2}}(0,-1,-i,0) \qquad \lambda : \text{helicity on traveling along the } z\text{-axis}$$

$$\varepsilon^\mu(\lambda=-)=\frac{1}{\sqrt{2}}(0,1,-i,0)$$

$$\varepsilon^\mu(\lambda=0)=(0,0,0,1) \qquad \varepsilon^\mu(\lambda=s)=(1,0,0,0)$$

$$\varepsilon_\mu(\lambda)\varepsilon^{\mu*}(\rho)=g_{\lambda\rho} \qquad (\lambda,\rho=s,+,-,0)$$

$$p^\mu=(E,0,0,p) \Rightarrow \varepsilon^\mu(p;\lambda=0)=(p,0,0,E)/M \qquad M^2=E^2-p^2$$

$$\varepsilon^\mu(p;\lambda=s)=(E,0,0,p)/M=p^\mu/M$$

$$\varepsilon^\mu(p;\lambda=\pm)=\varepsilon^\mu(\lambda=\pm)$$

$$\sum_{\lambda=\pm,0}\varepsilon^\mu(p;\lambda)\varepsilon^{\nu*}(p;\lambda)=-g^{\mu\nu}+p^\mu p^\nu/M^2$$

Rotation of Polarization vector

$$U_x(\xi)=\begin{pmatrix}1&0&0\\0&\cos\xi&-\sin\xi\\0&\sin\xi&\cos\xi\end{pmatrix} \quad U_y(\eta)=\begin{pmatrix}\cos\eta&0&\sin\eta\\0&1&0\\-\sin\eta&0&\cos\eta\end{pmatrix} \quad U_z(\zeta)=\begin{pmatrix}\cos\zeta&-\sin\zeta&0\\\sin\zeta&\cos\zeta&0\\0&0&1\end{pmatrix}$$

$$L_x=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix} \qquad L_y=\begin{pmatrix}0&0&i\\0&0&0\\-i&0&0\end{pmatrix} \qquad L_z=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}$$

γ matrixes

$$\gamma^0=\begin{pmatrix}1&0\\0&-1\end{pmatrix} \qquad \gamma^i=\begin{pmatrix}0&\sigma_i\\-\sigma_i&0\end{pmatrix} \qquad \gamma^5=\begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad 1-\gamma^5=\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$$

$$\{\gamma^\mu,\gamma^\nu\}=2\,g^{\mu\nu} \qquad g_{\mu\nu}g^{\mu\nu}=4$$

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix} \qquad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\gamma^5=i\,\gamma^0\gamma^1\gamma^2\gamma^3 \qquad (\gamma^5)^2=1 \qquad \{\gamma^5,\gamma^\mu\}=0 \qquad (1\pm\gamma^5)^2=2\,(1\pm\gamma^5)$$

$$\gamma^{0\dagger}=\gamma^0 \qquad \gamma^{i\dagger}=-\gamma^i \qquad \gamma^{5\dagger}=\gamma^5 \qquad \gamma^{\mu\dagger}=\gamma^0\gamma^\mu\gamma^0$$

$$\gamma_\mu\gamma^\mu=4 \qquad \gamma_\mu\not{a}\gamma^\mu=-2\not{a} \qquad \gamma_\mu\not{a}\not{b}\gamma^\mu=4a\cdot b \qquad \gamma_\mu\not{a}\not{b}\not{c}\gamma^\mu=-2\not{c}\not{b}\not{a}$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu]=4g^{\mu\nu} \qquad \mathrm{Tr}[A\,B\,C]=\mathrm{Tr}[B\,C\,A] \qquad \mathrm{Tr}[S^{-1}A\,S]=\mathrm{Tr}\,A \qquad \mathrm{Tr}\mathbf{1}=4$$

