$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad j,k = 1\sim 3$$

	T	T^3	Y	$Q=T^3+\frac{Y}{2}$
ν_L	$1/2$	$1/2$	-1	0
ℓ_L		$-1/2$		-1
ν_R	0	0	0	0
ℓ_R	0	0	-2	-1

	T	T^3	Y	Q
u_L	$1/2$	$1/2$	$1/3$	$2/3$
d_L		$-1/2$		$-1/3$
u_R	0	0	$4/3$	$2/3$
d_R	0	0	$-2/3$	$-1/3$

$$\Phi(x) \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = e^{i\left\{\frac{\tau^l}{2}\chi^l(x)\right\}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+\phi(x) \end{pmatrix}$$

	T	T^3	Y	Q
ϕ^+	$1/2$	$1/2$	1	1
ϕ^0		$-1/2$		0

$$\tilde{\Phi}(x) \equiv i\tau^2 \Phi^*(x)$$

$$\left[\frac{\tau^l}{2}, \frac{\tau^m}{2}\right] = i\epsilon_{lmn}\frac{\tau^n}{2} \qquad l,m,n = 1\sim 3 \qquad \left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i\, f_{abc}\frac{\lambda^c}{2} \qquad a,b,c = 1\sim 8$$

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}^{\text{kin}}_{\text{lepton}} + \mathcal{L}^{\text{mass}}_{\text{lepton}} + \mathcal{L}^{\text{kin}}_{\text{quark}} + \mathcal{L}^{\text{mass}}_{\text{quark}}$$

$$\begin{aligned}\mathcal{L}_{\text{gauge}} = & -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu A^l_\nu - \partial_\nu A^l_\mu - g\epsilon_{lmn}A^m_\mu A^n_\nu)^2 \\ & -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f_{abc} G^b_\mu G^c_\nu)^2\end{aligned}$$

$$\mathcal{L}_{\text{Higgs}} = \left|\left(i\partial_\mu - g\prime\frac{1}{2}Y_H B_\mu - g\frac{\tau^l}{2}A^l_\mu\right)\Phi\right|^2 + \mu^2\Phi^\dagger\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2$$

$$\mathcal{L}^{\text{kin}}_{\text{lepton}} = \bar{L}^j_L\gamma^\mu\left[i\partial_\mu - g\prime\frac{Y_{\ell L}}{2}B_\mu - g\frac{\tau^l}{2}A^l_\mu\right]L^j_L + \bar{\ell}^j_R\gamma^\mu\left[i\partial_\mu - g\prime\frac{Y_{\ell R}}{2}B_\mu\right]\ell^j_R + \bar{\nu}^j_R\gamma^\mu i\partial_\mu\nu^j_R$$

$$\begin{aligned}\mathcal{L}^{\text{mass}}_{\text{lepton}} &= -\left(\bar{L}\prime^j_L\Phi\right)Y^{jk}_\ell \ell\prime^k_R - \left(\bar{L}\prime^j_L\tilde{\Phi}\right)Y^{jk}_\nu \nu\prime^k_R + h.c. \\ &= -y^j_\ell\left(\bar{L}^j_L\Phi\right)\ell^j_R - y^j_\nu\left[(\bar{L}_L U_\nu)^j\tilde{\Phi}\right]\nu^j_R + h.c.\end{aligned}$$

$$V_{\ell L}\, Y_\ell V^\dagger_{\ell R} \equiv \text{diag}\left(y^j_\ell\right) \qquad V_{\nu L}\, Y_\nu V^\dagger_{\nu R} \equiv \text{diag}\left(y^j_\nu\right) \qquad V_{\ell L}V^\dagger_{\nu L} \equiv U_\nu$$

$$L^j_L \equiv V^{jk}_{\ell L}L\prime^k_L \qquad \ell^j_R \equiv V^{jk}_{\ell R}\ell\prime^k_R \qquad \nu^j_R \equiv V^{jk}_{\ell R}\nu\prime^k_R$$

