$$\mathcal{L}=\mathcal{L}_{\rm gauge}+\mathcal{L}_{\rm Higgs}+\mathcal{L}^{\rm kin}_{\rm lepton}+\mathcal{L}^{\rm mass}_{\rm lepton}+\mathcal{L}^{\rm kin}_{\rm quark}+\mathcal{L}^{\rm mass}_{\rm quark}$$

$$j,k=1\sim 3 \qquad L_L=\begin{pmatrix}\nu_L\\ \ell_L\end{pmatrix} \qquad Q_L=\begin{pmatrix}u_L\\ d_L\end{pmatrix}$$

	T	T^3	Y	$Q=T^3+\frac{Y}{2}$
ν_L	$1/2$	$1/2$	-1	0
ℓ_L		$-1/2$		-1
ν_R	0	0	0	0
ℓ_R	0	0	-2	-1

	T	T^3	Y	Q
u_L	$1/2$	$1/2$	$1/3$	$2/3$
d_L		$-1/2$		$-1/3$
u_R	0	0	$4/3$	$2/3$
d_R	0	0	$-2/3$	$-1/3$

	T	T^3	Y	Q
ϕ^+	$1/2$	$1/2$	1	1
ϕ^0		$-1/2$		0

$$\left[\frac{\tau^l}{2},\frac{\tau^m}{2}\right]=i\epsilon_{lmn}\frac{\tau^n}{2} \qquad \left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]=i\,f_{abc}\frac{\lambda^c}{2} \qquad \begin{matrix}lmn=1\sim 3\\ abc=1\sim 8\end{matrix}$$

EWSB in Higgs field

$$\Phi(x)\equiv\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\equiv e^{i\left\{\frac{\tau^l}{2}\chi^l(x)\right\}}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\phi(x)\end{pmatrix}\stackrel{\rm EWSB}{\longrightarrow}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\phi(x)\end{pmatrix}$$

$$\frac{v}{\sqrt{2}}\equiv\frac{\mu}{\sqrt{\lambda}}$$

$$\tilde{\Phi}\equiv i\tau^2\Phi^*\to\frac{1}{\sqrt{2}}\begin{pmatrix}v+\phi(x)\\ 0\end{pmatrix}$$

Gauge field

$$W_\mu\equiv\frac{1}{\sqrt{2}}(A^1_\mu-i\,A^2_\mu)\,:\,W^+\text{-field} \qquad W^\dagger_\mu\,:\,W^-\text{-field}$$

$$A^1_\mu=\frac{1}{\sqrt{2}}(W_\mu+W^\dagger_\mu) \qquad A^2_\mu=\frac{i}{\sqrt{2}}(W_\mu-W^\dagger_\mu)$$

$$\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix}\equiv\begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}A^3_\mu\\ B_\mu\end{pmatrix} \qquad \begin{pmatrix}A^3_\mu\\ B_\mu\end{pmatrix}=\begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix}$$

$$\cos\theta_W\equiv\frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W\equiv\frac{g'}{\sqrt{g^2+g'^2}}$$

$$e\equiv\frac{g\,g'}{\sqrt{g^2+g'^2}}=g\sin\theta_W=g'\cos\theta_W$$

$$gA^3_\mu=eA_\mu+g\cos\theta_W Z_\mu \qquad g'B_\mu=eA_\mu-\frac{g}{\cos\theta_W}\sin^2\theta_W Z_\mu$$

$$\mathcal{D}_\mu W_\nu\equiv(\partial_\mu+igA^3_\mu)W_\nu=(\partial_\mu+ieA_\mu+ig\cos\theta_W Z_\mu)W_\nu$$

$$F^A_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu \qquad F^Z_{\mu\nu}\equiv\partial_\mu Z_\nu-\partial_\nu Z_\mu$$

$$G^a_{\mu\nu}\equiv\partial_\mu G^a_\nu-\partial_\nu G^a_\mu-g_s\,f_{abc}\,G^b_\mu\,G^c_\nu$$

