

# **Neutrino-nucleus interactions in the DIS region**

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**Workshop on “Neutrino frontier in 2014”**

**MEXT Kakenhi (#2504), Unification and Development of the Neutrino Science Frontier  
December 21-23, 2014, Fuji-Calm, Fuji-Yoshida, Japan**  
**<http://hep-www.px.tsukuba.ac.jp/cgi-bin/nfws2014/nfws2014.pl>**

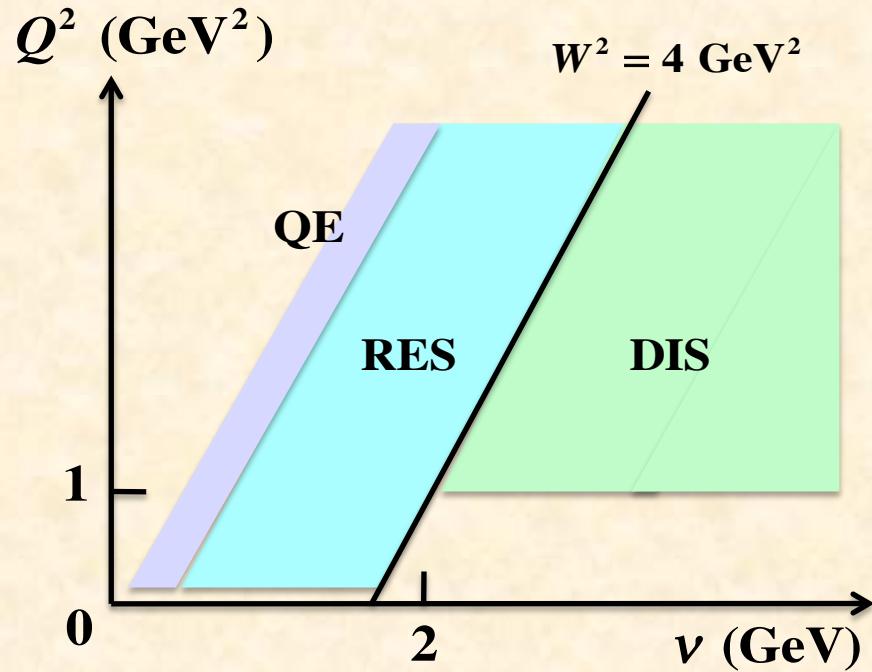
**December 22, 2014**

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- 2. Parton distributions functions (PDFs)**
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# **Introduction to neutrino deep inelastic scattering (DIS)**

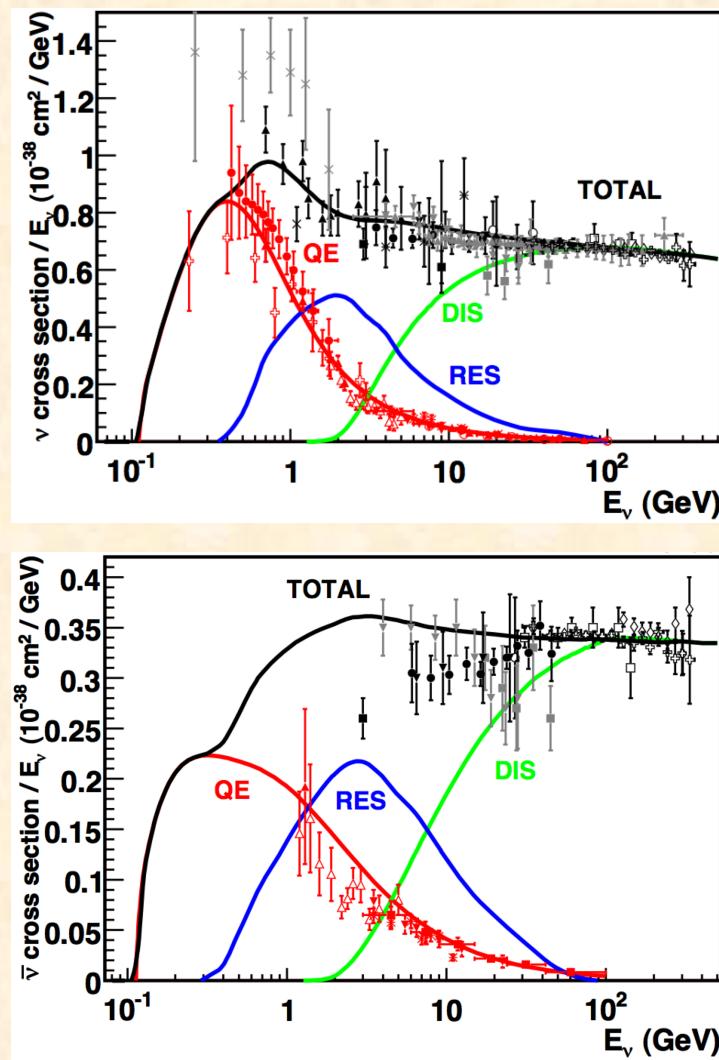
# Kinematical regions of neutrino-nucleus scattering



Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic)

Activities at the J-PARC branch, KEK theory center  
<http://j-parc-th.kek.jp/html/English/e-index.html>



J.L. Hewett *et al.*, arXiv:1205.2671,  
Proceedings of the 2011 workshop  
on Fundamental Physics at the Intensity Frontier

# Durham HEP server: <http://durpdg.dur.ac.uk/pdfs>

**Unpolarized Parton Distributions**

Access the parton distribution code, on-line calculation and graphical display of the distributions, from CTEQ, GRV, MRST/MSTW, Alekhin, ZEUS, H1, HERAPDF, BBG and NNPDF.

**CTEQ** fortran code and grids  
**GRV/GJR** fortran code and grids  
**MRST** fortran code and grids, C++ code  
**MSTW** fortran, C++ and Mathematica codes + grids etc.  
**ALEKHIN** fortran,C++,Mathematica code, and grids  
**ZEUS** ZEUS 2002 PDFs, ZEUS 2005 jet fit PDFs  
**HERAPDF** Combined H1/ZEUS page, HERAPDF1.0 paper  
**H1** H1 2000  
**BBG** BBG06\_NS  
**NNPDF** Non Singlet PDF code - hep-ph/0701127

**Polarized Parton Distributions**

Currently available parametrizations

LSS2001 E.Leader, A.V.Sidorov and D.B.Stamenov, Eur.Phys.J.C23 (2002) 479  
LSS2005 E.Leader, A.V.Sidorov and D.B.Stamenov, Phys.Rev.D73 (2006) 034023  
LSS2006 E.Leader, A.V.Sidorov and D.B.Stamenov, Phys.Rev.D75 (2007) 074027  
LSS2010 E.Leader, A.V.Sidorov and D.B.Stamenov, Phys.Rev.D82 (2010) 114018  
GRSV M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D53 (1996) 4775  
GRSV2000 M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D63 (2001) 094005  
GS T. Gehrmann and W.J. Stirling, Phys. Rev. D53 (1996) 6100  
BB J. Bluemlein and H. Boettcher - Nucl.Phys.B636(2002)225  
AAC Asymmetry Analysis Collaboration - M. Hirai et al- Phys. Rev. D69 (2004) 054021  
DS2000 D. de Florian and R. Sassot, Phys. Rev. D62 (2000) 094025  
DNS2005 D. de Florian, G.A. Navarro and R. Sassot, Phys. Rev. D71 (2005) 094018

**Diffractive Parton Distributions**

MRW2006 A.D.Martin, M.G.Ryskin and G.Watt

**Pion Parton Distributions**

MRS fortran code and grids

**PDFs from nuclei**

HKM M. Hirai, S. Kumano and M. Miyama - Phys. Rev. D64 (2001) 034003  
EKS98 K.J.Eskola, V.J.Kolhinen and C.A. Salgado - Eur.Phys.J C9(1999)61  
and K.J.Eskola, V.J.Kolhinen and P.V.Ruuskanen - Nuc.Phys.B535(1998)351  
nDS D. de Florian and R. Sassot, Phys.Rev.D69(2004)074028  
FGS10 L. Frankfurt, V. Guzey, M. Strikman, arXiv:1106.2091 [hep-ph]

**Generalized Parton Distributions**

GPD GPD code of Vinnikov

**Deeply Virtual Compton Scattering**

DVCS DVCS code of Freund and McDermott

**Fragmentation Functions**

FF Fragmentation Distribution database site compiled by Marco Radici and Rainer Jakob

Our contributions  
on nuclear PDFs

Our home page:  
[http://research.kek.jp/people/kumanos/  
nuclp.html](http://research.kek.jp/people/kumanos/nuclp.html)

# Hepforge (LHAPDF): <http://www.hepforge.org/>

## LHAPDF the Les Houches Accord PDF Interface

LHAPDF provides a unified and easy to use interface to modern PDF sets. It is designed to work not only with individual PDF sets but also with the more recent multiple "error" sets. It can be viewed as the successor to PDFLIB, incorporating many of the older sets found in the latter, including pion and photon PDFs. In LHAPDF the computer code and input parameters/grids are separated thus allowing more easy updating and no limit to the expansion possibilities. The code and data sets can be downloaded together or individually as desired. From version 4.1 onwards a configuration script facilitates the installation of LHAPDF.

**2013-10-09: C++ LHAPDF6 6.0.4 patch version is now available**

See the [LHAPDF6 announcement talk](#) from PDF4LHC (some small details have changed since).

Code tarball for download from [here](#).

New PDF data files for download from [here](#).

Note: from version 5.7.1 onwards the PDF grid files are not bundled with the tarball.