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=4[g^{\mu\nu}g^{\rho\sigma}+g^{\mu\sigma}g^{\nu\rho}-g^{\mu\rho}g^{\nu\sigma}]$$

$$\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma]=-4i\,\varepsilon^{\mu\nu\rho\sigma} \qquad (\varepsilon^{0123}=-\varepsilon_{0123}=1 \text{ convention})$$

$$\mathrm{Tr}[\gamma^1\gamma^2\cdots\gamma^{2n+1}]=0$$

$$\mathrm{Tr}[\gamma^5]=\mathrm{Tr}[\gamma^5\gamma^\mu]=\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu]=\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho]=0$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\mu\nu}{}^{\gamma\delta}=-2(g^{\rho\gamma}g^{\sigma\delta}-g^{\rho\delta}g^{\sigma\gamma})$$

$$\gamma^\mu\gamma^\nu\gamma^\rho=g^{\mu\nu}\gamma^\rho-g^{\mu\rho}\gamma^\nu+g^{\nu\rho}\gamma^\mu-i\gamma^5\varepsilon^{\mu\nu\rho\sigma}\gamma_\sigma$$

Dirac Field

$$\mathcal{L}=\bar{\psi}(\gamma^\mu i\overleftrightarrow{\partial}_\mu-m)\psi=\frac{1}{2}[\bar{\psi}\gamma^\mu(i\partial_\mu\psi)-(i\partial_\mu\bar{\psi})\gamma^\mu\psi]-m\bar{\psi}\psi \qquad \bar{\psi}\equiv\psi^\dagger\gamma^0$$

$$(\gamma^\mu i\partial_\mu-m)\psi(x)=0$$

$$\mathcal{L}(\psi,\partial_\mu\psi)=\bar{\psi}(x)(\gamma^\mu i\partial_\mu-m)\psi(x) : \text{Use this hereafter}$$

$$\pi = \frac{\partial\mathcal{L}}{\partial[\partial_t\psi]} = i\psi^\dagger$$

$$\mathcal{H} = \pi\partial_t\psi - \mathcal{L} = -\bar{\psi}\gamma^i i\partial_i\psi + m\bar{\psi}\psi = \psi^\dagger i\partial_t\psi$$

$$\psi(x) = \int\frac{d^3p}{(2\pi)^3\,2E}\sum_{s=1,2}\,[c_s(\vec{p})u_s(\vec{p})e^{-ipx} + d_s^\dagger(\vec{p})v_s(\vec{p})e^{ipx}]$$

$$\bar{\psi}(x) = \int\frac{d^3p}{(2\pi)^3\,2E}\sum_{s=1,2}\,[c_s^\dagger(\vec{p})\bar{u}_s(\vec{p})e^{ipx} + d_s(\vec{p})\bar{v}_s(\vec{p})e^{-ipx}]$$

$$u_s(\vec{p}) = N\begin{pmatrix}\phi_s\\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\phi_s\end{pmatrix} \qquad v_s(\vec{p}) = N\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s\\ \chi_s\end{pmatrix} \qquad N = \sqrt{E+m}$$

$$c_s(\vec{p}) = \int d^3x\,e^{-i\vec{p}\cdot\vec{x}}u_s^\dagger(\vec{p})\psi(0,\vec{x}) \qquad d_s(\vec{p}) = \int d^3x\,e^{-i\vec{p}\cdot\vec{x}}\psi^\dagger(0,\vec{x})v_s(\vec{p})$$

$$\{\psi_\alpha(t,\vec{x}),\psi_\beta^\dagger(t,\vec{x}')\} = \delta_{\alpha\beta}\delta(\vec{x}-\vec{x}') \qquad \alpha,\beta = 1\sim 4\text{: Dirac spinor suffix}$$

$$\{\psi,\psi\} = \{\psi^\dagger,\psi^\dagger\} = 0$$

$$\{c_s(\vec{p}),c_r^\dagger(\vec{p}')\} = \{d_s(\vec{p}),d_r^\dagger(\vec{p}')\} = (2\pi)^3\,2E\,\delta_{sr}\delta(\vec{p}-\vec{p}') \qquad s,r = 1,2$$

$$\{c,c\} = \{c,d\} = \{c,d^\dagger\} = \{d,d\} = 0$$

$$\begin{aligned} H &= \int\frac{d^3p}{2(2\pi)^3}\sum_s\,[c_s^\dagger(\vec{p})c_s(\vec{p}) - d_s(\vec{p})d_s^\dagger(\vec{p})]\\ &= \int\frac{d^3p}{2(2\pi)^3}\sum_s\,[c_s^\dagger(\vec{p})c_s(\vec{p}) + d_s^\dagger(\vec{p})d_s(\vec{p})] - \sum_s\int\frac{d^3pd^3x}{(2\pi)^3}E \end{aligned}$$

