

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{lepton}}^{\text{kin}} + \mathcal{L}_{\text{lepton}}^{\text{mass}} + \mathcal{L}_{\text{quark}}^{\text{kin}} + \mathcal{L}_{\text{quark}}^{\text{mass}}$$

$$j, k = 1 \sim 3 \quad L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} lmn = 1 \sim 3 \\ abc = 1 \sim 8 \end{matrix}$$

$$\left[\frac{\tau^l}{2}, \frac{\tau^m}{2} \right] = i \epsilon_{lmn} \frac{\tau^n}{2} \quad \left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right] = i f_{abc} \frac{\lambda^c}{2}$$

	T	T^3	Y	$Q = T^3 + \frac{Y}{2}$
ν_L	1/2	1/2	-1	0
ℓ_L	1/2	-1/2	-1	-1
ν_R	0	0	0	0
ℓ_R	0	0	-2	-1

	T	T^3	Y	Q
u_L	1/2	1/2	1/3	2/3
d_L	1/2	-1/2	1/3	-1/3
u_R	0	0	4/3	2/3
d_R	0	0	-2/3	-1/3

	T	T^3	Y	Q
ϕ^+	1/2	1/2	1	1
ϕ^0	1/2	-1/2	1	0

$$\begin{aligned} \mathcal{L}_{\text{gauge}} &= -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu A_\nu^l - \partial_\nu A_\mu^l - g\epsilon_{lmn}A_\mu^m A_\nu^n)^2 \\ &\quad - \frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc}G_\mu^b G_\nu^c)^2 \\ &= -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu A_\nu^l - \partial_\nu A_\mu^l)^2 \\ &\quad + \frac{1}{2}g(\partial_\mu A_\nu^l - \partial_\nu A_\mu^l)\epsilon_{lmn}A^{m\mu}A^{n\nu} - \frac{1}{4}(g\epsilon_{lmn}A_\mu^m A_\nu^n)^2 \\ &\quad - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} \\ &= -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger (\partial^\mu W^\nu - \partial^\nu W^\mu) \\ &\quad + \frac{1}{2}g(\partial_\mu A_\nu^1 - \partial_\nu A_\mu^1)(A^{2\mu}A^{3\nu} - A^{3\mu}A^{2\nu}) \\ &\quad + \frac{1}{2}g(\partial_\mu A_\nu^2 - \partial_\nu A_\mu^2)(A^{3\mu}A^{1\nu} - A^{1\mu}A^{3\nu}) \\ &\quad + \frac{1}{2}g(\partial_\mu A_\nu^l - \partial_\nu A_\mu^l)(A^{1\mu}A^{2\nu} - A^{2\mu}A^{1\nu}) \\ &\quad - \frac{g^2}{4}(A_\mu^2 A_\nu^3 - A_\mu^3 A_\nu^2)^2 - \frac{g^2}{4}(A_\mu^3 A_\nu^1 - A_\mu^1 A_\nu^3)^2 - \frac{g^2}{4}(A_\mu^1 A_\nu^2 - A_\mu^2 A_\nu^1)^2 \\ &\quad - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} \\ &= -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger (\partial^\mu W^\nu - \partial^\nu W^\mu) \\ &\quad + \frac{i}{4}g\{\partial_\mu(W_\nu + W_\nu^\dagger) - \partial_\nu(W_\mu + W_\mu^\dagger)\}\{(W^\mu - W^{\mu\dagger})A^{3\nu} - A^{3\mu}(W^\nu - W^{\nu\dagger})\} \\ &\quad + \frac{i}{4}g\{\partial_\mu(W_\nu - W_\nu^\dagger) - \partial_\nu(W_\mu - W_\mu^\dagger)\}\{A^{3\mu}(W^\nu + W^{\nu\dagger}) - (W^\mu + W^{\mu\dagger})A^{3\nu}\} \\ &\quad + \frac{i}{4}g(\partial_\mu A_\nu^3 - \partial_\nu A_\mu^3)\{(W^\mu + W^{\mu\dagger})(W^\nu - W^{\nu\dagger}) - (W^\mu - W^{\mu\dagger})(W^\nu + W^{\nu\dagger})\} \\ &\quad + \frac{g^2}{8}\{(W_\mu - W_\mu^\dagger)A_\nu^3 - A_\mu^3(W_\nu - W_\nu^\dagger)\}^2 \\ &\quad - \frac{g^2}{8}\{A_\mu^3(W_\nu + W_\nu^\dagger) - (W_\mu + W_\mu^\dagger)A_\nu^3\}^2 \\ &\quad + \frac{g^2}{16}\{(W_\mu + W_\mu^\dagger)(W_\nu - W_\nu^\dagger) - (W_\mu - W_\mu^\dagger)(W_\nu + W_\nu^\dagger)\}^2 \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} \\