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} =& -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu A^l_\nu - \partial_\nu A^l_\mu - g\epsilon_{lmn}A^m_\mu A^n_\nu)^2 \\
& -\frac{1}{4}\,(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s\, f_{abc}\, G^b_\mu\, G^c_\nu)^2 \\
=& -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu A^l_\nu - \partial_\nu A^l_\mu)^2 \\
& +\frac{1}{2}g(\partial_\mu A^l_\nu - \partial_\nu A^l_\mu)\epsilon_{lmn}A^{m\mu}A^{n\nu} - \frac{1}{4}(g\epsilon_{lmn}A^m_\mu A^n_\nu)^2 \\
& -\frac{1}{4}\,G^a_{\mu\nu}\,G^{a\mu\nu} \\
=& -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& +\frac{1}{2}g(\partial_\mu A^1_\nu - \partial_\nu A^1_\mu)(A^{2\mu}A^{3\nu} - A^{3\mu}A^{2\nu}) \\
& +\frac{1}{2}g(\partial_\mu A^2_\nu - \partial_\nu A^2_\mu)(A^{3\mu}A^{1\nu} - A^{1\mu}A^{3\nu}) \\
& +\frac{1}{2}g(\partial_\mu A^l_\nu - \partial_\nu A^l_\mu)(A^{1\mu}A^{2\nu} - A^{2\mu}A^{1\nu}) \\
& -\frac{g^2}{4}(A^2_\mu A^3_\nu - A^3_\mu A^2_\nu)^2 - \frac{g^2}{4}(A^3_\mu A^1_\nu - A^1_\mu A^3_\nu)^2 - \frac{g^2}{4}(A^1_\mu A^2_\nu - A^2_\mu A^1_\nu)^2 \\
& -\frac{1}{4}\,G^a_{\mu\nu}\,G^{a\mu\nu} \\
=& -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& +\frac{i}{4}g\{\partial_\mu(W_\nu + W^\dagger_\nu) - \partial_\nu(W_\mu + W^\dagger_\mu)\}\{(W^\mu - W^{\mu\dagger})A^{3\nu} - A^{3\mu}(W^\nu - W^{\nu\dagger})\} \\
& +\frac{i}{4}g\{\partial_\mu(W_\nu - W^\dagger_\nu) - \partial_\nu(W_\mu - W^\dagger_\mu)\}\{A^{3\mu}(W^\nu + W^{\nu\dagger}) - (W^\mu + W^{\mu\dagger})A^{3\nu}\} \\
& +\frac{i}{4}g(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu)\{(W^\mu + W^{\mu\dagger})(W^\nu - W^{\nu\dagger}) - (W^\mu - W^{\mu\dagger})(W^\nu + W^{\nu\dagger})\} \\
& +\frac{g^2}{8}\{(W_\mu - W^\dagger_\mu)A^3_\nu - A^3_\mu(W_\nu - W^\dagger_\nu)\}^2 \\
& -\frac{g^2}{8}\{A^3_\mu(W_\nu + W^\dagger_\nu) - (W_\mu + W^\dagger_\mu)A^3_\nu\}^2 \\
& +\frac{g^2}{16}\{(W_\mu + W^\dagger_\mu)(W_\nu - W^\dagger_\nu) - (W_\mu - W^\dagger_\mu)(W_\nu + W^\dagger_\nu)\}^2 \\
& -\frac{1}{4}\,G^a_{\mu\nu}\,G^{a\mu\nu} \\
=& -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& +\frac{i}{4}g(\partial_\mu W_\nu - \partial_\nu W_\mu + \partial_\mu W^\dagger_\nu - \partial_\nu W^\dagger_\mu)(W^\mu A^{3\nu} - A^{3\mu}W^\nu - W^{\mu\dagger}A^{3\nu} + A^{3\mu}W^{\nu\dagger}) \\
& +\frac{i}{4}g(\partial_\mu W_\nu - \partial_\nu W_\mu - \partial_\mu W^\dagger_\nu + \partial_\nu W^\dagger_\mu)(-W^\mu A^{3\nu} + A^{3\mu}W^\nu - W^{\mu\dagger}A^{3\nu} + A^{3\mu}W^{\nu\dagger}) \\
& +\frac{i}{2}g(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu)(W^{\mu\dagger}W^\nu - W^{\nu\dagger}W^\mu) \\
& +\frac{g^2}{8}(W_\mu A^3_\nu - W^\dagger_\mu A^3_\nu - A^3_\mu W_\nu + A^3_\mu W^\dagger_\nu)^2 \\
& -\frac{g^2}{8}(A^3_\mu W_\nu + A^3_\mu W^\dagger_\nu - W_\mu A^3_\nu - W^\dagger_\mu A^3_\nu)^2
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{g^2}{4}(W_\mu^\dagger W_\nu - W_\mu W_\nu^\dagger)^2 - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}\\
=&-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger(\partial^\mu W^\nu - \partial^\nu W^\mu)\\
&-\frac{i}{2}g\{(\partial_\mu W_\nu - \partial_\nu W_\mu)(W^{\mu\dagger}A^{3\nu} - A^{3\mu}W^{\nu\dagger}) - (\partial_\mu W_\nu^\dagger - \partial_\nu W_\mu^\dagger)(W^\mu A^{3\nu} - A^{3\mu}W^\nu)\}\\
&+\frac{i}{2}g(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu)(W^{\mu\dagger}W^\nu - W^{\nu\dagger}W^\mu)\\
&-\frac{g^2}{2}(W_\mu A^3_\nu - A^3_\mu W_\nu)(W^{\mu\dagger}A^{3\nu} - A^{3\mu}W^{\nu\dagger})\\
&+\frac{g^2}{2}(W_\mu^\dagger W^{\dagger\mu}W_\nu W^\nu - W_\mu W^\nu W^{\dagger\mu}W_\nu^\dagger) - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}\\
=&-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu}\\
&-\frac{1}{2}\{\partial_\mu W_\nu - \partial_\nu W_\mu - ig(W_\mu A^3_\nu - A^3_\mu W_\nu)\}^\dagger\{\partial^\mu W^\nu - \partial^\nu W^\mu - ig(W^\mu A^{3\nu} - A^{3\mu}W^\nu)\}\\
&+ig(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu)W^{\mu\dagger}W^\nu\\
&+\frac{g^2}{2}(|W_\mu W^\mu|^2 - |W_\mu W^\nu|^2) - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}\\
=&-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu} - \frac{1}{2}(\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu)^\dagger(\mathcal{D}^\mu W^\nu - \mathcal{D}^\nu W^\mu)\\
&+i(eF^A_{\mu\nu} + g\cos\theta_W F^Z_{\mu\nu})W^{\mu\dagger}W^\nu + \frac{g^2}{2}(|W_\mu W^\mu|^2 - |W_\mu W^\nu|^2) - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}
\end{aligned}
$$