$$J^\mu = q\bar{\psi}\gamma^\mu\psi \qquad \partial_\mu J^\mu = 0$$

$$\begin{aligned} Q &= \int d^3x\,J^0 = q\int d^3x\,\psi^\dagger\psi = q\int\frac{d^3p}{(2\pi)^3\,2E}\sum_s\,[c_s^\dagger(\vec{p})c_s(\vec{p}) + d_s(\vec{p})d_s^\dagger(\vec{p})]\\ &= q\int\frac{d^3p}{(2\pi)^3\,2E}\sum_s\,[c_s^\dagger(\vec{p})c_s(\vec{p}) - d_s^\dagger(\vec{p})d_s(\vec{p})] + q\sum_s\int\frac{d^3pd^3x}{(2\pi)^3} \end{aligned}$$

Two-component Spinor

$$\phi_\uparrow(z) = \begin{pmatrix}1\\0\end{pmatrix} \quad \phi_\downarrow(z) = \begin{pmatrix}0\\1\end{pmatrix} \qquad \chi_\uparrow(z) = \begin{pmatrix}0\\1\end{pmatrix} \quad \chi_\downarrow(z) = \begin{pmatrix}-1\\0\end{pmatrix}$$

$$\phi_\uparrow(\theta,\varphi) = \begin{pmatrix}e^{-i\varphi/2}\cos\dfrac{\theta}{2}\\ e^{i\varphi/2}\sin\dfrac{\theta}{2}\end{pmatrix} \quad \phi_\downarrow(\theta,\varphi) = \begin{pmatrix}-e^{-i\varphi/2}\sin\dfrac{\theta}{2}\\ e^{i\varphi/2}\cos\dfrac{\theta}{2}\end{pmatrix}\text{: particle}$$

$$\chi_\uparrow(\theta,\varphi) = \begin{pmatrix}-e^{-i\varphi/2}\sin\dfrac{\theta}{2}\\ e^{i\varphi/2}\cos\dfrac{\theta}{2}\end{pmatrix} \quad \chi_\downarrow(\theta,\varphi) = \begin{pmatrix}-e^{-i\varphi/2}\cos\dfrac{\theta}{2}\\ -e^{i\varphi/2}\sin\dfrac{\theta}{2}\end{pmatrix}\text{: anti-particle}$$

$$\sigma(\theta,\varphi)\phi_\uparrow(\theta,\varphi) = \phi_\uparrow(\theta,\varphi) \qquad \sigma(\theta,\varphi)\phi_\downarrow(\theta,\varphi) = -\phi_\downarrow(\theta,\varphi)$$

$$\sigma(\theta,\varphi)\chi_\uparrow(\theta,\varphi) = -\chi_\uparrow(\theta,\varphi) \qquad \sigma(\theta,\varphi)\chi_\downarrow(\theta,\varphi) = \chi_\downarrow(\theta,\varphi)$$

$$\sigma(\theta,\varphi) \equiv \sin\theta\cos\varphi\sigma_1 + \sin\theta\sin\varphi\sigma_2 + \cos\theta\sigma_3 = \begin{pmatrix}\cos\theta & \sin\theta e^{-i\varphi}\\ \sin\theta e^{i\varphi} & -\cos\theta\end{pmatrix}$$

Rotation of two-component spinor

$$u_i(\theta) = e^{-i\sigma_i\theta/2} \qquad \text{(Rotation of coordinates: } \tilde{u}_i(\theta) = e^{i\sigma_i\theta/2}\text{)}$$

$$u_1(\xi)=\begin{pmatrix} \cos\frac{\xi}{2} & -i\sin\frac{\xi}{2} \\ -i\sin\frac{\xi}{2} & \cos\frac{\xi}{2} \end{pmatrix} \qquad u_2(\eta)=\begin{pmatrix} \cos\frac{\eta}{2} & -\sin\frac{\eta}{2} \\ \sin\frac{\eta}{2} & \cos\frac{\eta}{2} \end{pmatrix} \qquad u_3(\zeta)=\begin{pmatrix} e^{-i\zeta/2} & 0 \\ 0 & e^{i\zeta/2} \end{pmatrix}$$

$$R(\theta,\varphi)=R_3(\varphi)R_2(\theta)=\begin{pmatrix} e^{-i\varphi/2}\cos\frac{\theta}{2} & -e^{-i\varphi/2}\sin\frac{\theta}{2} \\ e^{i\varphi/2}\sin\frac{\theta}{2} & e^{i\varphi/2}\cos\frac{\theta}{2} \end{pmatrix}$$