& = -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2} (\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger (\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& \quad + \frac{i}{4} g (\partial_\mu W_\nu - \partial_\nu W_\mu + \partial_\mu W_\nu^\dagger - \partial_\nu W_\mu^\dagger) (W^\mu A^{3\nu} - A^{3\mu} W^\nu - W^{\mu\dagger} A^{3\nu} + A^{3\mu} W^{\nu\dagger}) \\
& \quad + \frac{i}{4} g (\partial_\mu W_\nu - \partial_\nu W_\mu - \partial_\mu W_\nu^\dagger + \partial_\nu W_\mu^\dagger) (-W^\mu A^{3\nu} + A^{3\mu} W^\nu - W^{\mu\dagger} A^{3\nu} + A^{3\mu} W^{\nu\dagger}) \\
& \quad + \frac{i}{2} g (\partial_\mu A_\nu^3 - \partial_\nu A_\mu^3) (W^{\mu\dagger} W^\nu - W^{\nu\dagger} W^\mu) \\
& \quad + \frac{g^2}{8} (W_\mu A_\nu^3 - W_\mu^\dagger A_\nu^3 - A_\mu^3 W_\nu + A_\mu^3 W_\nu^\dagger)^2 \\
& \quad - \frac{g^2}{8} (A_\mu^3 W_\nu + A_\mu^3 W_\nu^\dagger - W_\mu A_\nu^3 - W_\mu^\dagger A_\nu^3)^2 \\
& \quad + \frac{g^2}{4} (W_\mu^\dagger W_\nu - W_\mu W_\nu^\dagger)^2 - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} \\
& = -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2} (\partial_\mu W_\nu - \partial_\nu W_\mu)^\dagger (\partial^\mu W^\nu - \partial^\nu W^\mu) \\
& \quad - \frac{i}{2} g \{ (\partial_\mu W_\nu - \partial_\nu W_\mu) (W^{\mu\dagger} A^{3\nu} - A^{3\mu} W^{\nu\dagger}) - (\partial_\mu W_\nu^\dagger - \partial_\nu W_\mu^\dagger) (W^\mu A^{3\nu} - A^{3\mu} W^\nu) \} \\
& \quad + \frac{i}{2} g (\partial_\mu A_\nu^3 - \partial_\nu A_\mu^3) (W^{\mu\dagger} W^\nu - W^{\nu\dagger} W^\mu) \\
& \quad - \frac{g^2}{2} (W_\mu A_\nu^3 - A_\mu^3 W_\nu) (W^{\mu\dagger} A^{3\nu} - A^{3\mu} W^{\nu\dagger}) \\
& \quad + \frac{g^2}{2} (W_\mu^\dagger W_\nu^\dagger W_\nu W^\nu - W_\mu W^\nu W^{\mu\dagger} W_\nu^\dagger) - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} \\
& = -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} F_{\mu\nu}^Z F^{Z\mu\nu} \\
& \quad - \frac{1}{2} \{ \partial_\mu W_\nu - \partial_\nu W_\mu - i g (W_\mu A_\nu^3 - A_\mu^3 W_\nu) \}^\dagger \{ \partial^\mu W^\nu - \partial^\nu W^\mu - i g (W^\mu A^{3\nu} - A^{3\mu} W^\nu) \} \\
& \quad + i g (\partial_\mu A_\nu^3 - \partial_\nu A_\mu^3) W^{\mu\dagger} W^\nu \\
& \quad + \frac{g^2}{2} (|W_\mu W^\mu|^2 - |W_\mu W^\nu|^2) - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} \\
& = -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} F_{\mu\nu}^Z F^{Z\mu\nu} - \frac{1}{2} (\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu)^\dagger (\mathcal{D}^\mu W^\nu - \mathcal{D}^\nu W^\mu) \\
& \quad + i (e F_{\mu\nu}^A + g \cos\theta_W F_{\mu\nu}^Z) W^{\mu\dagger} W^\nu + \frac{g^2}{2} (|W_\mu W^\mu|^2 - |W_\mu W^\nu|^2) - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}
\end{aligned}$$