Higgs field

$$
\begin{aligned}
V(\Phi) \equiv& -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 = -\frac{1}{2}\mu^2(v+\phi)^2 + \frac{\lambda}{8}(v+\phi)^4\\
=& -\frac{1}{2}\mu^2(v^2 + 2v\phi + \phi^2) + \frac{\lambda}{8}(v^4 + 4v^3\phi + 6v^2\phi^2 + 4v\phi^3 + \phi^4)\\
=& -\frac{1}{2}\mu^2 v^2 + \frac{\lambda}{8}v^4 + \left(-\mu^2 v + \frac{\lambda}{2}v^3\right)\phi + \left(-\frac{1}{2}\mu^2 + \frac{3\lambda}{4}v^2\right)\phi^2 + \frac{\lambda}{2}v\phi^3 + \frac{\lambda}{8}\phi^4
\end{aligned}
$$

$$
V(v) \equiv -\frac{1}{2}\mu^2 v^2 + \frac{\lambda}{8}v^4 \qquad V'(v) = -\mu^2 v + \frac{\lambda}{2}v^3 = 0 \;\rightarrow\; \frac{v^2}{2} = \frac{\mu^2}{\lambda}
$$

$$
M_W \equiv \frac{g\,v}{2} \qquad M_Z \equiv \frac{1}{2}\sqrt{g^2 + g'^2}\,v = \frac{M_W}{\cos\theta_W} \qquad m \equiv \sqrt{2}\mu = v\sqrt{\lambda} : \text{Higgs mass}
$$