Formula

$$\begin{pmatrix} \phi_\uparrow^\dagger\sigma_1\chi_\uparrow & \phi_\uparrow^\dagger\sigma_2\chi_\uparrow & \phi_\uparrow^\dagger\sigma_3\chi_\uparrow \\ \phi_\uparrow^\dagger\sigma_1\chi_\downarrow & \phi_\uparrow^\dagger\sigma_2\chi_\downarrow & \phi_\uparrow^\dagger\sigma_3\chi_\downarrow \\ \phi_\downarrow^\dagger\sigma_1\chi_\uparrow & \phi_\downarrow^\dagger\sigma_2\chi_\uparrow & \phi_\downarrow^\dagger\sigma_3\chi_\uparrow \\ \phi_\downarrow^\dagger\sigma_1\chi_\downarrow & \phi_\downarrow^\dagger\sigma_2\chi_\downarrow & \phi_\downarrow^\dagger\sigma_3\chi_\downarrow \end{pmatrix}=\begin{pmatrix} \cos\theta\cos\varphi+i\sin\varphi & \cos\theta\sin\varphi-i\cos\varphi & -\sin\theta \\ -\sin\theta\cos\varphi & -\sin\theta\sin\varphi & -\cos\theta \\ -\sin\theta\cos\varphi & -\sin\theta\sin\varphi & -\cos\theta \\ -\cos\theta\cos\varphi+i\sin\varphi & -\cos\theta\sin\varphi-i\cos\varphi & \sin\theta \end{pmatrix}$$

Dirac Spinor

$$u_s^\dagger(\vec{p})u_r(\vec{p})=v_s^\dagger(\vec{p})v_r(\vec{p})=2E\delta_{sr} \qquad u_s^\dagger(\vec{p})v_r(-\vec{p})=v_s^\dagger(\vec{p})u_r(-\vec{p})=0$$

$$\bar{u}_s(\vec{p})u_r(\vec{p})=-\bar{v}_s(\vec{p})v_r(\vec{p})=2m\delta_{sr} \qquad \bar{u}_s(\vec{p})v_r(\vec{p})=\bar{v}_s(\vec{p})u_r(\vec{p})=0$$

$$\sum_{s=1,2}u_s(p)\bar{u}_s(p)=\not{p}+m \qquad \sum_{s=1,2}v_s(p)\bar{v}_s(p)=\not{p}-m$$

$$(\not{p}-m)u=0 \qquad (\not{p}+m)v=0 \qquad \bar{u}(\not{p}-m)=0 \qquad \bar{v}(\not{p}+m)=0$$

Magnetic moment of dirac particle (gyromagnetic ratio)

$$qe\bar{\psi}\gamma^\mu\psi A_\mu \to qe\bar{u}_s(p)u_s(p)\frac{p^\mu}{m}A_\mu(x)-\frac{qe}{2m}\bar{u}_s(p)\sigma^{\mu\nu}u_s(p)\partial_\nu A_\mu(x)$$

on conditions of $|\vec{p}|\ll E\sim m$, $u_s\sim N\begin{pmatrix} \phi_s \\ \vec{\sigma}\cdot\vec{v}\phi_s/2 \end{pmatrix}$, $N=1$

$$\to qe(\phi-\vec{v}\cdot\vec{A})-\frac{qe}{m}\vec{s}\cdot(\vec{B}-\vec{v}\times\vec{E}),\text{ where }\vec{s}\equiv\frac{1}{2}\phi_s^\dagger\vec{\sigma}\phi_s$$

$$\vec{\mu}\equiv g\frac{qe}{2m}\vec{s}=gq\mu_B\vec{s} \qquad g=2 \qquad \mu_B\equiv\frac{e}{2m}\text{: Bohr magneton}$$