$$W_\mu \equiv \frac{1}{\sqrt{2}} (A_\mu^1 - i A_\mu^2)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix} \quad \cos\theta_W \equiv \frac{g}{\sqrt{g^2 + g'^2}} \quad \sin\theta_W \equiv \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$F_{\mu\nu}^A \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \quad F_{\mu\nu}^Z \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu$$

$$e \equiv \frac{g g'}{\sqrt{g^2 + g'^2}} = g \sin\theta_W = g' \cos\theta_W$$

$$\mathcal{D}_\mu W_\nu \equiv (\partial_\mu + i g A_\mu^3) W_\nu = (\partial_\mu + i e A_\mu + i g \cos\theta_W Z_\mu) W_\nu$$

$$A_\mu^1 = \frac{1}{\sqrt{2}} (W_\mu + W_\mu^\dagger) \quad A_\mu^2 = \frac{i}{\sqrt{2}} (W_\mu - W_\mu^\dagger)$$

$$\begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

$$g A_\mu^3 = e A_\mu + g \cos\theta_W Z_\mu \quad g' B_\mu = e A_\mu - \frac{g}{\cos\theta_W} \sin^2\theta_W Z_\mu$$

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &= \left| \left(i\partial_\mu - g' \frac{1}{2} Y_H B_\mu - g \frac{\tau^l}{2} A_\mu^l \right) \Phi \right|^2 + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 \\ &= \frac{1}{2} \left| \begin{pmatrix} i\partial_\mu - g' \frac{1}{2} B_\mu - \frac{g}{2} A_\mu^3 & -\frac{g}{\sqrt{2}} W_\mu \\ -\frac{g}{\sqrt{2}} W_\mu^\dagger & i\partial_\mu - g' \frac{1}{2} B_\mu + \frac{g}{2} A_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v + \phi \end{pmatrix} \right|^2 + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 \\ &= \frac{1}{2} \left| \begin{pmatrix} -\frac{g}{\sqrt{2}} W_\mu (v + \phi) \\ \left(i\partial_\mu - \frac{g'}{2} B_\mu + \frac{g}{2} A_\mu^3 \right) (v + \phi) \end{pmatrix} \right|^2 + \frac{1}{2} \mu^2 (v + \phi)^2 - \frac{\lambda}{8} (v + \phi)^4 \\ &= \frac{g^2}{4} W_\mu^\dagger W^\mu (v + \phi)^2 + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{8} |-g' B_\mu + g A_\mu^3|^2 (v + \phi)^2 \\ &\quad + \mu^2 \left(\frac{v}{\sqrt{2}} \right)^2 - \frac{\lambda}{2} \left(\frac{v}{\sqrt{2}} \right)^4 + v \lambda \left(\frac{\mu^2}{\lambda} - \frac{v^2}{2} \right) \phi + \left(\frac{\mu^2}{2} - \frac{3\lambda v^2}{4} \right) \phi^2 - \frac{\lambda v}{2} \phi^3 - \frac{\lambda}{8} \phi^4 \\ &= \frac{g^2}{4} W_\mu^\dagger W^\mu (v + \phi)^2 + \frac{1}{8 \cos^2\theta_W} Z_\mu^2 (v + \phi)^2 \\ &\quad + \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda v^2}{2} \phi^2 - \frac{\lambda v}{2} \phi^3 - \frac{\lambda}{8} \phi^4 - V \left(\frac{v}{\sqrt{2}} \right) \\ &= \frac{g^2 v^2}{4} W_\mu^\dagger W^\mu + \frac{g^2 v^2}{8 \cos^2\theta_W} Z_\mu^2 \\ &\quad + \frac{g^2 (2v\phi + \phi^2)}{4} \left(W_\mu^\dagger W^\mu + \frac{1}{2 \cos^2\theta_W} Z_\mu^2 \right) \\ &\quad + \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda v^2}{2} \phi^2 - \frac{\lambda v}{2} \phi^3 - \frac{\lambda}{8} \phi^4 - V \left(\frac{v}{\sqrt{2}} \right) \\ &= M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu^2 + \left(g M_W \phi + \frac{g^2}{4} \phi^2 \right) \left(W_\mu^\dagger W^\mu + \frac{1}{2 \cos^2\theta_W} Z_\mu^2 \right) \\ &\quad + \frac{1}{2} [(\partial_\mu \phi)^2 - m^2 \phi^2] - \frac{m\sqrt{\lambda}}{2} \phi^3 - \frac{\lambda}{8} \phi^4 - V \left(\frac{v}{\sqrt{2}} \right) \end{aligned}$$