Note: Problems compiling on MacOS (particularly v10.6)

Latest: new version of bin/lhapdf-getdata script needed for all versions

### Contents:

- [Installing LHAPDF.](#)
- [Configuration options.](#)
- [List \(and download\) of PDF sets.](#)
- [On-line user manual.](#)
- [PDF set numbers](#)
- [A wrapper for C++.](#)
- [A wrapper for C++. \(old version\)](#)
- [A little bit of theory.](#)

### Downloads:

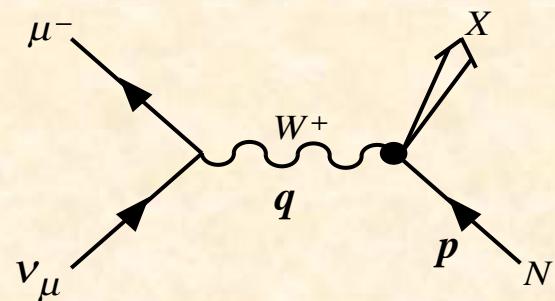
- Latest version 6 release (09/10/2013):
  - 6.0.4: [LHAPDF-6.0.4.tar.gz](#) (PDF sets)
  - 6.0.3: [LHAPDF-6.0.3.tar.gz](#) (PDF sets)
  - 6.0.2: [LHAPDF-6.0.2.tar.gz](#) (PDF sets)
  - 6.0.1: [LHAPDF-6.0.1.tar.gz](#) (PDF sets)
  - 6.0.0: [LHAPDF-6.0.0.tar.gz](#) (PDF sets)
- Latest (& final) version 5 release (24/09/2013):

← Our nuclear PDFs are included.

# Deep inelastic scattering

A nucleon is broken up by a high-energy neutrino.

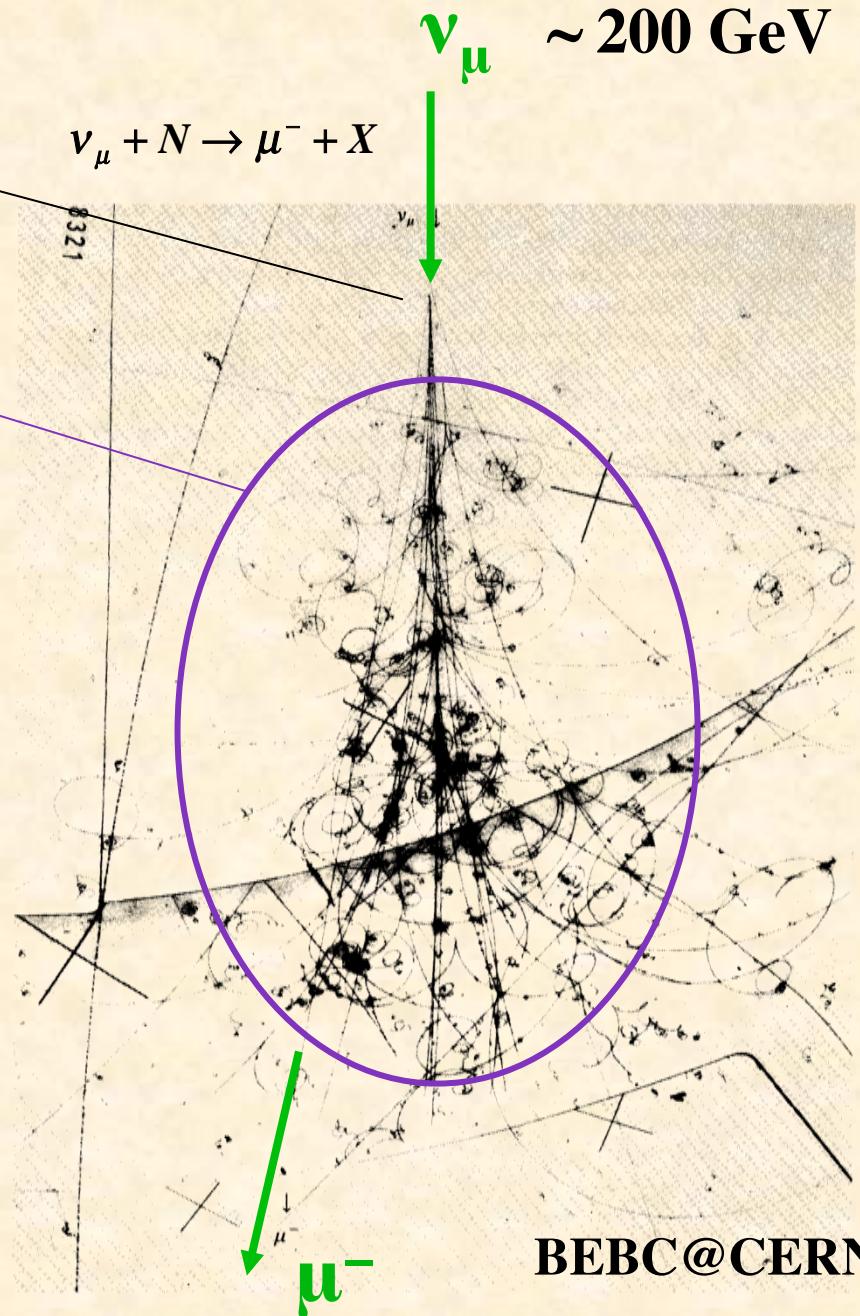
Hadrons are produced; however, these are not usually measured.  
(inclusive reaction)



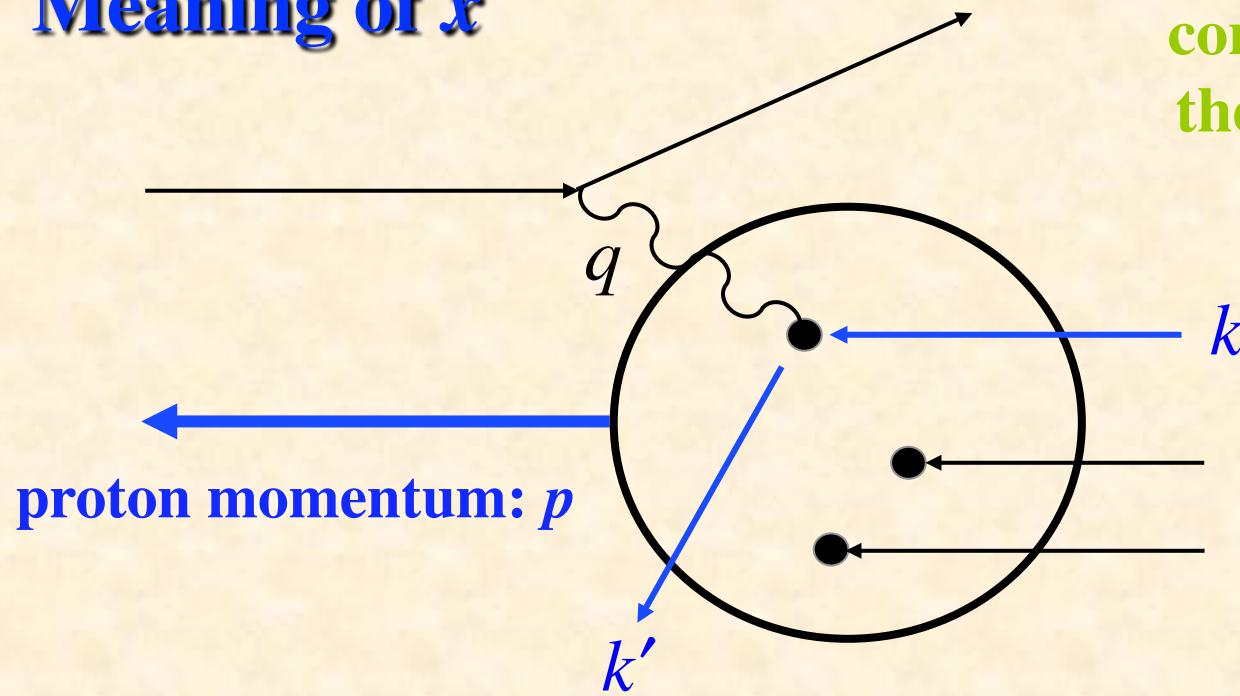
Momentum transfer:  $q^2 = (k - k')^2 = -Q^2$

Bjorken scaling variable:  $x = \frac{Q^2}{2p \cdot q}$

Invariant mass:  $W^2 = p_x^2 = (p + q)^2$



## Meaning of $x$



proton momentum:  $p$

consider the frame where  
the proton is moving fast

$$(k + q)^2 = k'^2$$

$$(k + q)^2 = m_q^2 + 2k \cdot q - Q^2$$

$$k'^2 = m_q^2$$

$$\longrightarrow 2k \cdot q = Q^2$$

$$\text{if } k = \xi p, \quad 2\xi p \cdot q = Q^2$$

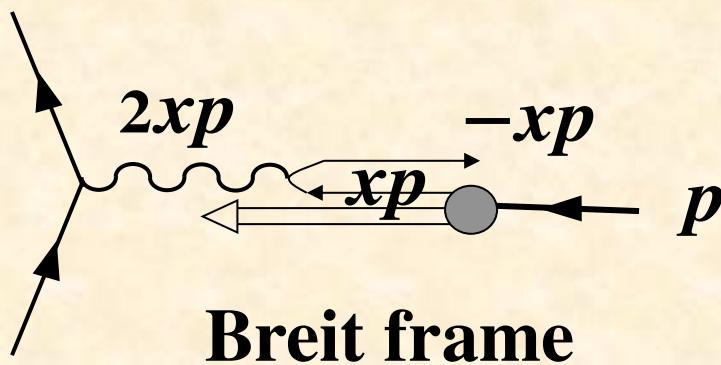
$$\xi = \frac{Q^2}{2p \cdot q} = x$$

$x$  = momentum fraction carried by the struck parton

For example,  $x=0.5$  means that the struck parton carries 50% momentum of the proton.