$$
\tau^l A^l_\mu = \begin{pmatrix} A^3_\mu & \sqrt{2}W_\mu \\ \sqrt{2}W_\mu^\dagger & -A^3_\mu \end{pmatrix}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{Higgs}} =& \left|\left(i\partial_\mu - g'\frac{1}{2}Y_H B_\mu - g\frac{\tau^l}{2}A^l_\mu\right)\Phi\right|^2 + \mu^2\Phi^\dagger\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2\\
=& \frac{1}{2}\left|\begin{pmatrix} i\partial_\mu - g'\frac{1}{2}B_\mu - \frac{g}{2}A^3_\mu & -\frac{g}{\sqrt{2}}W_\mu \\ -\frac{g}{\sqrt{2}}W_\mu^\dagger & i\partial_\mu - g'\frac{1}{2}B_\mu + \frac{g}{2}A^3_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v+\phi \end{pmatrix}\right|^2 + \mu^2\Phi^\dagger\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2\\
=& \frac{1}{2}\left|\begin{pmatrix} -\frac{g}{\sqrt{2}}W_\mu(v+\phi) \\ \left(i\partial_\mu - \frac{g'}{2}B_\mu + \frac{g}{2}A^3_\mu\right)(v+\phi) \end{pmatrix}\right|^2 + \frac{1}{2}\mu^2(v+\phi)^2 - \frac{\lambda}{8}(v+\phi)^4
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{g^2}{4}W_\mu^\dagger W^\mu(v+\phi)^2+\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{8}|-g'B_\mu+gA_\mu^3|^2(v+\phi)^2\\
&+\mu^2\left(\frac{v}{\sqrt{2}}\right)^2-\frac{\lambda}{2}\left(\frac{v}{\sqrt{2}}\right)^4+v\lambda\left(\frac{\mu^2}{\lambda}-\frac{v^2}{2}\right)\phi+\left(\frac{\mu^2}{2}-\frac{3\lambda v^2}{4}\right)\phi^2-\frac{\lambda v}{2}\phi^3-\frac{\lambda}{8}\phi^4\\
=&\frac{g^2}{4}W_\mu^\dagger W^\mu(v+\phi)^2+\frac{1}{8}\frac{g^2}{\cos^2\theta_W}Z_\mu^2(v+\phi)^2\\
&+\frac{1}{2}(\partial_\mu\phi)^2-\frac{\lambda v^2}{2}\phi^2-\frac{\lambda v}{2}\phi^3-\frac{\lambda}{8}\phi^4-V\left(\frac{v}{\sqrt{2}}\right)\\
=&\frac{g^2v^2}{4}W_\mu^\dagger W^\mu+\frac{g^2v^2}{8\cos^2\theta_W}Z_\mu^2\\
&+\frac{g^2(2v\phi+\phi^2)}{4}\left(W_\mu^\dagger W^\mu+\frac{1}{2\cos^2\theta_W}Z_\mu^2\right)\\
&+\frac{1}{2}(\partial_\mu\phi)^2-\frac{\lambda v^2}{2}\phi^2-\frac{\lambda v}{2}\phi^3-\frac{\lambda}{8}\phi^4-V\left(\frac{v}{\sqrt{2}}\right)\\
=&M_W^2W_\mu^\dagger W^\mu+\frac{1}{2}M_Z^2Z_\mu^2+\left(gM_W\phi+\frac{g^2}{4}\phi^2\right)\left(W_\mu^\dagger W^\mu+\frac{1}{2\cos^2\theta_W}Z_\mu^2\right)\\
&+\frac{1}{2}[(\partial_\mu\phi)^2-m^2\phi^2]-\frac{m\sqrt{\lambda}}{2}\phi^3-\frac{\lambda}{8}\phi^4-V\left(\frac{v}{\sqrt{2}}\right)
\end{aligned}
$$

Lepton kinematic

$\ell^j\equiv\ell_L^j+\ell_R^j \qquad \nu^j\equiv\nu_L^j+\nu_R^j$

$J_\ell^\mu\equiv\bar{\ell}_L^j\gamma^\mu\nu_L^j$: Weak charged current

$J_{\ell,\mathrm{EM}}^\mu\equiv(-)\bar{\ell}^j\gamma^\mu\ell^j$: EM current

$J_\ell^{3\mu}\equiv\bar{L}_L^j\gamma^\mu\frac{\tau^3}{2}L_L^j=\frac{1}{2}\bar{\nu}_L^j\gamma^\mu\nu_L^j-\frac{1}{2}\bar{\ell}_L^j\gamma^\mu\ell_L^j$