$$\Phi(x) \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \equiv e^{i \left\{ \frac{\tau^l}{2} \chi^l(x) \right\}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \phi(x) \end{pmatrix} \xrightarrow{(\text{unitary gauge})} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \phi(x) \end{pmatrix}$$

$$\frac{v}{\sqrt{2}} \equiv \frac{\mu}{\sqrt{\lambda}}$$

$$\tilde{\Phi} \equiv i\tau^2 \Phi^* \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} v + \phi(x) \\ 0 \end{pmatrix}$$

$$\tau^l A_\mu^l = \begin{pmatrix} A_\mu^3 & \sqrt{2} W_\mu \\ \sqrt{2} W_\mu^\dagger & -A_\mu^3 \end{pmatrix}$$

$$V(\Phi) \equiv -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 = -\frac{1}{2} \mu^2 (v + \phi)^2 + \frac{\lambda}{8} (v + \phi)^4$$

$$\begin{aligned}
&= -\frac{1}{2}\mu^2(v^2 + 2v\phi + \phi^2) + \frac{\lambda}{8}(v^4 + 4v^3\phi + 6v^2\phi^2 + 4v\phi^3 + \phi^4) \\
&= -\frac{1}{2}\mu^2v^2 + \frac{\lambda}{8}v^4 + \left(-\mu^2v + \frac{\lambda}{2}v^3\right)\phi + \left(-\frac{1}{2}\mu^2 + \frac{3\lambda}{4}v^2\right)\phi^2 + \frac{\lambda}{2}v\phi^3 + \frac{\lambda}{8}\phi^4 \\
V(v) &\equiv -\frac{1}{2}\mu^2v^2 + \frac{\lambda}{8}v^4 \quad V'(v) = -\mu^2v + \frac{\lambda}{2}v^3 = 0 \rightarrow \frac{v^2}{2} = \frac{\mu^2}{\lambda}
\end{aligned}$$

$$M_W \equiv \frac{gv}{2} \quad M_Z \equiv \frac{1}{2}\sqrt{g^2 + g'^2}v = \frac{M_W}{\cos\theta_W} \quad m \equiv \sqrt{2}\mu = v\sqrt{\lambda}$$

