

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad j, k = 1 \sim 3$$

	T	T^3	Y	$Q = T^3 + \frac{Y}{2}$
ν_L	$1/2$	$1/2$	-1	0
ℓ_L	$1/2$	$-1/2$	-1	-1
ν_R	0	0	0	0
ℓ_R	0	0	-2	-1

	T	T^3	Y	Q
u_L	$1/2$	$1/2$	$1/3$	$2/3$
d_L	$1/2$	$-1/2$	$1/3$	$-1/3$
u_R	0	0	$4/3$	$2/3$
d_R	0	0	$-2/3$	$-1/3$

$$\Phi(x) \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = e^{i\left\{\frac{\tau^l}{2}\chi^l(x)\right\}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \phi(x) \end{pmatrix}$$

	T	T^3	Y	Q
ϕ^+	$1/2$	$1/2$	1	1
ϕ^0	$1/2$	$-1/2$	1	0

$$\tilde{\Phi}(x) \equiv i\tau^2 \Phi^*(x)$$

$$\left[\frac{\tau^l}{2}, \frac{\tau^m}{2} \right] = i\epsilon_{lmn} \frac{\tau^n}{2} \quad l, m, n = 1 \sim 3 \quad \left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right] = i f_{abc} \frac{\lambda^c}{2} \quad a, b, c = 1 \sim 8$$

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{lepton}}^{\text{kin}} + \mathcal{L}_{\text{lepton}}^{\text{mass}} + \mathcal{L}_{\text{quark}}^{\text{kin}} + \mathcal{L}_{\text{quark}}^{\text{mass}}$$

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = & -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu A_\nu^l - \partial_\nu A_\mu^l - g\epsilon_{lmn} A_\mu^m A_\nu^n)^2 \\ & -\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc} G_\mu^b G_\nu^c)^2 \end{aligned}$$

$$\mathcal{L}_{\text{Higgs}} = \left| \left(i\partial_\mu - g' \frac{1}{2} Y_H B_\mu - g \frac{\tau^l}{2} A_\mu^l \right) \Phi \right|^2 + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{2} (\Phi^\dagger \Phi)^2$$

$$\mathcal{L}_{\text{lepton}}^{\text{kin}} = \bar{L}_L^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{\ell L}}{2} B_\mu - g \frac{\tau^l}{2} A_\mu^l \right] L_L^j + \bar{\ell}_R^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{\ell R}}{2} B_\mu \right] \ell_R^j + \bar{\nu}_R^j \gamma^\mu i\partial_\mu \nu_R^j$$

$$\begin{aligned} \mathcal{L}_{\text{lepton}}^{\text{mass}} = & -(\bar{L}_L^j \Phi) Y_\ell^{jk} \ell_R^k - (\bar{L}_L^j \tilde{\Phi}) Y_\nu^{jk} \nu_R^k + h.c. \\ = & -y_\ell^j (\bar{L}_L^j \Phi) \ell_R^j - y_\nu^j [(\bar{L}_L U_\nu)^j \tilde{\Phi}] (U_\nu^\dagger \nu_R)^j + h.c. \end{aligned}$$

$$V_\ell Y_\ell V_\ell^\dagger \equiv \text{diag}(y_\ell^j) \quad V_\nu Y_\nu V_\nu^\dagger \equiv \text{diag}(y_\nu^j) \quad V_\nu V_\ell^\dagger \equiv U_\nu^\dagger$$

$$L_L^j \equiv V_\ell^{jk} L_L^k \quad \ell_R^j \equiv V_\ell^{jk} \ell_R^k \quad \nu_R^j \equiv V_\ell^{jk} \nu_R^k$$


$$\begin{aligned} \mathcal{L}_{\text{quark}}^{\text{kin}} = & \bar{Q}_L^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{QL}}{2} B_\mu - g \frac{\tau^l}{2} A_\mu^l - g_s \frac{\lambda^a}{2} G_\mu^a \right] Q_L^j \\ & + \bar{u}_R^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{uR}}{2} B_\mu - g_s \frac{\lambda^a}{2} G_\mu^a \right] u_R^j + \bar{d}_R^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{dR}}{2} B_\mu - g_s \frac{\lambda^a}{2} G_\mu^a \right] d_R^j \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{quark}}^{\text{mass}} = & -(\bar{Q}_L^j \Phi) Y_d^{jk} d_R^k - (\bar{Q}_L^j \tilde{\Phi}) Y_u^{jk} u_R^k + h.c. \\ = & -y_d^j [(\bar{Q}_L U_d)^j \Phi] (U_d^\dagger d_R)^j - y_u^j (\bar{Q}_L \tilde{\Phi}) u_R^j + h.c. \end{aligned}$$

$$V_u Y_u V_u^\dagger \equiv \text{diag}(y_u^j) \quad V_d Y_d V_d^\dagger \equiv \text{diag}(y_d^j) \quad V_d V_u^\dagger \equiv U_d^\dagger$$

$$Q_L^j \equiv V_u^{jk} Q_L^k \quad u_R^j \equiv V_u^{jk} u_R^k \quad d_R^j \equiv V_u^{jk} d_R^k$$