$J_{\ell,Z}^\mu\equiv J_\ell^{3\mu}-\sin^2\theta_W J_{\ell,\mathrm{EM}}^\mu$: Weak neutral current

$$
\begin{aligned}
\mathcal{L}_{\text{lepton}}^{\text{kin}}=&\bar{L}_L^j\gamma^\mu\left[i\partial_\mu-g'\frac{Y_{\ell L}}{2}B_\mu-g\frac{\tau^l}{2}A_\mu^l\right]L_L^j+\bar{\ell}_R^j\gamma^\mu\left[i\partial_\mu-g'\frac{Y_{\ell R}}{2}B_\mu\right]\ell_R^j+\bar{\nu}_R^j\gamma^\mu i\partial_\mu\nu_R^j\\
=&\bar{\ell}^j i\partial\!\!\!/\ell^j+\bar{\nu}^j i\partial\!\!\!/\nu^j+\frac{1}{2}\bar{L}_L^j\gamma^\mu\begin{pmatrix} g'B_\mu-gA_\mu^3 & -\sqrt{2}gW_\mu\\ -\sqrt{2}gW_\mu^\dagger & g'B_\mu+gA_\mu^3\end{pmatrix}L_L^j+g'B_\mu\bar{\ell}_R^j\gamma^\mu\ell_R^j\\
=&\bar{\ell}^j i\partial\!\!\!/\ell^j+\bar{\nu}^j i\partial\!\!\!/\nu^j+g'B_\mu\bar{\ell}_R^j\gamma^\mu\ell_R^j\\
&+\frac{1}{2}\bar{L}_L^j\gamma^\mu\begin{pmatrix} -\frac{g}{\cos\theta_W}Z_\mu & -\sqrt{2}gW_\mu\\ -\sqrt{2}gW_\mu^\dagger & 2e\,A_\mu+\frac{g}{\cos\theta_W}(1-2\sin^2\theta_W)Z_\mu\end{pmatrix}L_L^j\\
=&\bar{\ell}^j i\partial\!\!\!/\ell^j+\bar{\nu}^j i\partial\!\!\!/\nu^j+\left(e\,A_\mu-\frac{g}{\cos\theta_W}\sin^2\theta_W Z_\mu\right)\bar{\ell}_R^j\gamma^\mu\ell_R^j-\frac{g}{\sqrt{2}}(\bar{\nu}_L^j\gamma^\mu\ell_L^jW_\mu+\bar{\ell}_L^j\gamma^\mu\nu_L^jW_\mu^\dagger)\\
&-\frac{g}{2\cos\theta_W}\bar{\nu}_L^j\gamma^\mu\nu_L^jZ_\mu+\left\{e\,A_\mu+\frac{g}{2\cos\theta_W}(1-2\sin^2\theta_W)Z_\mu\right\}\bar{\ell}_L^j\gamma^\mu\ell_L^j\\
=&\bar{\ell}^j i\partial\!\!\!/\ell^j+\bar{\nu}^j i\partial\!\!\!/\nu^j-\frac{g}{\sqrt{2}}\{(\bar{\ell}_L^j\gamma^\mu\nu_L^j)^\dagger W_\mu+\bar{\ell}_L^j\gamma^\mu\nu_L^jW_\mu^\dagger\}
\end{aligned}
$$

$$+e\bar{\ell}^j\gamma^\mu\ell^j A_\mu - \frac{g}{\cos\theta_W}\left(\frac{1}{2}\bar{\nu}_L^j\gamma^\mu\nu_L^j - \frac{1}{2}\bar{\ell}_L^j\gamma^\mu\ell_L^j + \sin^2\theta_W\bar{\ell}^j\gamma^\mu\ell^j\right)Z_\mu$$

$$= \bar{\ell}^j i\partial\!\!\!/\ell^j + \bar{\nu}^j i\partial\!\!\!/\nu^j - \frac{g}{\sqrt{2}}(J_\ell^{\mu\dagger}W_\mu + J_\ell^\mu W_\mu^\dagger) - e\,J_{\ell,\mathrm{EM}}^\mu A_\mu - \frac{g}{\cos\theta_W}J_{\ell,Z}^\mu Z_\mu$$