$$\begin{aligned}
\mathcal{L}_{\text{lepton}}^{\text{kin}} &= \bar{L}_L^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{\ell L}}{2} B_\mu - g \frac{\tau^l}{2} A_\mu^l \right] L_L^j + \bar{\ell}_R^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{\ell R}}{2} B_\mu \right] \ell_R^j + \bar{\nu}_R^j \gamma^\mu i\partial_\mu \nu_R^j \\
&= \bar{\ell}^j i\not{\partial} \ell^j + \bar{\nu}^j i\not{\partial} \nu^j + \frac{1}{2} \bar{L}_L^j \gamma^\mu \begin{pmatrix} g' B_\mu - g A_\mu^3 & -\sqrt{2} g W_\mu \\ -\sqrt{2} g W_\mu^\dagger & g' B_\mu + g A_\mu^3 \end{pmatrix} L_L^j + g' B_\mu \bar{\ell}_R^j \gamma^\mu \ell_R^j \\
&= \bar{\ell}^j i\not{\partial} \ell^j + \bar{\nu}^j i\not{\partial} \nu^j + g' B_\mu \bar{\ell}_R^j \gamma^\mu \ell_R^j \\
&\quad + \frac{1}{2} \bar{L}_L^j \gamma^\mu \begin{pmatrix} -\frac{g}{\cos\theta_W} Z_\mu & -\sqrt{2} g W_\mu \\ -\sqrt{2} g W_\mu^\dagger & 2e A_\mu + \frac{g}{\cos\theta_W} (1 - 2\sin^2\theta_W) Z_\mu \end{pmatrix} L_L^j \\
&= \bar{\ell}^j i\not{\partial} \ell^j + \bar{\nu}^j i\not{\partial} \nu^j + \left(e A_\mu - \frac{g}{\cos\theta_W} \sin^2\theta_W Z_\mu \right) \bar{\ell}_R^j \gamma^\mu \ell_R^j - \frac{g}{\sqrt{2}} (\bar{\nu}_L^j \gamma^\mu \ell_L^j W_\mu + \bar{\ell}_L^j \gamma^\mu \nu_L^j W_\mu^\dagger) \\
&\quad - \frac{g}{2\cos\theta_W} \bar{\nu}_L^j \gamma^\mu \nu_L^j Z_\mu + \left\{ e A_\mu + \frac{g}{2\cos\theta_W} (1 - 2\sin^2\theta_W) Z_\mu \right\} \bar{\ell}_L^j \gamma^\mu \ell_L^j \\
&= \bar{\ell}^j i\not{\partial} \ell^j + \bar{\nu}^j i\not{\partial} \nu^j - \frac{g}{\sqrt{2}} \{ (\bar{\ell}_L^j \gamma^\mu \nu_L^j)^\dagger W_\mu + \bar{\ell}_L^j \gamma^\mu \nu_L^j W_\mu^\dagger \} \\
&\quad + e \bar{\ell}^j \gamma^\mu \ell^j A_\mu - \frac{g}{\cos\theta_W} \left(\frac{1}{2} \bar{\nu}_L^j \gamma^\mu \nu_L^j - \frac{1}{2} \bar{\ell}_L^j \gamma^\mu \ell_L^j + \sin^2\theta_W \bar{\ell}^j \gamma^\mu \ell^j \right) Z_\mu \\
&= \bar{\ell}^j i\not{\partial} \ell^j + \bar{\nu}^j i\not{\partial} \nu^j - \frac{g}{\sqrt{2}} (J_\ell^{\mu\dagger} W_\mu + J_\ell^\mu W_\mu^\dagger) - e J_{\ell, \text{EM}}^\mu A_\mu - \frac{g}{\cos\theta_W} J_{\ell, Z}^\mu Z_\mu
\end{aligned}$$

$$\begin{aligned}
J_\ell^\mu &\equiv \bar{\ell}_L^j \gamma^\mu \nu_L^j & J_{\ell, \text{EM}}^\mu &\equiv (-) \bar{\ell}^j \gamma^\mu \ell^j & J_\ell^{3\mu} &\equiv \bar{L}_L^j \gamma^\mu \frac{\tau^3}{2} L_L^j = \frac{1}{2} \bar{\nu}_L^j \gamma^\mu \nu_L^j - \frac{1}{2} \bar{\ell}_L^j \gamma^\mu \ell_L^j \\
J_{\ell, Z}^\mu &\equiv J_\ell^{3\mu} - \sin^2\theta_W J_{\ell, \text{EM}}^\mu
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\text{lepton}}^{\text{mass}} &= -(\bar{L}_L^j \Phi) Y_\ell^{jk} \ell_R^k - (\bar{L}_L^j \tilde{\Phi}) Y_\nu^{jk} \nu_R^k + h.c. \\
&= -(\bar{L}_L^j \Phi) (V_\ell Y_\ell V_\ell^\dagger)^{jk} \ell_R^k - (\bar{L}_L^j \tilde{\Phi}) (V_\ell V_\nu^\dagger V_\nu Y_\nu V_\nu^\dagger V_\nu V_\ell^\dagger)^{jk} \nu_R^k + h.c. \\
&= -y_\ell^j (\bar{L}_L^j \Phi) \ell_R^j - y_\nu^j [(\bar{L}_L U_\nu)^j \tilde{\Phi}] (U_\nu^\dagger \nu_R)^j + h.c.
\end{aligned}$$

$$V_\ell Y_\ell V_\ell^\dagger \equiv \text{diag}(y_\ell^j) \quad V_\nu Y_\nu V_\nu^\dagger \equiv \text{diag}(y_\nu^j) \quad V_\nu V_\ell^\dagger \equiv U_\nu^\dagger \quad (U_\nu: \text{PMNS matrix})$$