$$\begin{aligned}\mathcal{L}^{\text{kin}}_{\text{quark}} = & \; \bar{Q}^j_L\gamma^\mu\left[i\partial_\mu - g\prime\frac{Y_{QL}}{2}B_\mu - g\frac{\tau^l}{2}A^l_\mu - g_s\frac{\lambda^a}{2}G^a_\mu\right]Q^j_L \\ & + \bar{u}^j_R\gamma^\mu\left[i\partial_\mu - g\prime\frac{Y_{uR}}{2}B_\mu - g_s\frac{\lambda^a}{2}G^a_\mu\right]u^j_R + \bar{d}^j_R\gamma^\mu\left[i\partial_\mu - g\prime\frac{Y_{dR}}{2}B_\mu - g_s\frac{\lambda^a}{2}G^a_\mu\right]d^j_R\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{\text{mass}}_{\text{quark}} &= -\left(\bar{Q}\prime^j_L\Phi\right)Y^{jk}_d d\prime^k_R - \left(\bar{Q}\prime^j_L\tilde{\Phi}\right)Y^{jk}_u u\prime^k_R + h.c. \\ &= -y^j_d[(\bar{Q}_L U_d)^j\Phi]d^j_R - y^j_u\left(\bar{Q}^j_L\tilde{\Phi}\right)u^j_R + h.c.\end{aligned}$$

$$V_{uL}\, Y_u V^\dagger_{uR} \equiv \text{diag}\left(y^j_u\right) \qquad V_{dL}\, Y_d V^\dagger_{dR} \equiv \text{diag}\left(y^j_d\right) \qquad V_{uL}V^\dagger_{dL} \equiv U_d$$

$$Q^j_L \equiv V^{jk}_{uL}Q\prime^k_L \qquad u^j_R \equiv V^{jk}_{uR}u\prime^k_R \qquad d^j_R \equiv V^{jk}_{uR}d\prime^k_R$$

$$\Phi(x) \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\phi(x) \end{pmatrix} \qquad \tilde{\Phi}(x) \to \frac{1}{\sqrt{2}}\begin{pmatrix} v+\phi(x) \\ 0 \end{pmatrix} \qquad V(\Phi) \equiv -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2$$

$$\cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}} \qquad e \equiv \frac{g\,g'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

$$\frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{\lambda}} \qquad m \equiv \sqrt{2}\mu \qquad M_W \equiv \frac{g\,v}{2} \qquad M_Z \equiv \frac{1}{2}\sqrt{g^2+g'^2}\,v = \frac{M_W}{\cos\theta_W}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} \qquad \sin^2\theta_W \approx 0.23$$

$$W_\mu \equiv \frac{1}{\sqrt{2}}(A^1_\mu - iA^2_\mu) \qquad F^A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \qquad F^Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu$$

$$\mathcal{D}_\mu W_\nu \equiv (\partial_\mu + igA^3_\mu)W_\nu = (\partial_\mu + i\,eA_\mu + ig\cos\theta_W Z_\mu)W_\nu$$

$$m^j_f \equiv \frac{y^j_f v}{\sqrt{2}} \qquad y^j_f = \sqrt{2}\,\frac{m^j_f}{v} = g\,\frac{m^j_f}{\sqrt{2}M_W} \qquad \text{where} \quad f = \ell,\nu,u,d$$

$$\nu^j \equiv U^{jk}_\nu \hat{\nu}^k \ (U_\nu\text{: PMNS matrix}) \qquad d^j \equiv U^{jk}_d \hat{d}^k \ (U_d\text{: CKM matrix})$$

$$J^\mu \equiv \frac{1}{2}\bar{\ell}^{\,j}\gamma^\mu(1-\gamma^5)\nu^j + \frac{1}{2}\bar{d}^{\,j}\gamma^\mu(1-\gamma^5)u^j = \frac{1}{2}\bar{\ell}^{\,j}\gamma^\mu(1-\gamma^5)U^{jk}_\nu\hat{\nu}^k + \frac{1}{2}U^{jk*}_d\bar{\hat{d}}^{\,k}\gamma^\mu(1-\gamma^5)u^j$$

$$J^\mu_{\rm EM} \equiv (-)\bar{\ell}^{\,j}\gamma^\mu\ell^j + \left(\frac{2}{3}\right)\bar{u}^{\,j}\gamma^\mu u^j + \left(-\frac{1}{3}\right)\bar{d}^{\,j}\gamma^\mu d^j$$