## Meaning of $Q^2$

Breit frame is defined as the frame in which exchanged boson is completely spacelike:  $q = (0, 0, 0, q)$ .



$q^0=0$ : photon does not transfer any energy

Spatial resolution = reduced wavelength  $\lambda = \frac{1}{|\vec{q}|} = \frac{1}{\sqrt{Q^2}}$

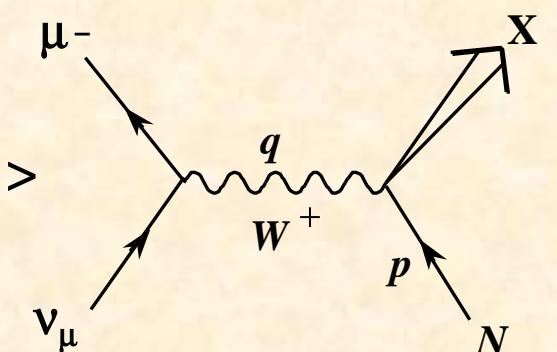
$Q^2$  corresponds to the “spatial resolution”.

# Neutrino deep inelastic scattering (CC: Charged Current)

$$d\sigma = \frac{1}{4k \cdot p} \frac{1}{2} \sum_{spins} \sum_X (2\pi)^4 \delta^4(k + p - k' - p_X) |M|^2 \frac{d^3 k'}{(2\pi)^3 2E'}$$

$$M = \frac{1}{1+Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k', \lambda') \gamma^\mu (1-\gamma_5) u(k, \lambda) \langle X | J_\mu^{CC} | p, \lambda_p \rangle$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$



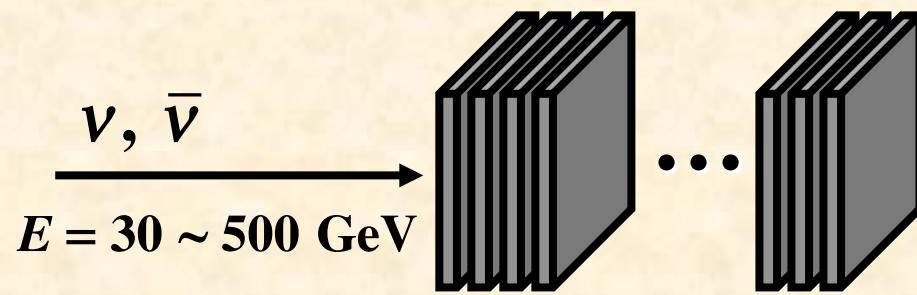
$$L^{\mu\nu} = 8 \left[ k^\mu k'^\nu + k'^\mu k^\nu - k \cdot k' g^{\mu\nu} + i \epsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \right], \quad \epsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1 \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left( p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left( p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) + \frac{i}{2M^2} W_3 \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma$$

$$MW_1 = F_1, \quad vW_2 = F_2, \quad vW_3 = F_3, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

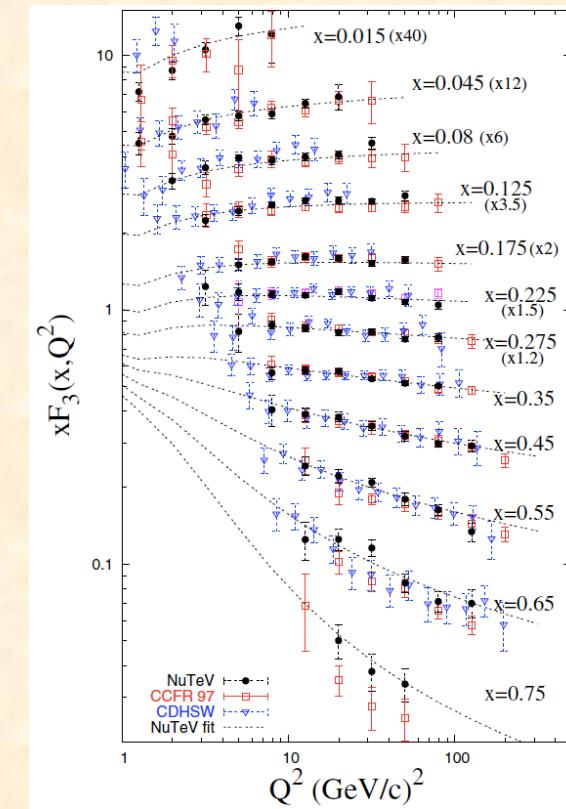
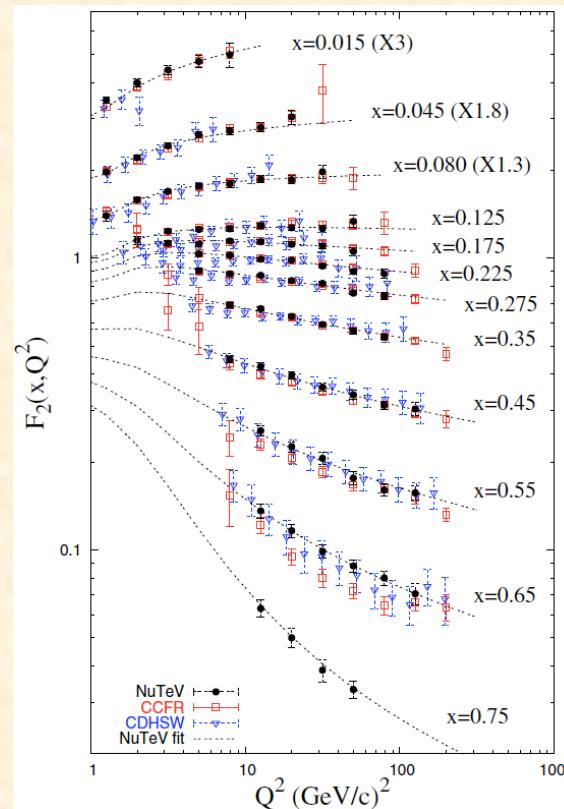
$$\frac{d\sigma_{v,\bar{v}}^{CC}}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi (1+Q^2/M_W^2)^2} \left[ x y^2 F_1^{CC} + \left( 1 - y - \frac{M}{2E} \frac{x y}{x + y} \right) F_2^{CC} \pm x y \left( 1 - \frac{y}{2} \right) F_3^{CC} \right]$$

# Neutrino DIS experiments



Huge Fe target (690 ton)

M. Tzanov *et al.* (NuTeV),  
PRD74 (2006) 012008.

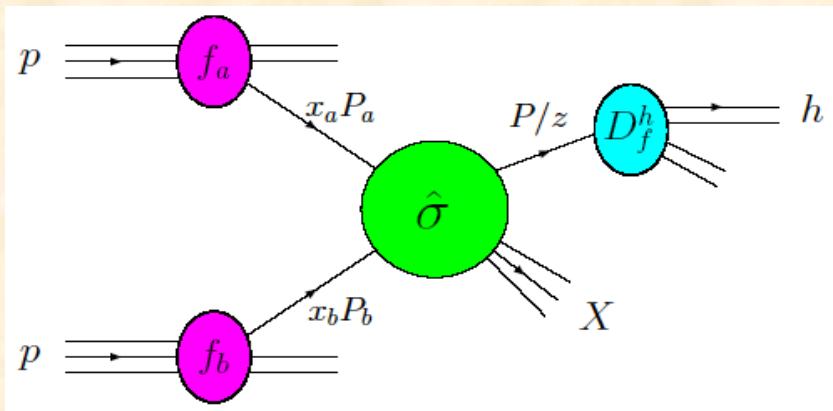


Experiment	Target	energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

# **Parton Distribution Functions in the Nucleon**

# High-energy nuclear reactions

Nuclear PDFs are needed for describing high-energy nuclear reactions in order to find any new phenomena.



$$\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \rightarrow cX) \otimes D_c^h(z, Q^2)$$

$f_a(x_a, Q^2)$ : parton distribution functions

$\hat{\sigma}(ab \rightarrow cX)$ : partonic cross sections

$D_c^h(z, Q^2)$ : fragmentation functions

# Recent works on unpolarized PDFs

**ABKM** (Alekhin, Blümlein, Klein, Moch)

ABKM-2010, 2011, S. Alekhin *et al.*, Phys. Rev. D 81 (2010) 014032; Phys. Rev. D86 (2012) 054009;  
ABM-2014, S. Alekhin *et al.*, Phys. Rev. D89 (2014) 054028.

**CTEQ** (Coordinated Theoretical-Experimental Project on QCD)

CTEQ6.6, P. M. Nadolsky *et al.*, Phys. Rev. D 78 (2008) 013004.  
CT10, H.-L. Lau *et al.*, Phys. Rev. D 82 (2010) 074024.  
CT12, J. F. Owens *et al.*, Phys. Rev. D 87 (2013) 094012.

**GJR** (Glück, Jimenez-Delgado, Reya)

GJR-2008, M. Gluck *et al.*, Eur. Phys. J. C 53 (2008) 355; PRD79 (2009) 074023;  
JR-2014, Phys.Rev. D89 (2014) 074049.

**HERA** (H1 and ZEUS collaborations)

HERAPDF, F. D. Aaron *et al.*, JHEP 01 (2010) 109; Eur. Phys. J. C73 (2013) 2311.