Lepton mass

$$V_{\ell L}\,Y_\ell V_{\ell R}^\dagger \equiv \mathrm{diag}(y_\ell^j) \qquad V_{\nu L}\,Y_\nu V_{\nu R}^\dagger \equiv \mathrm{diag}(y_\nu^j) \qquad y_\ell^j \geqslant 0 \qquad y_\nu^j \geqslant 0$$

$V_{\ell L}V_{\nu L}^\dagger \equiv U_\nu$ (U_ν: PMNS matrix)

$$L_L^j \equiv V_{\ell L}^{jk}L'^k_L \qquad \ell_R^j \equiv V_{\ell R}^{jk}\ell'^k_R \qquad \nu_R^j \equiv (U_\nu V_{\nu R})^{jk}\nu'^k_R$$

$$L'^j_L = V_{\ell L}^{\dagger jk}L_L^k \qquad \ell'^j_R = V_{\ell R}^{\dagger jk}\ell_R^k \qquad \nu'^j_R = (U_\nu V_{\nu R})^{\dagger jk}\nu_R^k = (V_{\nu R}^\dagger U_\nu^\dagger)^{jk}\nu_R^k$$

$\hat{\nu}^j \equiv U_\nu^{\dagger jk}(\nu_L + \nu_R)^k = U_\nu^{\dagger jk}\nu^k$: mass eigenstate neutrino

$$m_\ell^j \equiv \frac{y_\ell^j v}{\sqrt{2}} \qquad m_\nu^j \equiv \frac{y_\nu^j v}{\sqrt{2}}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{lepton}}^{\mathrm{mass}} &= -(\bar{L}'^j_L\Phi)Y_\ell^{jk}\ell'^k_R - (\bar{L}'^j_L\tilde{\Phi})Y_\nu^{jk}\nu'^k_R + h.c.\\
&= -(\bar{L}_L^j\Phi)(V_{\ell L}\,Y_\ell V_{\ell R}^\dagger)^{jk}\ell_R^k - (\bar{L}_L^j\tilde{\Phi})(V_{\ell L}\,Y_\nu V_{\nu R}^\dagger U_\nu^\dagger)^{jk}\nu_R^k + h.c.\\
&= -(\bar{L}_L^j\Phi)(V_{\ell L}\,Y_\ell V_{\ell R}^\dagger)^{jk}\ell_R^k - (\bar{L}_L^j\tilde{\Phi})(V_{\ell L}V_{\nu L}^\dagger V_{\nu L}Y_\nu V_{\nu R}^\dagger U_\nu^\dagger)^{jk}\nu_R^k + h.c.\\
&= -y_\ell^j(\bar{L}_L^j\Phi)\ell_R^j - y_\nu^j[(\bar{L}_L U_\nu)^j\tilde{\Phi}](U_\nu^\dagger\nu_R)^j + h.c.\\
&= -\frac{1}{\sqrt{2}}y_\ell^j\bar{\ell}_L^j(v+\phi)\ell_R^j - \frac{1}{\sqrt{2}}y_\nu^j(\bar{\nu}_L U_\nu)^j(v+\phi)(U_\nu^\dagger\nu_R)^j + h.c.\\
&= -\frac{1}{\sqrt{2}}y_\ell^j\bar{\ell}_L^j(v+\phi)\ell_R^j - \frac{1}{\sqrt{2}}y_\nu^j\bar{\hat{\nu}}_L^j(v+\phi)\hat{\nu}_R^j + h.c.\\
&= -\left(m_\ell^j + \frac{y_\ell^j}{\sqrt{2}}\phi\right)\bar{\ell}^j\ell^j - \left(m_\nu^j + \frac{y_\nu^j}{\sqrt{2}}\phi\right)\bar{\hat{\nu}}^j\hat{\nu}^j
\end{aligned}$$