Covariance of Dirac spinor

$$x^\mu\to x'^\mu=a^\mu{}_\nu x^\nu$$

$$\psi(x)\to\psi'(x')=S(a)\psi(x) \qquad \bar{\psi}(x)\to\bar{\psi}(x)S(a)^{-1}$$

$$S^{-1}\gamma^\mu S=a^\mu{}_\nu\gamma^\nu \qquad S^{-1}=\gamma^0S^\dagger\gamma^0$$

boost with 4 momentum $p^\mu=(E,\vec{p})$, $m=\sqrt{p^2}$

$$S(\vec{p})=\sqrt{\frac{E+m}{2\,m}}\begin{pmatrix} 1 & \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} & 1 \end{pmatrix} \qquad \vec{\sigma}\cdot\vec{p}=\begin{pmatrix} p_3 & p_1-i\,p_2 \\ p_1+i\,p_2 & -p_3 \end{pmatrix}$$

boost with $\hat{n}$, $\cosh\alpha=\gamma$

$$S(\alpha)=\begin{pmatrix} \cosh\frac{\alpha}{2} & \hat{n}\cdot\vec{\sigma}\sinh\frac{\alpha}{2} \\ \hat{n}\cdot\vec{\sigma}\sinh\frac{\alpha}{2} & \cosh\frac{\alpha}{2} \end{pmatrix}=\sqrt{\frac{\gamma+1}{2}}\begin{pmatrix} 1 & \hat{n}\cdot\vec{\sigma}\frac{\gamma\beta}{\gamma+1} \\ \hat{n}\cdot\vec{\sigma}\frac{\gamma\beta}{\gamma+1} & 1 \end{pmatrix}$$

charge conjugation of Dirac spinor

$$C=i\gamma^2\gamma^0=\begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}$$

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu \qquad C\{\gamma^\mu(1-\gamma^5)\}^T C^{-1} = -\gamma^\mu(1+\gamma^5)$$

$$C = C^* = -C^{-1} = -C^T = -C^\dagger$$

$$\psi^C = C\bar{\psi}^T = i\gamma^2\psi^* \qquad \bar{\psi}^C = -\psi^T C^{-1} = -i\bar{\psi}^*\gamma^2$$

$$u = \begin{pmatrix} \phi_s \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\phi_s \end{pmatrix} \to u^C = i\gamma^2 u^* = \begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s \\ \chi_s \end{pmatrix}, \text{ where } \chi_s = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\phi_s^*$$

$$v = \begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s \\ \chi_s \end{pmatrix} \to v^C = i\gamma^2 v^* = \begin{pmatrix} \phi_\alpha \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\phi_s \end{pmatrix}, \text{ where } \phi_s = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\chi_s^*$$

parity transformation of Dirac spinor

$$P = \eta_P\gamma^0 = \eta_P\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad P^2 = 1 \qquad \eta_P^2 = 1 \qquad P^{-1} = \eta_P\gamma^0$$

$$P^{-1}\gamma^\mu P = (\gamma^0; -\gamma^i) = \gamma_\mu$$

$$Pu_s(\vec{p}) = u_s(-\vec{p}) \qquad Pv_s(\vec{p}) = -v_s(-\vec{p}) \qquad \eta_P \equiv 1$$

$$P_L = \frac{1-\gamma^5}{2} \qquad P_R = \frac{1+\gamma^5}{2}$$

$$P_L + P_R = 1 \qquad P_{L,R}^2 = P_{L,R} \qquad P_L P_R = P_R P_L = 0 \qquad P^{-1}P_{L,R}P = P_{R,L}$$

Chirality(Weyl) Representation

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\psi_L \equiv P_L\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \qquad \psi_R \equiv P_R\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \qquad P_L\gamma^\mu = \gamma^\mu P_R$$

$$\bar{\psi}_L = \bar{\psi}P_R \qquad \bar{\psi}_L\psi_L = \bar{\psi}_R\psi_R = 0 \qquad \bar{\psi}_L\gamma^\mu\psi_R = \bar{\psi}_R\gamma^\mu\psi_L = 0$$

$$\psi_D = T\psi_W \qquad \gamma_D = T\gamma_W T^{-1} \qquad T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad T^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$S_W(\vec{p}) = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} 1 - \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} & 0 \\ 0 & 1 + \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}$$

Bilinear Expression

$\bar{\psi}\psi$	$\bar{\psi}\gamma^5\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma^5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$
(S)	(P)	(V)	(A)	(T)
1	1	4	4	6

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] = i[\gamma^\mu\gamma^\nu - g^{\mu\nu}]$$

$$\bar{u}(p')\gamma^\mu u(p) = \frac{1}{m+m'}\bar{u}(p')[(p+p')^\mu - i\sigma^{\mu\nu}q_\nu]u(p) \qquad q \equiv p - p'$$