**MSTW** (Martin, Stirling, Thorne, Watt, L. A. Harland-Lang, P. Motylinski)

MSTW2008, A. D. Martin *et al.*, Eur. Phys. J. C 63 (2009) 189;  
MMHT2014, A. Harland-Lang *et al.*, arXiv:1412.3989.

**Neural Network** (Ball, Bertone, Carrazza, Del Debbio, Forte, Guffanti, Hartland, Latorre, Rojo, Ubiali, ...)

NNPDF2.0, R. D. Ball *et al.*, Nucl. Phys. B 838 (2010) 136; B855 (2012) 153;  
B867 (2013) 244; B874 (2013) 36; B877 (2013) 290; arXiv:1410.8849.

## Parton distribution functions are determined by fitting various experimental data.

- electron/muon:  $\mu + p \rightarrow \mu + X$
- neutrino:  $\nu_\mu + p \rightarrow \mu + X$
- Drell-Yan:  $p + p \rightarrow \mu^+ \mu^- + X$
- ...

(1) assume functional form of PDFs at fixed  $Q^2 (\equiv Q_0^2)$ :

$$\text{e.g. } f_i(x, Q_0^2) = A_i x^{\alpha_i} (1-x)^{\beta_i} (1+\gamma_i x),$$

where  $i = u_\nu, d_\nu, \bar{u}, \bar{d}, \bar{s}, g$

(2) calculate observables at their experimental  $Q^2$  points.

(3) then, the parameters  $A_i, \alpha_i, \beta_i, \gamma_i$  are determined so as to minimize  $\chi^2$  in comparison with data.

## Determination of each distribution

**Valence quark**  $q_v(x) \equiv q(x) - \bar{q}(x)$

$$M = \frac{1}{1+Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k', \lambda') \gamma^\mu (1 - \underline{\gamma_5}) u(k, \lambda) \langle X | J_\mu^{CC} | p, \lambda_p \rangle$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$

$$L^{\mu\nu} = 8 \left[ k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k \cdot k' + \underline{i\varepsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma} \right] \quad \text{where } \varepsilon_{0123} = +1$$

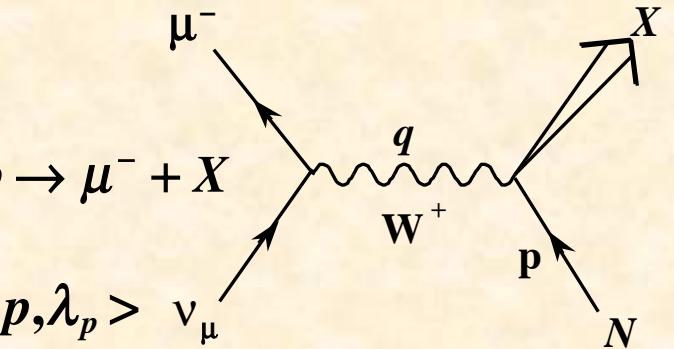
$$W_{\mu\nu} = -W_1 \left( g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M_N^2} \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) + \boxed{\frac{i}{2M_N^2} W_3 \varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma}$$

$$MW_1 = F_1, \quad \nu W_2 = F_2, \quad \nu W_3 = F_3, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

$$\frac{d\sigma_{\nu,\bar{\nu}}^{CC}}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi (1+Q^2/M_W^2)^2} \left[ x y^2 F_1^{CC} + \left( 1 - y - \frac{M x y}{2E} \right) F_2^{CC} \pm \underline{x y \left( 1 - \frac{y}{2} \right) F_3^{CC}} \right]$$

$$\frac{1}{2} [F_3^{\nu p} + F_3^{\bar{\nu} p}]_{CC} = \underline{u_v + d_v} + s - \bar{s} + c - \bar{c}$$

Valence: also  $F_2$  at large  $x$



**Note:** Nuclear corrections  
in CCFR/NuTeV ( $\nu$ +Fe).

# Sea quark

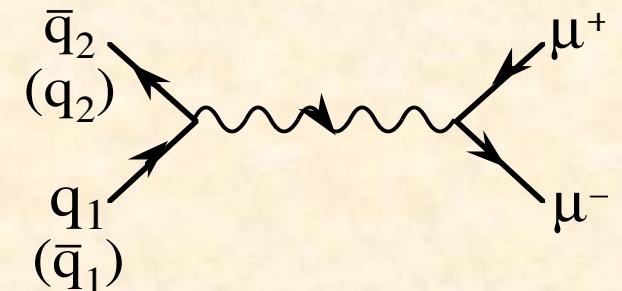
## e/ $\mu$ scattering

$$\begin{aligned}
 F_2^N &= \frac{F_2^p + F_2^n}{2} = \frac{x}{2} \left[ \frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d} + s + \bar{s}) + \frac{4}{9}(d + \bar{d}) + \frac{1}{9}(u + \bar{u} + s + \bar{s}) \right] = \frac{x}{2} \left[ \frac{5}{9}(u + \bar{u} + d + \bar{d}) + \frac{2}{9}(s + \bar{s}) \right] \\
 &= \frac{x}{2} \left[ \frac{5}{9}(u_v + d_v) + \frac{10}{9}(\bar{u} + \bar{d}) + \frac{2}{9}(s + \bar{s}) \right] = \frac{5}{18}x(u_v + d_v) + \frac{2}{18}x \underline{\left[ 5(\bar{u} + \bar{d}) + (s + \bar{s}) \right]}
 \end{aligned}$$

## Drell-Yan (lepton-pair production)

$$p_1 + p_2 \rightarrow \mu^+ \mu^- + X$$

$$d\sigma \propto q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)$$



at large  $x_F = x_1 - x_2$

projectile target

$$d\sigma \propto q_v(x_1)\bar{q}(x_2)$$

$\bar{q}(x_2)$  can be obtained if  $q_v(x_1)$  is known.

# Gluon

## scaling violation of $F_2$

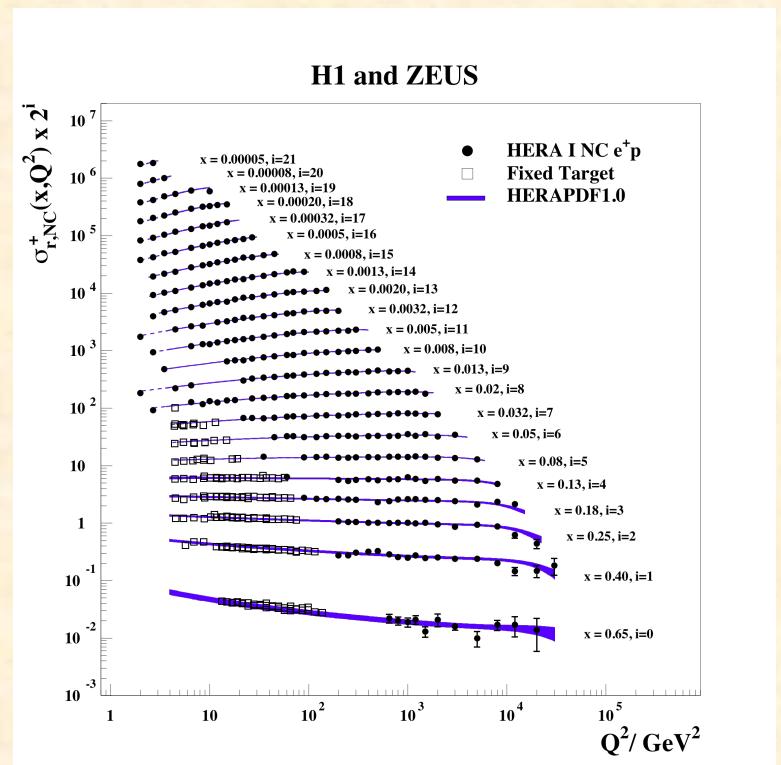
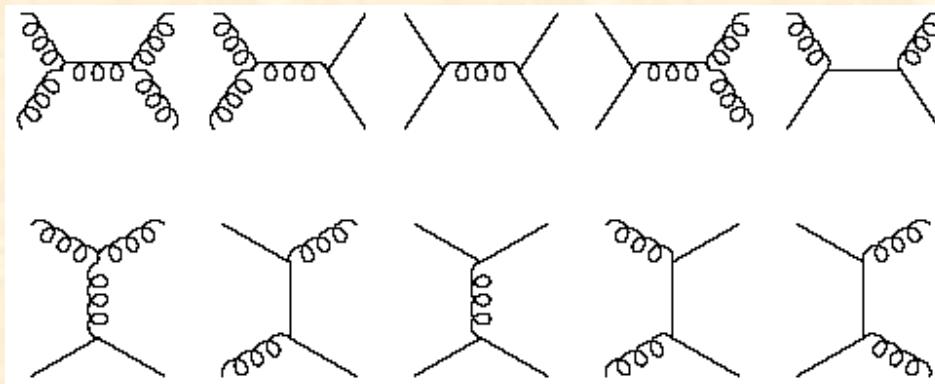
$$\frac{\partial}{\partial(\ln Q^2)} \begin{pmatrix} q_s(x,t) \\ g(x,t) \end{pmatrix} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{qq}(x/y) & P_{qg}(x/y) \\ P_{gq}(x/y) & P_{gg}(x/y) \end{pmatrix} \begin{pmatrix} q_s(y,t) \\ g(y,t) \end{pmatrix}$$

H1 and ZEUS  
JHEP01(2010)109

$$\text{at small } x \quad \frac{\partial F_2}{\partial(\ln Q^2)} \approx \frac{10 \alpha_s}{27\pi} g$$