$$\begin{aligned}
L_L^j &\equiv V_\ell^{jk} L_L^k & \ell_R^j &\equiv V_\ell^{jk} \ell_R^k & \nu_R^j &\equiv V_\ell^{jk} \nu_R^k \\
&= -\frac{1}{\sqrt{2}} y_\ell^j \bar{\ell}_L^j (v + \phi) \ell_R^j - \frac{1}{\sqrt{2}} y_\nu^j \bar{\nu}_L^j (v + \phi) \nu_R^j + h.c. \\
&= -\left(m_\ell^j + \frac{y_\ell^j}{\sqrt{2}} \phi \right) \bar{\ell}^j \ell^j - \left(m_\nu^j + \frac{y_\nu^j}{\sqrt{2}} \phi \right) \bar{\nu}^j \nu^j
\end{aligned}$$

$$\nu^j \equiv U_\nu^{jk} \hat{\nu}^k \quad m_\ell^j \equiv \frac{y_\ell^j \nu}{\sqrt{2}} \quad m_\nu^j \equiv \frac{y_\nu^j \nu}{\sqrt{2}}$$

$$\begin{aligned} \mathcal{L}_{\text{quark}}^{\text{kin}} &= Q_L^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{QL}}{2} B_\mu - g \frac{\tau^l}{2} A_\mu^l - g_s \frac{\lambda^a}{2} G_\mu^a \right] Q_L^j \\ &\quad + \bar{u}_R^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{uR}}{2} B_\mu - g_s \frac{\lambda^a}{2} G_\mu^a \right] u_R^j \\ &\quad + \bar{d}_R^j \gamma^\mu \left[i\partial_\mu - g' \frac{Y_{dR}}{2} B_\mu - g_s \frac{\lambda^a}{2} G_\mu^a \right] d_R^j \\ &= \bar{u}^j i \not{\partial} u^j + \bar{d}^j i \not{\partial} d^j - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G_\mu^a - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G_\mu^a \\ &\quad + \frac{1}{2} \bar{Q}_L^j \gamma^\mu \begin{pmatrix} -\frac{1}{3} g' B_\mu - g A_\mu^3 & -\sqrt{2} g W_\mu \\ -\sqrt{2} g W_\mu^\dagger & -\frac{1}{3} g' B_\mu + g A_\mu^3 \end{pmatrix} Q_L^j \\ &\quad - \frac{2}{3} g' B_\mu \bar{u}_R^j \gamma^\mu u_R^j + \frac{1}{3} g' B_\mu \bar{d}_R^j \gamma^\mu d_R^j \\ &= \bar{u}^j i \not{\partial} u^j + \bar{d}^j i \not{\partial} d^j - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G_\mu^a - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G_\mu^a \\ &\quad + \bar{Q}_L^j \gamma^\mu \begin{pmatrix} -\frac{2}{3} e A_\mu + \frac{g(\frac{4}{3} \sin^2 \theta_W - 1)}{2 \cos \theta_W} Z_\mu & -\frac{g}{\sqrt{2}} W_\mu \\ -\frac{g}{\sqrt{2}} W_\mu^\dagger & \frac{1}{3} e A_\mu + \frac{g(-\frac{2}{3} \sin^2 \theta_W + 1)}{2 \cos \theta_W} Z_\mu \end{pmatrix} Q_L^j \\ &\quad - \frac{2}{3} \left(e A_\mu - \frac{g}{\cos \theta_W} \sin^2 \theta_W Z_\mu \right) \bar{u}_R^j \gamma^\mu u_R^j + \frac{1}{3} \left(e A_\mu - \frac{g}{\cos \theta_W} \sin^2 \theta_W Z_\mu \right) \bar{d}_R^j \gamma^\mu d_R^j \\ &= \bar{u}^j i \not{\partial} u^j + \bar{d}^j i \not{\partial} d^j - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G_\mu^a - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G_\mu^a \\ &\quad - \frac{g}{\sqrt{2}} \bar{u}_L^j \gamma^\mu d_L^j W_\mu - \frac{g}{\sqrt{2}} \bar{d}_L^j \gamma^\mu u_L^j W_\mu^\dagger \\ &\quad - \frac{2}{3} e \bar{u}^j \gamma^\mu u^j A_\mu + \frac{1}{3} e \bar{d}^j \gamma^\mu d^j A_\mu \\ &\quad - \frac{g}{\cos \theta_W} \left\{ \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \bar{u}_L^j \gamma^\mu u_L^j - \frac{2}{3} \sin^2 \theta_W \bar{u}_R^j \gamma^\mu u_R^j \right\} Z_\mu \\ &\quad - \frac{g}{\cos \theta_W} \left\{ \left(\frac{1}{3} \sin^2 \theta_W - \frac{1}{2} \right) \bar{d}_L^j \gamma^\mu d_L^j + \frac{1}{3} \sin^2 \theta_W \bar{d}_R^j \gamma^\mu d_R^j \right\} Z_\mu \\ &= \bar{u}^j i \not{\partial} u^j + \bar{d}^j i \not{\partial} d^j - \frac{g}{\sqrt{2}} (J_q^{\mu\dagger} W_\mu + J_q^\mu W_\mu^\dagger) - e J_{q,\text{EM}}^\mu A_\mu - \frac{g}{\cos \theta_W} J_{q,Z}^\mu Z_\mu \\ &\quad - g_s \bar{u}^j \gamma^\mu \frac{\lambda^a}{2} u^j G_\mu^a - g_s \bar{d}^j \gamma^\mu \frac{\lambda^a}{2} d^j G_\mu^a \end{aligned}$$