Propagator

$$\begin{aligned} i\,\Delta_F(x'-x) &= \langle 0|\mathcal{T}(\phi(x')\phi^\dagger(x))|0\rangle \\ &= \theta(t'-t)\langle 0|\phi(x')\phi^\dagger(x)|0\rangle \underset{\substack{\text{boson} \\ \text{fermion}}}{\pm} \theta(t-t')\langle 0|\phi^\dagger(x)\phi(x')|0\rangle \end{aligned}$$

$$\theta(t) = \frac{-1}{2\pi i}\int dk^0 \frac{e^{-ik^0 t}}{k^0 + i\varepsilon}$$

$$i\Delta_F(x' - x) = i\int \frac{d^4k}{(2\pi)^4}\, i\tilde{\Delta}_F(k)\, e^{-ik(x'-x)}$$

Klein-Gordon: $i\tilde{\Delta}_F(p) = \dfrac{i}{p^2 - m^2 + i\,\varepsilon}$

Gauge field: $i\tilde{D}_F(p) = \dfrac{-ig^{\mu\nu}}{p^2 - m^2 + i\,\varepsilon}$

Dirac field: $i\tilde{S}_F(p) = \dfrac{i(\not{p} + m)}{p^2 - m^2 + i\,\varepsilon}$

Local Gauge Transformation: scalar field with U(1)

$$D_\mu = \partial_\mu + iqA_\mu \qquad (p^\mu \to p^\mu - qA^\mu)$$

$$\begin{aligned}\mathcal{L} &= (D^\mu\phi)^\dagger(D_\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &= (\partial^\mu\phi)^\dagger(\partial_\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j_\mu A^\mu\end{aligned} \qquad j_\mu = qJ_\mu = iq[\phi^\dagger\partial_\mu\phi - (\partial_\mu\phi)^\dagger\phi]$$

Gauge Field

U(1): $e^{ig_1\beta(x)\cdot Y/2}$ $\qquad B_\mu$: Abelian

SU(2): $e^{ig_2\vec{\alpha}(x)\cdot\vec{\tau}/2}$ $\qquad W_\mu$: non-Abelian $\qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = if_{abc}\frac{\tau_c}{2}$

$$D_\mu = \partial_\mu + i\,g_1\frac{Y}{2}B_\mu + i\,g_2\frac{\tau_a}{2}W_{a\mu}$$

$$B_\mu \to B'_\mu = B_\mu - \partial_\mu\beta \qquad W_{a\mu} \to W'_{a\mu} = W_{a\mu} - \partial_\mu\alpha_a + g_2 f_{abc}W_{b\mu}\alpha_c$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \qquad W_{a\mu\nu} \equiv \partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} - g_2 f_{abc}W_{b\mu}W_{c\nu}$$

SU(3)

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if_{abc}\frac{\lambda_c}{2} \qquad \mathrm{Tr}(\lambda_a) = 0 \qquad \mathrm{Tr}(\lambda_a\lambda_b) = 2\delta_{ab} \qquad f_{abc} = \frac{1}{4i}\mathrm{Tr}([\lambda_a, \lambda_b]\lambda_c)$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$f_{123} = 1$$

$$f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

Jacobi identity $f_{abc}f_{ck\ell} + f_{bkc}f_{ca\ell} + f_{kac}f_{cb\ell} = 0$

S-Matrix

$$H_0|n,t\rangle = i\frac{\partial}{\partial t}|n,t\rangle \qquad |n,t\rangle = U_0(t)|n\rangle$$

$$(H_0 + V)|\psi,t\rangle = i\frac{\partial}{\partial t}|\psi,t\rangle \qquad |\psi,t\rangle = U(t)|\psi\rangle \qquad V(t) = -\int d^3x\mathcal{L}_{\mathrm{int}}(x)$$

$$U(t) = U_0(t)\mathcal{T}\exp\left[-i\int_{t_0}^{t} dt' U_0^\dagger(t')\, V(t') U_0(t')\right] U_0^\dagger(t_0)$$
$$S \equiv U(\infty)U^\dagger(-\infty) = U_0(\infty)\mathcal{T}\exp\left[i\int d^4x' U_0^\dagger(t')\mathcal{L}_{\rm int}(x')U_0(t')\right]U_0^\dagger(-\infty)$$