**K. Prytz, Phys. Lett. B311 (1993) 286.**

# jet production





## Structure functions in parton model for neutrino-nucleon scattering

$$F_2 = 2 \times F_1$$

$$F_2^{vp} = 2 \times (d + s + \bar{u} + \bar{c})$$

$$F_2^{\bar{v}p} = 2 \times (u + c + \bar{d} + \bar{s})$$

$$F_2^{vn} = 2 \times (u + s + \bar{d} + \bar{c})$$

$$F_2^{\bar{v}n} = 2 \times (d + c + \bar{u} + \bar{s})$$

$$xF_3^{vp} = 2 \times (d + s - \bar{u} - \bar{c})$$

$$xF_3^{\bar{v}p} = 2 \times (u + c - \bar{d} - \bar{s})$$

$$xF_3^{vn} = 2 \times (u + s - \bar{d} - \bar{c})$$

$$xF_3^{\bar{v}n} = 2 \times (d + c - \bar{u} - \bar{s})$$

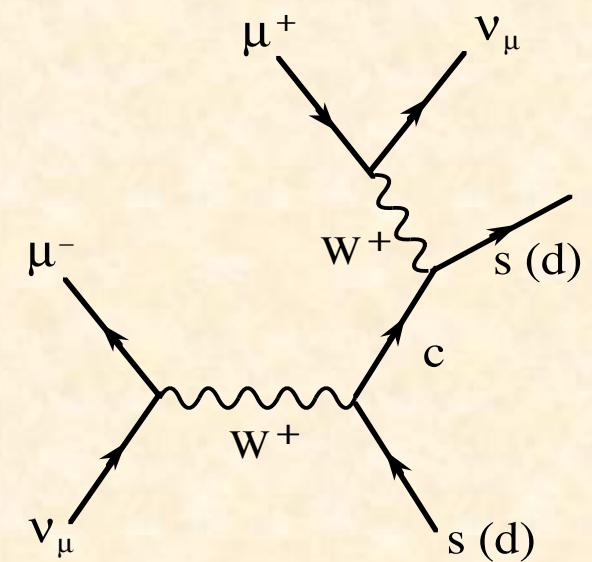
also  $\nu p \rightarrow \mu^- \mu^+ X$  for finding  $2 \bar{s} / (\bar{u} + \bar{d})$



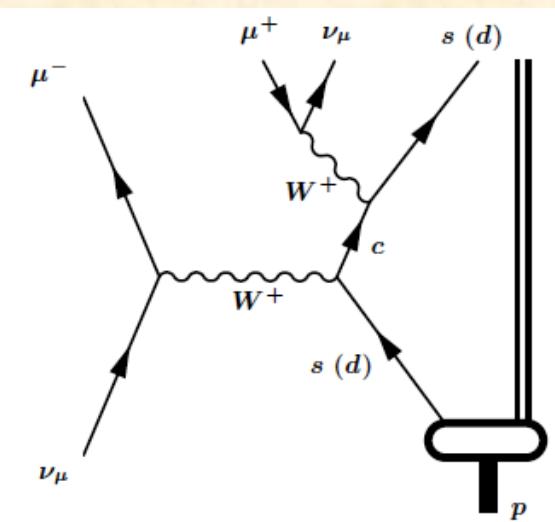
$$F_3^{vp} + F_3^{\bar{v}p} = 2 (u_v + d_v) + 2 (s - \bar{s}) + 2 (c - \bar{c})$$

**valence-quark distributions**

$$F_3^{v(p+n)/2} - F_3^{\bar{v}(p+n)/2} = 2 (s + \bar{s}) - 2 (c + \bar{c})$$



## $s(x)$ from neutrino-induced opposite-sign dimuon events



A. Kayis-Topaksu *et al.*, NPB7 98 (2008) 1.  
U. Dore, arXiv: 1103.4572 [hep-ex].

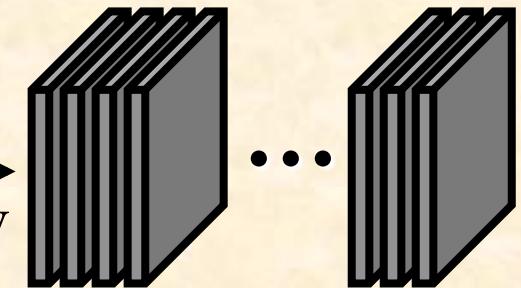
$$\kappa = \frac{\int dx x [s(x, Q^2) + \bar{s}(x, Q^2)]}{\int dx x [\bar{u}(x, Q^2) + \bar{d}(x, Q^2)]}$$

$$Q^2 = 20 \text{ GeV}^2$$

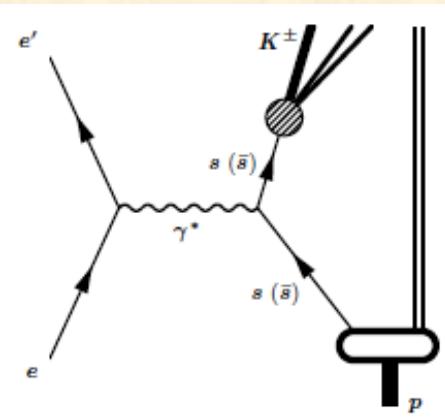
CCFR, NuTeV

$\nu, \bar{\nu}$   
 $E = 30 \sim 500 \text{ GeV}$

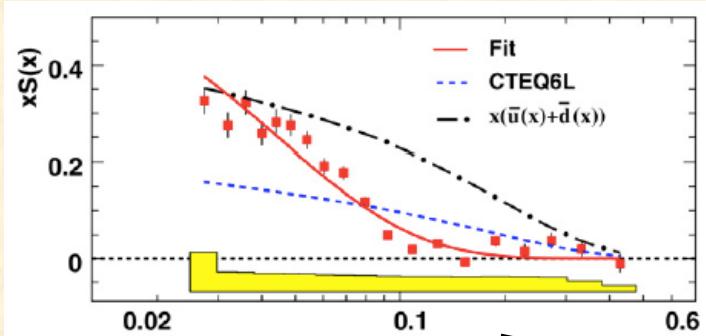
Experiment	$\kappa$
This analysis	$0.33 \pm 0.07$
CDHS [1]	$0.47 \pm 0.09$
CCFR [2]	$0.44 \pm 0.09$
CHARM II [3]	$0.39 \pm 0.09$
NOMAD [4]	$0.48 \pm 0.17$
NuTeV [5]	$0.38 \pm 0.08$