Quark kinematic

$$u^j \equiv u_L^j + u_R^j \qquad d^j \equiv d_L^j + d_R^j$$

$$J_q^\mu \equiv \bar{d}_L^j\gamma^\mu u_L^j$$

$$J_{q,\mathrm{EM}}^\mu \equiv \left(\frac{2}{3}\right)\bar{u}^j\gamma^\mu u^j + \left(-\frac{1}{3}\right)\bar{d}^j\gamma^\mu d^j$$

$$J_q^{3\mu} \equiv \bar{Q}_L^j\gamma^\mu\frac{\tau^3}{2}Q_L^j = \frac{1}{2}\bar{u}_L^j\gamma^\mu u_L^j - \frac{1}{2}\bar{d}_L^j\gamma^\mu d_L^j$$

$$J_{q,Z}^\mu \equiv J_q^{3\mu} - \sin^2\theta_W J_{q,\mathrm{EM}}^\mu$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{quark}}^{\mathrm{kin}} = \bar{Q}_L^j\gamma^\mu&\left[i\partial_\mu - g'\frac{Y_{QL}}{2}B_\mu - g\frac{\tau^l}{2}A_\mu^l - g_s\frac{\lambda^a}{2}G_\mu^a\right]Q_L^j\\
&+\bar{u}_R^j\gamma^\mu\left[i\partial_\mu - g'\frac{Y_{uR}}{2}B_\mu - g_s\frac{\lambda^a}{2}G_\mu^a\right]u_R^j\\
&+\bar{d}_R^j\gamma^\mu\left[i\partial_\mu - g'\frac{Y_{dR}}{2}B_\mu - g_s\frac{\lambda^a}{2}G_\mu^a\right]d_R^j
\end{aligned}$$

$$
\begin{aligned}
=&\bar{u}^j i\partial\!\!\!/ u^j + \bar{d}^j i\partial\!\!\!/ d^j - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G^a_\mu - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G^a_\mu \\
&+\frac{1}{2}\bar{Q}^j_L \gamma^\mu \begin{pmatrix} -\frac{1}{3}g' B_\mu - g A^3_\mu & -\sqrt{2} g W_\mu \\ -\sqrt{2} g W^\dagger_\mu & -\frac{1}{3} g' B_\mu + g A^3_\mu \end{pmatrix} Q^j_L \\
&-\frac{2}{3} g' B_\mu \bar{u}^j_R \gamma^\mu u^j_R + \frac{1}{3} g' B_\mu \bar{d}^j_R \gamma^\mu d^j_R \\
=&\bar{u}^j i\partial\!\!\!/ u^j + \bar{d}^j i\partial\!\!\!/ d^j - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G^a_\mu - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G^a_\mu \\
&+\bar{Q}^j_L \gamma^\mu \begin{pmatrix} -\frac{2}{3} e A_\mu + \frac{g\left(\frac{4}{3}\sin^2\theta_W - 1\right)}{2\cos\theta_W} Z_\mu & -\frac{g}{\sqrt{2}} W_\mu \\ -\frac{g}{\sqrt{2}} W^\dagger_\mu & \frac{1}{3} e A_\mu + \frac{g\left(-\frac{2}{3}\sin^2\theta_W + 1\right)}{2\cos\theta_W} Z_\mu \end{pmatrix} Q^j_L \\
&-\frac{2}{3}\left(e A_\mu - \frac{g}{\cos\theta_W}\sin^2\theta_W Z_\mu \right) \bar{u}^j_R \gamma^\mu u^j_R + \frac{1}{3}\left(e A_\mu - \frac{g}{\cos\theta_W}\sin^2\theta_W Z_\mu \right) \bar{d}^j_R \gamma^\mu d^j_R \\
=&\bar{u}^j i\partial\!\!\!/ u^j + \bar{d}^j i\partial\!\!\!/ d^j - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G^a_\mu - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G^a_\mu \\
&-\frac{g}{\sqrt{2}} \bar{u}^j_L \gamma^\mu d^j_L W_\mu - \frac{g}{\sqrt{2}} \bar{d}^j_L \gamma^\mu u^j_L W^\dagger_\mu \\
&-\frac{2}{3} e \bar{u}^j \gamma^\mu u^j A_\mu + \frac{1}{3} e \bar{d}^j \gamma^\mu d^j A_\mu \\
&-\frac{g}{\cos\theta_W}\left\{ \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \bar{u}^j_L \gamma^\mu u^j_L - \frac{2}{3}\sin^2\theta_W \bar{u}^j_R \gamma^\mu u^j_R \right\} Z_\mu \\
&-\frac{g}{\cos\theta_W}\left\{ \left(\frac{1}{3}\sin^2\theta_W - \frac{1}{2} \right) \bar{d}^j_L \gamma^\mu d^j_L + \frac{1}{3}\sin^2\theta_W \bar{d}^j_R \gamma^\mu d^j_R \right\} Z_\mu \\
=&\bar{u}^j i\partial\!\!\!/ u^j + \bar{d}^j i\partial\!\!\!/ d^j - \frac{g}{\sqrt{2}}(J^{\mu\dagger}_q W_\mu + J^\mu_q W^\dagger_\mu) - e J^\mu_{q,\mathrm{EM}} A_\mu - \frac{g}{\cos\theta_W} J^\mu_{q,Z} Z_\mu \\
&-g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G^a_\mu - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G^a_\mu
\end{aligned}
$$