$$\begin{aligned}
\Phi(x) &\rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \phi(x) \end{pmatrix} & \tilde{\Phi}(x) &\rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} v + \phi(x) \\ 0 \end{pmatrix} & V(\Phi) &\equiv -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 \\
\cos\theta_W &\equiv \frac{g}{\sqrt{g^2 + g'^2}} & \sin\theta_W &\equiv \frac{g'}{\sqrt{g^2 + g'^2}} & e &\equiv \frac{g g'}{\sqrt{g^2 + g'^2}} = g \sin\theta_W = g' \cos\theta_W \\
\frac{v}{\sqrt{2}} &= \frac{\mu}{\sqrt{\lambda}} & m &\equiv \sqrt{2} \mu & M_W &\equiv \frac{g v}{2} & M_Z &\equiv \frac{1}{2} \sqrt{g^2 + g'^2} v = \frac{M_W}{\cos\theta_W} \\
\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} &\equiv \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix} & \sin^2\theta_W &\approx 0.23 \\
W_\mu &\equiv \frac{1}{\sqrt{2}} (A_\mu^1 - i A_\mu^2) & F_{\mu\nu}^A &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu & F_{\mu\nu}^Z &\equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu \\
\mathcal{D}_\mu W_\nu &\equiv (\partial_\mu + i g A_\mu^3) W_\nu = (\partial_\mu + i e A_\mu + i g \cos\theta_W Z_\mu) W_\nu \\
m_f^j &\equiv \frac{y_f^j v}{\sqrt{2}} & y_f^j &= \sqrt{2} \frac{m_f^j}{v} = g \frac{m_f^j}{\sqrt{2} M_W} & \text{where } f &= \ell, \nu, u, d \\
\nu^j &\equiv U_\nu^{jk} \hat{\nu}^k \text{ (} U_\nu \text{: PMNS matrix)} & d^j &\equiv U_d^{jk} \hat{d}^k \text{ (} U_d \text{: CKM matrix)} \\
J^\mu &\equiv \frac{1}{2} \bar{\ell}^j \gamma^\mu (1 - \gamma^5) \nu^j + \frac{1}{2} \bar{d}^j \gamma^\mu (1 - \gamma^5) u^j = \frac{1}{2} \bar{\ell}^j \gamma^\mu (1 - \gamma^5) U_\nu^{jk} \hat{\nu}^k + \frac{1}{2} U_d^{jk*} \bar{\hat{d}}^k \gamma^\mu (1 - \gamma^5) u^j \\
J_{\text{EM}}^\mu &\equiv (-) \bar{\ell}^j \gamma^\mu \ell^j + \left(\frac{2}{3}\right) \bar{u}^j \gamma^\mu u^j + \left(-\frac{1}{3}\right) \bar{d}^j \gamma^\mu d^j \\
J_Z^\mu &\equiv \frac{1}{4} \bar{\nu}^j \gamma^\mu (1 - \gamma^5) \nu^j - \frac{1}{4} \bar{\ell}^j \gamma^\mu (1 - \gamma^5) \ell^j + \frac{1}{4} \bar{u}^j \gamma^\mu (1 - \gamma^5) u^j - \frac{1}{4} \bar{d}^j \gamma^\mu (1 - \gamma^5) d^j \\
J_Z^\mu &\equiv J_Z^\mu - \sin^2\theta_W J_{\text{EM}}^\mu \\
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2} (\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu)^\dagger (\mathcal{D}^\mu W^\nu - \mathcal{D}^\nu W^\mu) \\
&\quad + i(e F_{\mu\nu}^A + g \cos\theta_W F_{\mu\nu}^Z) W^{\dagger\mu} W^\nu + \frac{g^2}{2} (|W_\mu W^\mu|^2 - |W_\mu W^\nu|^2) \\
&\quad - \frac{1}{4} (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc} G_\mu^b G_\nu^c)^2 \\
\mathcal{L}_{\text{Higgs}} &= M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu^2 + \left(g M_W \phi + \frac{g^2}{4} \phi^2\right) \left(W_\mu^\dagger W^\mu + \frac{1}{2 \cos^2\theta_W} Z_\mu^2\right) \\
&\quad + \frac{1}{2} [(\partial_\mu \phi)^2 - m^2 \phi^2] - \frac{m \sqrt{\lambda}}{2} \phi^3 - \frac{\lambda}{8} \phi^4 - V\left(\frac{v}{\sqrt{2}}\right) \\
\mathcal{L}_{\text{lepton}}^{\text{kin}} &= \bar{\ell}^j i \not{\partial} \ell^j + \bar{\nu}^j i \not{\partial} \nu^j + \mathcal{L}_{\text{EW int}}^{(\text{lepton})} \\
\mathcal{L}_{\text{lepton}}^{\text{mass}} &= -\left(m_\ell^j + \frac{y_\ell^j}{\sqrt{2}} \phi\right) \bar{\ell}^j \ell^j - \left(m_\nu^j + \frac{y_\nu^j}{\sqrt{2}} \phi\right) \bar{\nu}^j \nu^j \\
\mathcal{L}_{\text{quark}}^{\text{kin}} &= \bar{u}^j i \not{\partial} u^j + \bar{d}^j i \not{\partial} d^j - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G_\mu^a - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G_\mu^a + \mathcal{L}_{\text{EW int}}^{(\text{quark})} \\
\mathcal{L}_{\text{quark}}^{\text{mass}} &= -\left(m_d^j + \frac{y_d^j}{\sqrt{2}} \phi\right) \bar{d}^j d^j - \left(m_u^j + \frac{y_u^j}{\sqrt{2}} \phi\right) \bar{u}^j u^j \\
\mathcal{L}_{\text{EW int}} &= -\frac{g}{\sqrt{2}} (J^\mu{}^\dagger W_\mu + J^\mu W_\mu^\dagger) - e J_{\text{EM}}^\mu A_\mu - \frac{g}{\cos\theta_W} J_Z^\mu Z_\mu
\end{aligned}$$



$$\left(-i \frac{g}{\sqrt{2}}\right)^2 J_\mu^\dagger \frac{i(-g^{\mu\nu} + p^\mu p^\nu / M_W^2)}{p^2 - M_W^2} J_\nu \xrightarrow{p^2 \ll M_W^2} -i \frac{4G}{\sqrt{2}} J_\mu^\dagger J^\mu \quad \frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$