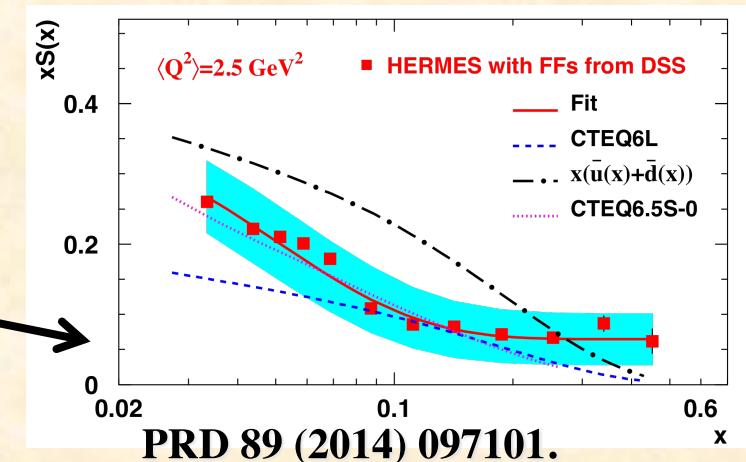
## HERMES semi-inclusive measurement



A. Airapetian *et al.*,  
PLB 666 (2008) 446.



Huge Fe target (690 ton)  
Issue: nuclear corrections



# MMLT-2014

L. A. Harland-Lang, A. D. Martin,  
P. Motylinkski, and R. S. Thorne,  
arXiv:1412.3989.

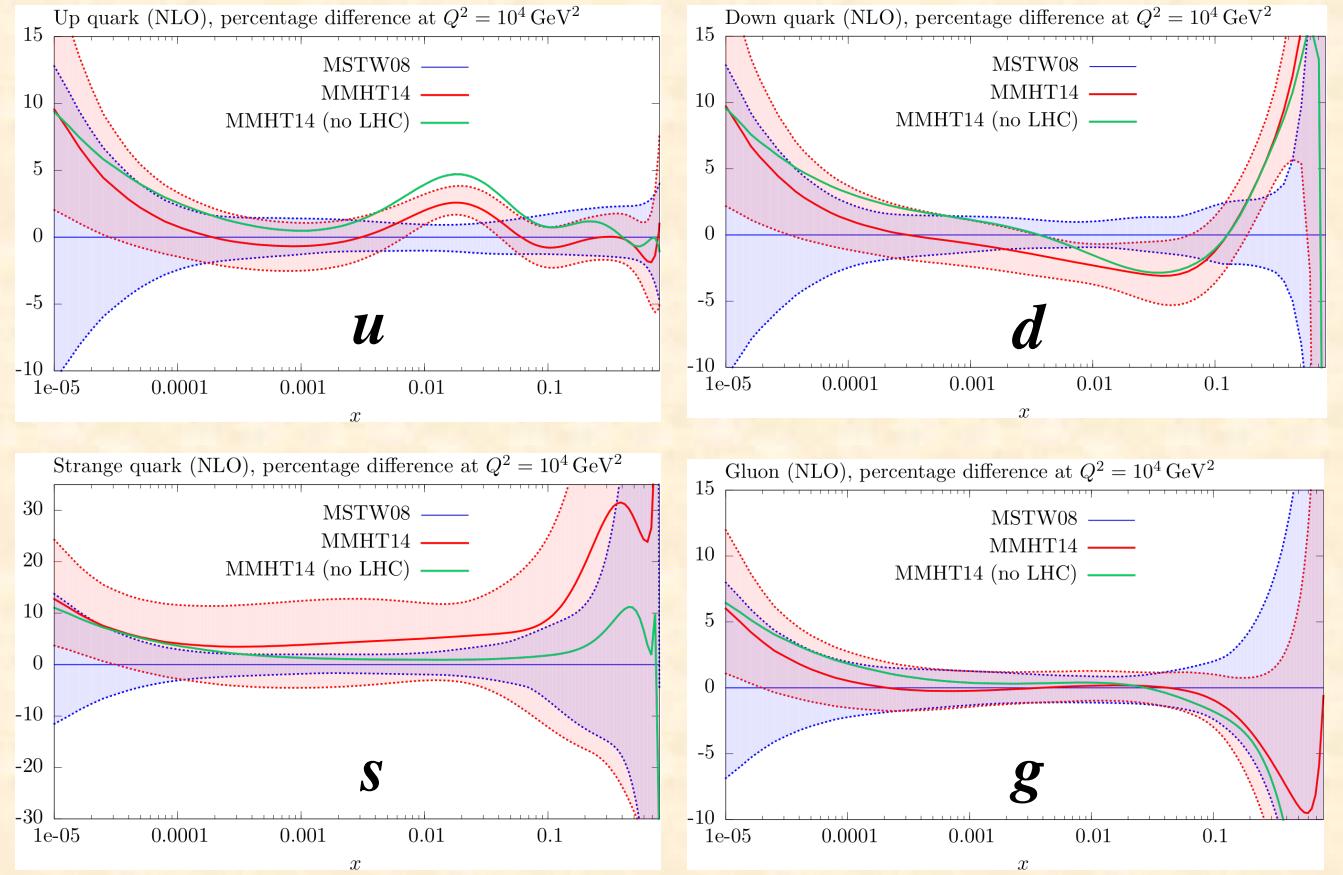
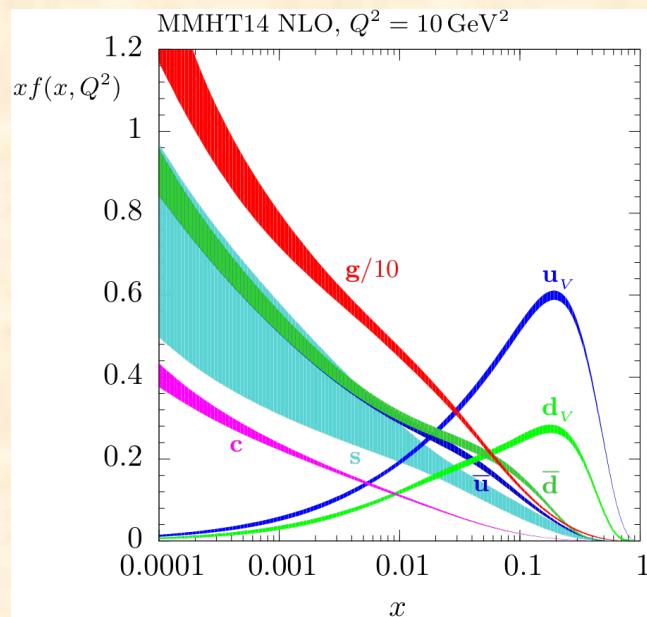
Functional form:  $Q_0^2 = 1 \text{ GeV}^2$

$$xf(x, Q_0^2) = Ax^\delta(1-x)^\eta \left[ 1 + \sum_{i=1}^n a_i T_i(y(x)) \right], \quad y = 1 - 2\sqrt{x}, \quad n = 4, \quad f = u_v, \quad d_v, \quad S, \quad s_+ = s + \bar{s}, \quad \delta_s = \delta_{s_+}$$

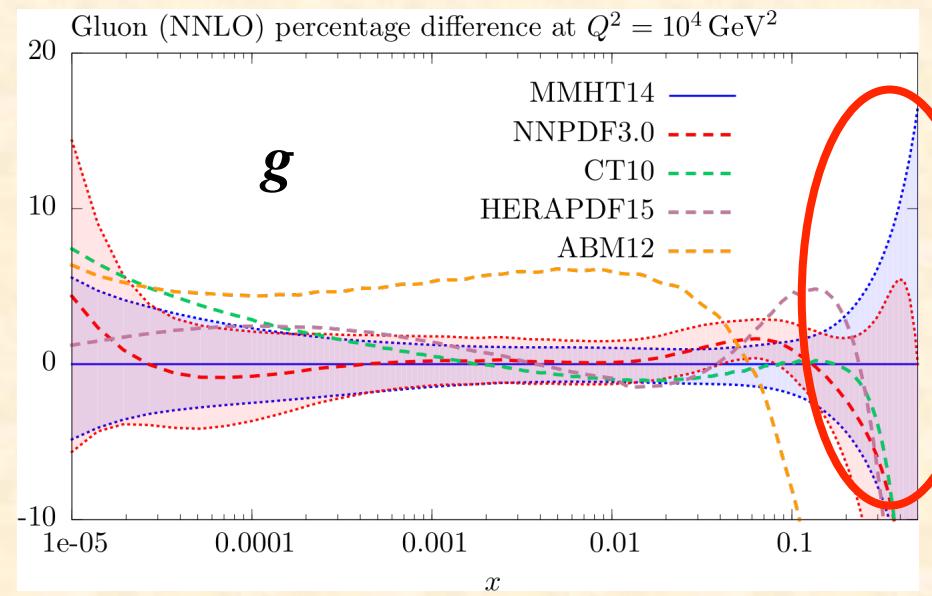
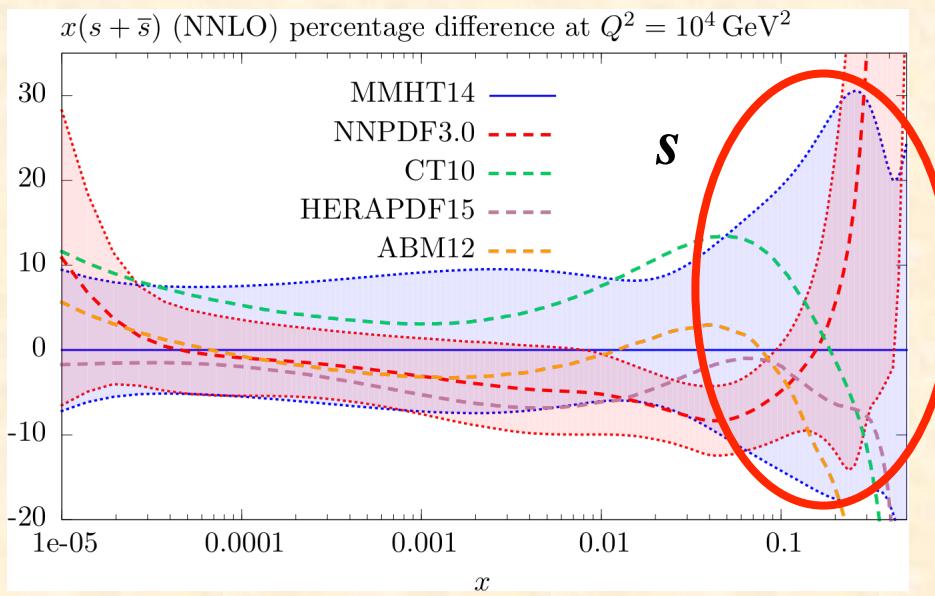
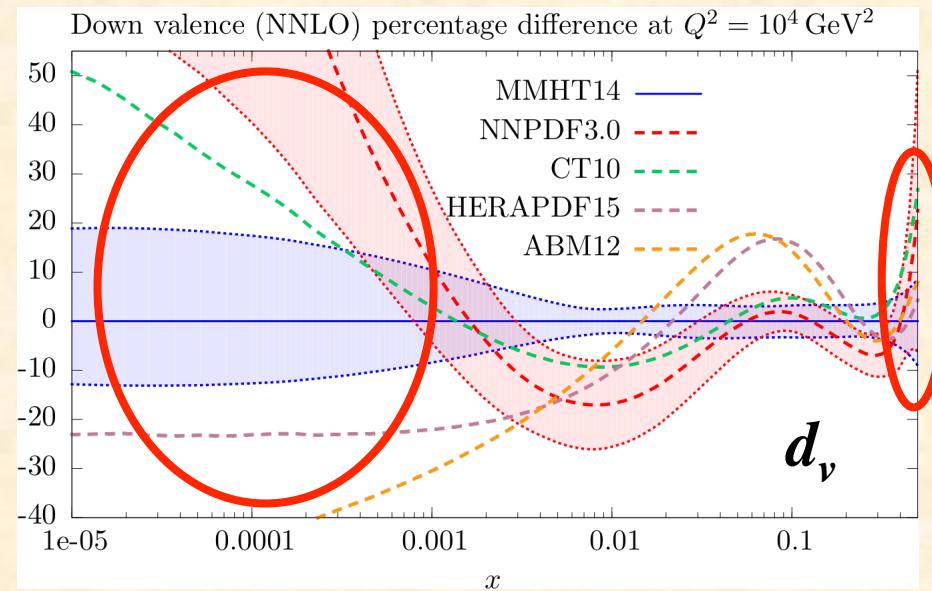
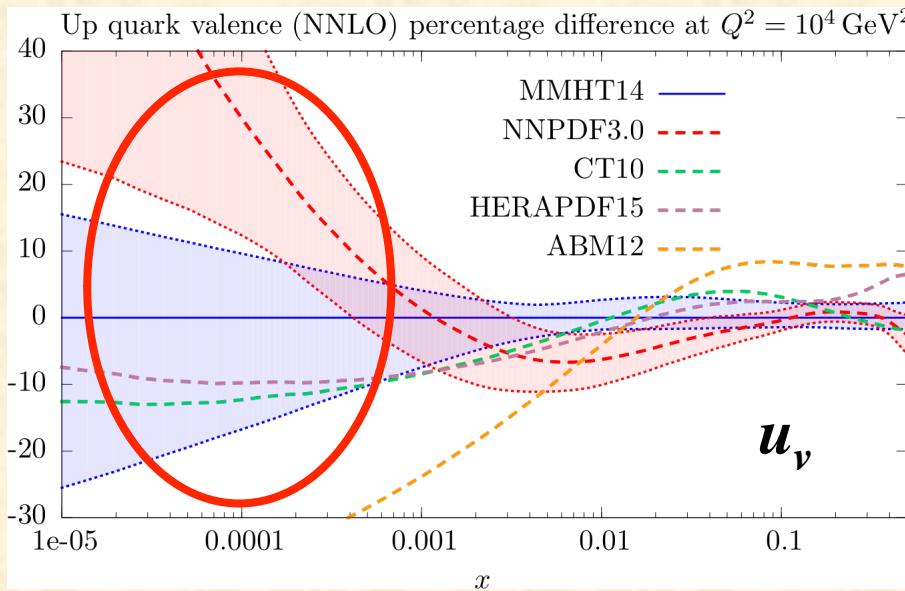
$$xg(x) = A_g x^{\delta_g} (1-x)^{\eta_g} \left[ 1 + \sum_{i=1}^2 a_{gi} T_i(y(x)) \right] + A'_g x^{\delta'_g} (1-x)^{\eta'_g}$$