Quark mass

$V_{uL} Y_u V^\dagger_{uR} \equiv \mathrm{diag}(y^j_u) \qquad V_{dL} Y_d V^\dagger_{dR} \equiv \mathrm{diag}(y^j_d)$

$V_{uL} V^\dagger_{dL} \equiv U_d$ (U_d: CKM matrix)

$Q^j_L \equiv V^{jk}_{uL} Q'^k_L \qquad u^j_R \equiv V^{jk}_{uR} u'^k_R \qquad d^j_R \equiv (U_d V_{dR})^{jk} d'^k_R$

$Q'^j_L = V^{\dagger jk}_{uL} Q^k_L \qquad u'^j_R = V^{\dagger jk}_{uR} u^k_R \qquad d'^j_R = (V^\dagger_{dR} U^\dagger_d)^{jk} d^k_R$

$\hat{d}^j \equiv U^{\dagger jk}_d (d_L + d_R)^k = U^{\dagger jk}_d d^k$: mass eigenstate down-type quark

$m^j_d \equiv \frac{y^j_d v}{\sqrt{2}} \qquad m^j_u \equiv \frac{y^j_u v}{\sqrt{2}}$

$$
\begin{aligned}
\mathcal{L}^{\mathrm{mass}}_{\mathrm{quark}} =& -(\bar{Q}'^j_L \Phi) Y^{jk}_d d'^k_R - (\bar{Q}'^j_L \tilde{\Phi}) Y^{jk}_u u'^k_R + h.c. \\
=& -(\bar{Q}^j_L \Phi)(V_{uL} Y_d V^\dagger_{dR} U^\dagger_d)^{jk} d^k_R - (\bar{Q}^j_L \tilde{\Phi})(V_{uL} Y_u V^\dagger_{uR})^{jk} u^k_R + h.c. \\
=& -(\bar{Q}^j_L \Phi)(V_{uL} V^\dagger_{dL} V_{dL} Y_d V^\dagger_{dR} U^\dagger_d)^{jk} d^k_R - (\bar{Q}^j_L \tilde{\Phi})(V_{uL} Y_u V^\dagger_{uR})^{jk} u^k_R + h.c.
\end{aligned}
$$

$$
\begin{aligned}
&= -y_d^j[(\bar{Q}_L V_{uL} V_{dL}^\dagger)^j \Phi](U_d^\dagger d_R)^j - y_u^j(\bar{Q}_L^j \tilde{\Phi}) u_R^j + h.c. \\
&= -y_d^j[(\bar{Q}_L U_d)^j \Phi](U_d^\dagger d_R)^j - y_u^j(\bar{Q}_L^j \tilde{\Phi}) u_R^j + h.c. \\
&= -\frac{1}{\sqrt{2}} y_d^j \bar{\hat{d}}_L^j (v+\phi) \hat{d}_R^j - \frac{1}{\sqrt{2}} y_u^j \bar{u}_L^j (v+\phi) u_R^j + h.c. \\
&= -\left(m_d^j + \frac{y_d^j}{\sqrt{2}} \phi \right) \bar{\hat{d}}^j \hat{d}^j - \left(m_u^j + \frac{y_u^j}{\sqrt{2}} \phi \right) \bar{u}^j u^j
\end{aligned}
$$