$$J^{3\mu} \equiv \frac{1}{4}\bar{\nu}^{\,j}\gamma^\mu(1-\gamma^5)\nu^j - \frac{1}{4}\bar{\ell}^{\,j}\gamma^\mu(1-\gamma^5)\ell^j + \frac{1}{4}\bar{u}^{\,j}\gamma^\mu(1-\gamma^5)u^j - \frac{1}{4}\bar{d}^{\,j}\gamma^\mu(1-\gamma^5)d^j$$

$$J^\mu_Z \equiv J^{3\mu} - \sin^2\theta_W J^\mu_{\rm EM}$$

$$\begin{aligned}\mathcal{L}_{\rm gauge} = \ & -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu} - \frac{1}{2}(\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu)^\dagger(\mathcal{D}^\mu W^\nu - \mathcal{D}^\nu W^\mu) \\ & +i(e\,F^A_{\mu\nu} + g\cos\theta_W F^Z_{\mu\nu})W^{\dagger\mu}W^\nu + \frac{g^2}{2}(|W_\mu W^\mu|^2 - |W_\mu W^\nu|^2) \\ & -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s\,f_{abc}\,G^b_\mu\,G^c_\nu)^2\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\rm Higgs} = \ & M^2_W W^\dagger_\mu W^\mu + \frac{1}{2}M^2_Z Z^2_\mu + \left(g\,M_W\phi + \frac{g^2}{4}\phi^2\right)\left(W^\dagger_\mu W^\mu + \frac{1}{2\cos^2\theta_W}Z^2_\mu\right) \\ & +\frac{1}{2}[(\partial_\mu\phi)^2 - m^2\phi^2] - \frac{m\sqrt{\lambda}}{2}\phi^3 - \frac{\lambda}{8}\phi^4 - V\left(\frac{v}{\sqrt{2}}\right)\end{aligned}$$

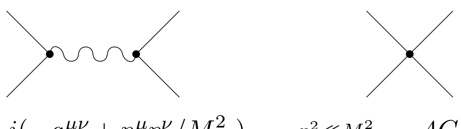
$$\mathcal{L}^{\rm kin}_{\rm lepton} = \bar{\ell}^{\,j}i\partial\!\!\!/\ell^j + \bar{\nu}^{\,j}i\partial\!\!\!/\nu^j + \mathcal{L}^{\rm (lepton)}_{\rm EW\,int}$$

$$\mathcal{L}^{\rm mass}_{\rm lepton} = -\left(m^j_\ell + \frac{y^j_\ell}{\sqrt{2}}\phi\right)\bar{\ell}^{\,j}\ell^j - \left(m^j_\nu + \frac{y^j_\nu}{\sqrt{2}}\phi\right)\bar{\hat{\nu}}^{\,j}\hat{\nu}^{\,j}$$

$$\mathcal{L}^{\rm kin}_{\rm quark} = \bar{u}^{\,j}i\partial\!\!\!/u^j + \bar{d}^{\,j}i\partial\!\!\!/d^j - g_s\bar{u}^{\,j}\gamma^\mu\frac{\lambda^a}{2}u^jG^a_\mu - g_s\bar{d}^{\,j}\gamma^\mu\,\frac{\lambda^a}{2}d^jG^a_\mu + \mathcal{L}^{\rm (quark)}_{\rm EW\,int}$$

$$\mathcal{L}^{\rm mass}_{\rm quark} = -\left(m^j_d + \frac{y^j_d}{\sqrt{2}}\phi\right)\bar{\hat{d}}^{\,j}\hat{d}^j - \left(m^j_u + \frac{y^j_u}{\sqrt{2}}\phi\right)\bar{u}^{\,j}u^j$$

$$\mathcal{L}_{\rm EW\,int} = -\frac{g}{\sqrt{2}}\big(J^{\mu\dagger}W_\mu + J^\mu W^\dagger_\mu\big) - e\,J^\mu_{\rm EM}A_\mu - \frac{g}{\cos\theta_W}J^\mu_Z Z_\mu$$



$$\left(-i\frac{g}{\sqrt{2}}\right)^2 J^\dagger_\mu\frac{i(-g^{\mu\nu} + p^\mu p^\nu/M^2_W)}{p^2 - M^2_W}J_\nu \quad \xrightarrow{p^2 \ll M^2_W} \ -i\frac{4G}{\sqrt{2}}J^\dagger_\mu J^\mu \qquad \frac{G}{\sqrt{2}} = \frac{g^2}{8M^2_W} = \frac{1}{2v^2}$$