$$x\Delta(x) \equiv x(\bar{d} - \bar{u}) = Ax^\delta(1-x)^\eta(1 + \gamma x + \varepsilon x^2), \quad xs_- = x[s(x) - \bar{s}(x)] = A_x \delta_{s_-} (1-x)^{\eta_{s_-}} (1 - x/x_0)$$

- NLO, NNLO



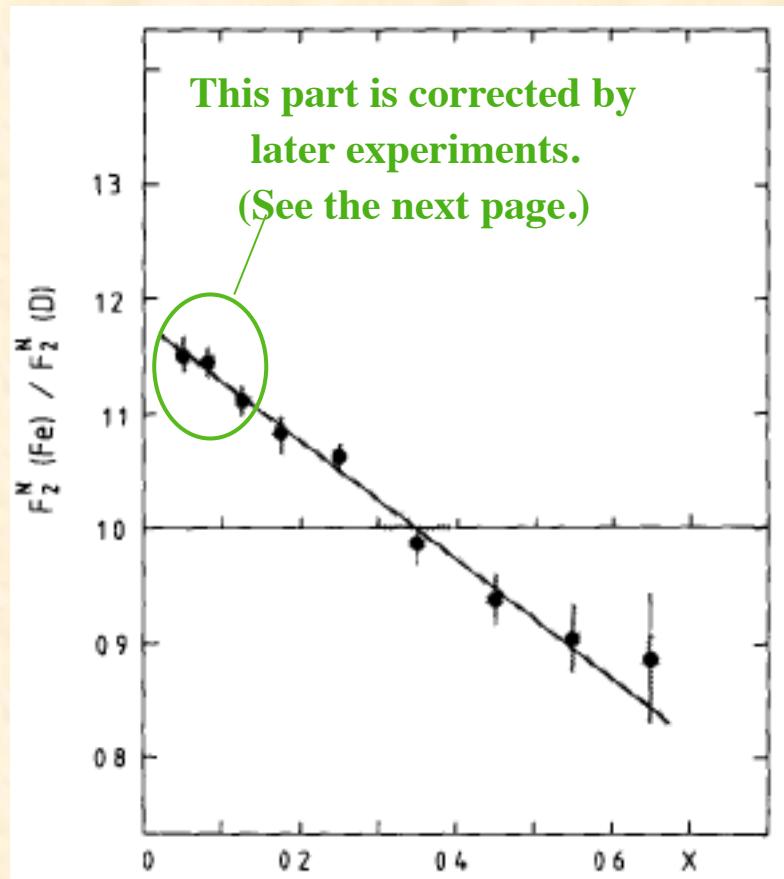
# Comparisons with various PDFs



# **Nuclear modifications of parton distribution functions: Physics mechanisms**

# EMC (European Muon Collaboration) effect

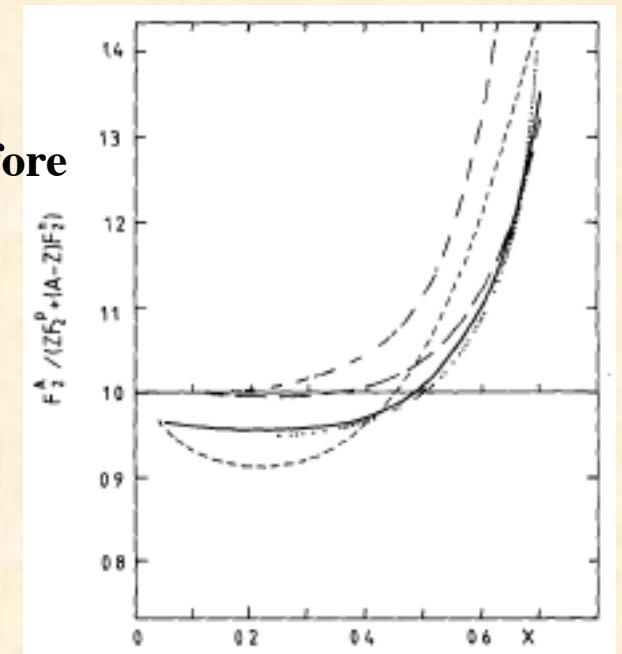
J. J. Aubert et al. (EMC),  
Phys. Lett. B123 (1983) 275.



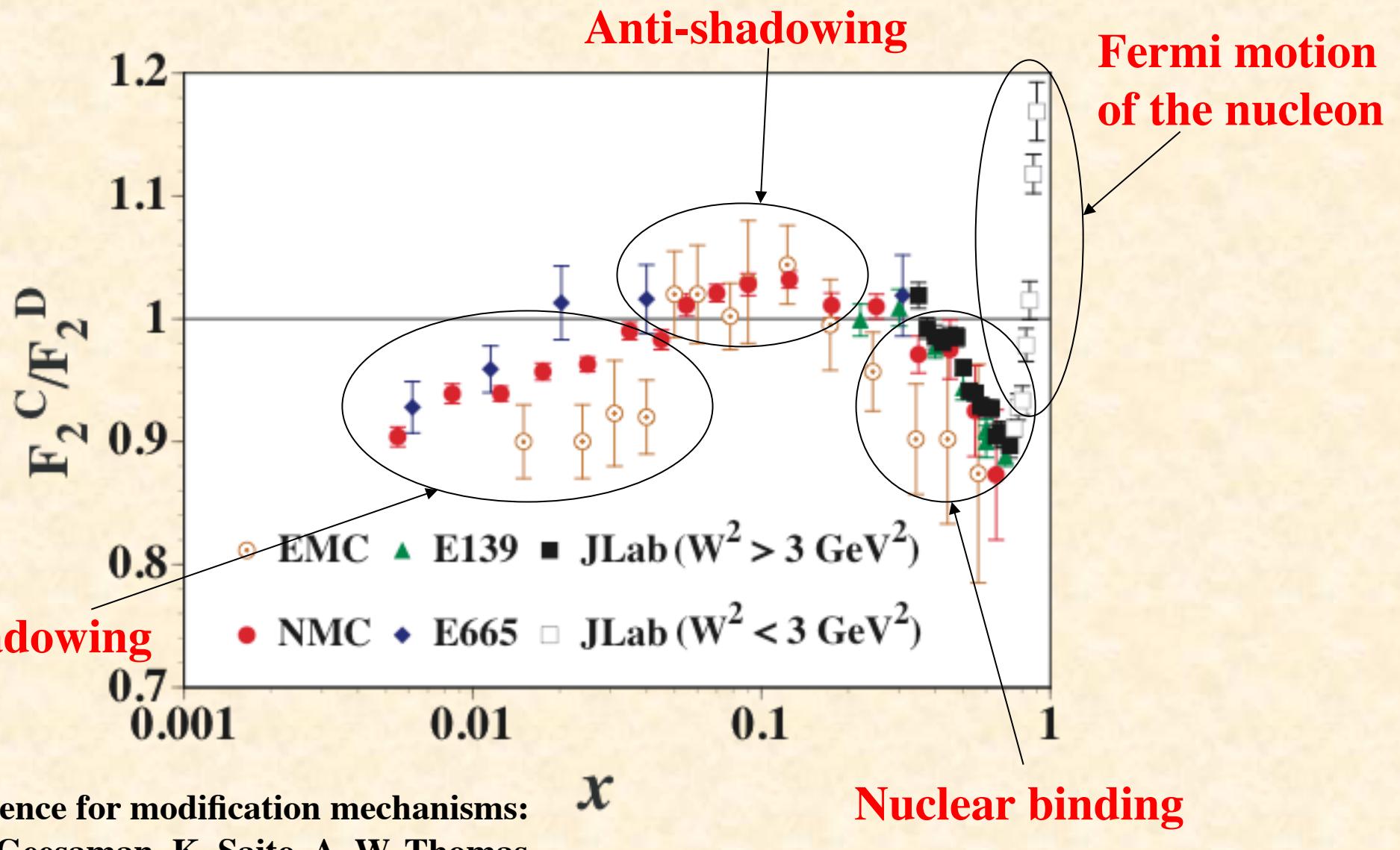
In the EMC paper of 1983, they pointed out that nuclear modifications exist in a deep Inelastic structure function  $F_2$ .