$$\begin{aligned}
T_{fi} &\equiv \langle f,\infty|S|i,-\infty\rangle = \left\langle f\left|\mathcal{T}\exp\left[i\int d^4x' U_0^\dagger(t')\mathcal{L}_{\rm int}(x')U_0(t')\right]\right|i\right\rangle \\
&= \delta_{fi} + i\int d^4x' \langle f,t'|\mathcal{L}_{\rm int}(x')|i,t'\rangle \\
&\quad + \frac{i^2}{2!}\int d^4x' \int d^4x'' \sum_n \mathcal{T}[\langle f,t'|\mathcal{L}_{\rm int}(x')|n,t'\rangle\langle n,t''|\mathcal{L}_{\rm int}(x'')|i,t''\rangle] + \cdots \\
&\equiv \delta_{fi} - i(2\pi)^4\delta^4(p_f - p_i)\mathcal{M}_{fi}
\end{aligned}$$

Matrix Element for $-i\mathcal{M}$

$$|\bar\psi_f\,\Gamma\,\psi_i|^2 = \bar\psi_f\,\Gamma\,\psi_i\,\bar\psi_i\,\gamma^0\,\Gamma^\dagger\,\gamma^0\,\psi_f = \mathrm{Tr}[\Gamma\,\psi_i\,\bar\psi_i\,\gamma^0\,\Gamma^\dagger\,\gamma^0\,\psi_f\bar\psi_f]$$
$$X = (x_i),\ Y = (y_i) \Longrightarrow X^T Y = \mathrm{Tr}[Y X^T]$$
$$\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu \qquad \gamma^0(1-\gamma^5)^\dagger\gamma^0 = 1+\gamma^5 \qquad \gamma^0(\psi_i\,\bar\psi_j)^\dagger\gamma^0 = \psi_j\,\bar\psi_i$$

External Lines

spin 1/2 incoming: $u(p,s)$ or $v(p,s)$ outgoing: $\bar u(p,s)$ or $\bar v(p,s)$

spin 1 incoming: $\varepsilon_\mu(p,\lambda)$ outgoing: $\varepsilon_\mu^*(p,\lambda)$

Internal Lines

spin 0 $\dfrac{i}{p^2 - m^2 + i\,\varepsilon}$

spin 1/2 $\dfrac{i(\not p + m)}{p^2 - m^2 + i\,\varepsilon}$

spin 1 $\dfrac{i}{k^2 + i\,\varepsilon}\left[-g^{\mu\nu} + (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right]$ $\xi = 1$ for Feynman gauge

$\dfrac{i}{p^2 - m^2 + i\,\varepsilon}\left(-g^{\mu\nu} + \dfrac{p^\mu p^\nu}{m^2}\right)$ Unitary gauge

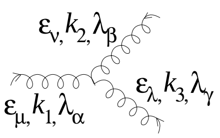
$\dfrac{i}{p^2 - m^2 + i\,\varepsilon}\left[-g^{\mu\nu} + (1-\xi)\dfrac{p^\mu p^\nu}{p^2 - \xi m^2}\right]$ R gauge

Vertexes

EM current for charge $+e$ $-ie\gamma^\mu$

EW charged current $-i\dfrac{g}{\sqrt{2}}\gamma^\mu\dfrac{1-\gamma^5}{2}$

3 gluons vertex $-g_s f_{\alpha\beta\gamma}[g^{\mu\nu}(k_1-k_2)^\lambda + g^{\nu\lambda}(k_2-k_3)^\mu + g^{\lambda\mu}(k_3-k_1)^\nu]$



$\gamma(k_\gamma,\varepsilon_\mu)$-$W^+(k_+,\varepsilon_\nu)$-$W^-(k_-,\varepsilon_\lambda)$ vertex

$$ie[g^{\nu\lambda}(k_+ - k_-)^\mu + g^{\lambda\mu}(k_- - k_\gamma)^\nu + g^{\mu\nu}(k_\gamma - k_+)^\lambda]$$

Conservation law

Q num\Int.	Strong	EM	Weak
Isospin I	O	X	X
I_3	O	O	X
Parity P	O	O	X
C-parity C	O	O	X
G-parity G	O	X	X