$$J_q^\mu \equiv \bar{d}_L^j \gamma^\mu u_L^j \quad J_{q,\text{EM}}^\mu \equiv \left(\frac{2}{3} \right) \bar{u}^j \gamma^\mu u^j + \left(-\frac{1}{3} \right) \bar{d}^j \gamma^\mu d^j$$

$$J_q^{3\mu} \equiv \bar{Q}_L^j \gamma^\mu \frac{\tau^3}{2} Q_L^j = \frac{1}{2} \bar{u}_L^j \gamma^\mu u_L^j - \frac{1}{2} \bar{d}_L^j \gamma^\mu d_L^j$$

$$J_{q,Z}^\mu \equiv J_q^{3\mu} - \sin^2 \theta_W J_{q,\text{EM}}^\mu$$

$$\begin{aligned}
\mathcal{L}_{\text{quark}}^{\text{mass}} &= -(\bar{Q}'^j_L \Phi) Y_d^{jk} d'^k_R - (\bar{Q}'^j_L \tilde{\Phi}) Y_u^{jk} u'^k_R + h.c. \\
&= -(\bar{Q}^j_L \Phi) (V_u V_d^\dagger V_d Y_d V_d^\dagger V_d V_u^\dagger)^{jk} d^k_R - (\bar{Q}^j_L \tilde{\Phi}) (V_u Y_u V_u^\dagger)^{jk} u^k_R + h.c. \\
&= -y_d^j [(\bar{Q}_L U_d)^j \Phi] (U_d^\dagger d_R)^j - y_u^j (\bar{Q}_L \tilde{\Phi}) u_R^j + h.c. \\
V_u Y_u V_u^\dagger &\equiv \text{diag}(y_u^j) \quad V_d Y_d V_d^\dagger \equiv \text{diag}(y_d^j) \quad V_d V_u^\dagger \equiv U_d^\dagger \text{ (} U_d \text{: CKM matrix)} \\
Q_L^j &\equiv V_u^{jk} Q'^k_L \quad u_R^j \equiv V_u^{jk} u'^k_R \quad d_R^j \equiv V_u^{jk} d'^k_R \\
&= -\frac{1}{\sqrt{2}} y_d^j \bar{\tilde{d}}^j_L (v + \phi) \hat{d}_R^j - \frac{1}{\sqrt{2}} y_u^j \bar{u}_L^j (v + \phi) u_R^j + h.c. \\
&= -\left(m_d^j + \frac{y_d^j}{\sqrt{2}} \phi\right) \bar{\tilde{d}}^j \hat{d}^j - \left(m_u^j + \frac{y_u^j}{\sqrt{2}} \phi\right) \bar{u}^j u^j \\
d^j &\equiv U_d^{jk} \hat{d}^k \quad m_d^j \equiv \frac{y_d^j v}{\sqrt{2}} \quad m_u^j \equiv \frac{y_u^j v}{\sqrt{2}}
\end{aligned}$$