In general, nuclear binding energies are negligible in comparison with typical DIS energies ( $Q, v$ ), so that such modifications were expected to be small.

Fermi motion effects were theoretically calculated before the EMC publication. →

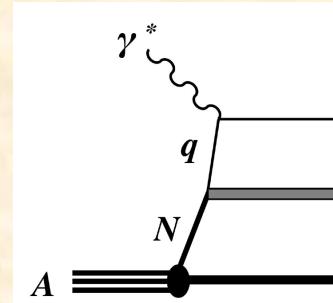


# Nuclear modifications of structure function $F_2$



# Binding and Fermi motion

**Convolution:**  $W_{\mu\nu}^A(p_A, q) = \int d^4 p S(p) W_{\mu\nu}^N(p_N, q)$



$S(p)$  = Spectral function = nucleon momentum distribution in a nucleus

In a simple shell model:  $S(p) = \sum_i |\phi_i(\vec{p})|^2 \delta(p_0 - M_N - \varepsilon_i)$

Separation energy:  $\varepsilon_i$

$$\boxed{\begin{aligned}\hat{P}_2^{\mu\nu} &= -\frac{M_N^2 v}{2\tilde{p}^2} \left( g^{\mu\nu} - \frac{3\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2} \right) \\ \hat{P}_2^{\mu\nu} W_{\mu\nu} &= F_2\end{aligned}}$$

Projecting out  $F_2$ :  $F_2^A(x, Q^2) = \sum_i \int dz f_i(z) F_2^N(x/z, Q^2)$

$$z = \frac{\mathbf{p} \cdot \mathbf{q}}{M_N v} \simeq \frac{\mathbf{p} \cdot \mathbf{q}}{p_A \cdot \mathbf{q}/A} \simeq \frac{p^+}{p_A^+/A} \quad \text{lightcone momentum fraction}$$

$$\mathbf{p} \cdot \mathbf{q} = p^+ q^- + p^- q^+ - \vec{p}_T \cdot \vec{q}_T \simeq p^+ q^-$$

$$\boxed{\begin{aligned}a^\pm &= \frac{a^0 \pm a^3}{\sqrt{2}} \\ \mathbf{q} &= (v, 0, 0, -\sqrt{v^2 + Q^2}) \\ q^+ &= -\frac{Mx}{\sqrt{2}}, \quad q^- = \frac{2v + Mx}{\sqrt{2}} = \sqrt{2}v \gg M\end{aligned}}$$

$$f_i(z) = \int d^3 p z \delta\left(z - \frac{\mathbf{p} \cdot \mathbf{q}}{M_N v}\right) |\phi_i(\vec{p})|^2 \quad \text{lightcone momentum distribution for a nucleon } i$$

$$F_2^A(x, Q^2) = \sum_i \int dz f_i(z) F_2^N(x/z, Q^2) \quad f_i(z) = \int d^3 p z \delta\left(z - \frac{\vec{p} \cdot \vec{q}}{M_N v}\right) |\phi_i(\vec{p})|^2$$

$$z = \frac{\vec{p} \cdot \vec{q}}{M_N v} = \frac{p^0 v - \vec{p} \cdot \vec{q}}{M_N v} = 1 - \frac{|\varepsilon_i|}{M_N} - \frac{\vec{p} \cdot \vec{q}}{M_N v} \approx 1.00 - 0.02 \pm 0.20 \text{ for a medium-size nucleus}$$

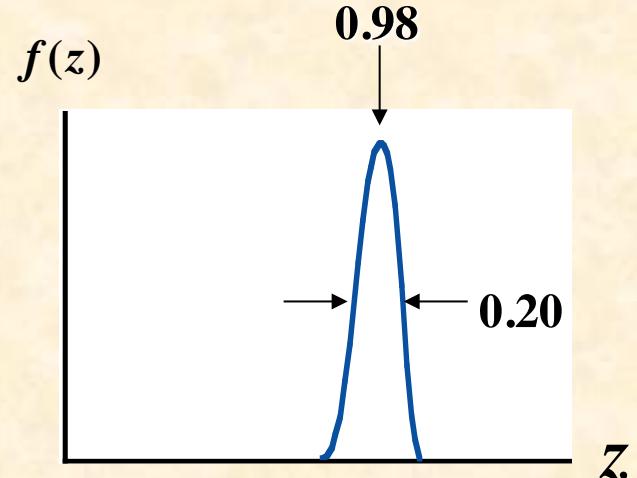
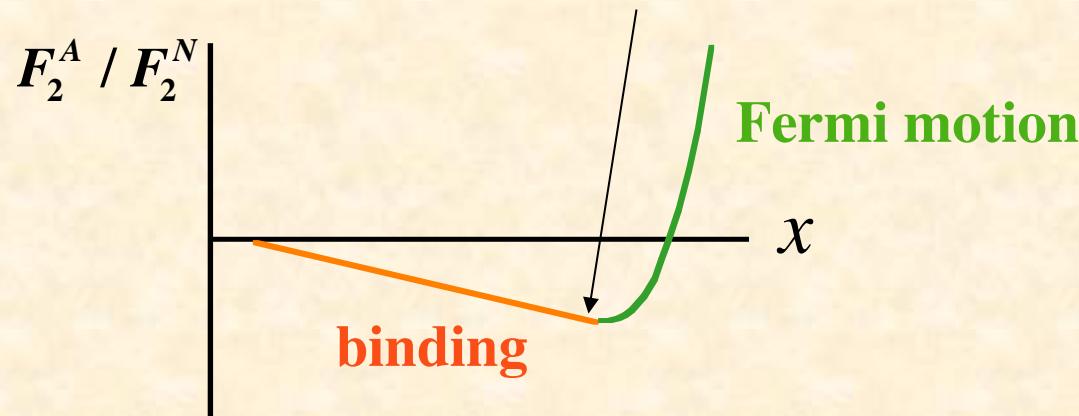
If  $f_i(z)$  were  $f_i(z) = \delta(z - 1)$ , there is no nuclear modification:  $F_2^A(x, Q^2) = F_2^N(x, Q^2)$ .

Because the peak shifts slightly ( $1 \rightarrow 0.98$ ), nuclear modification of  $F_2$  is created.

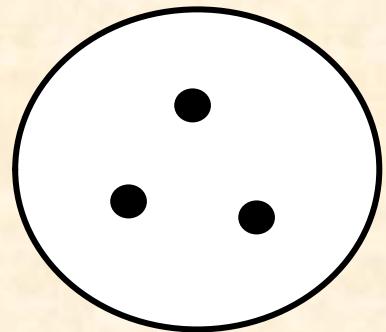
$$F_2^A(x, Q^2) \approx F_2^N(x/0.98, Q^2)$$

$$\text{For } x = 0.60, \quad x/0.98 = 0.61$$

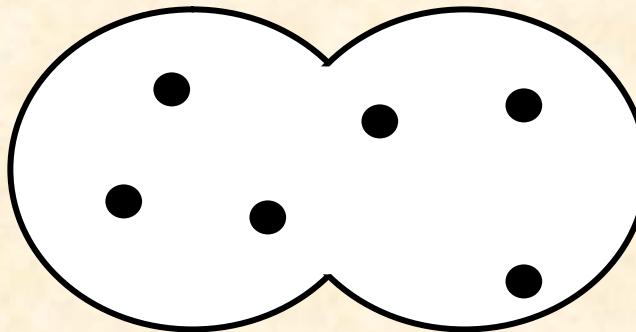
$$\frac{F_2^N(x=0.61)}{F_2^N(x=0.60)} = \frac{0.021}{0.024} = 0.88$$



# Theoretical Ideas at Medium $x$ : $Q^2$ Rescaling Model



Free nucleon



Nucleon may overlap in a nucleus.

Average nucleon separation  
 $(2 \text{ fm}) \approx \text{Nucleon diameter}$

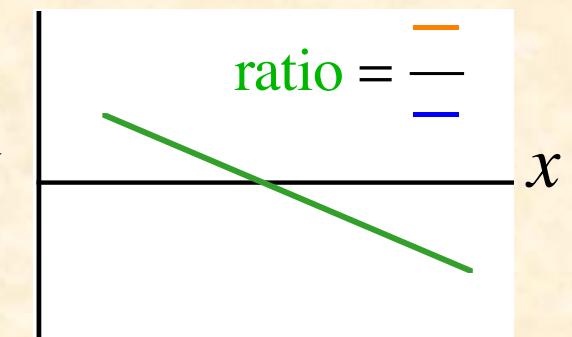
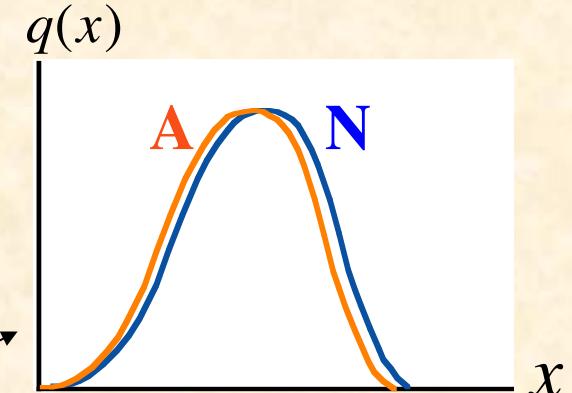
Confinement radius changes:  $\lambda_A > \lambda_N$



Quark momentum distribution changes.



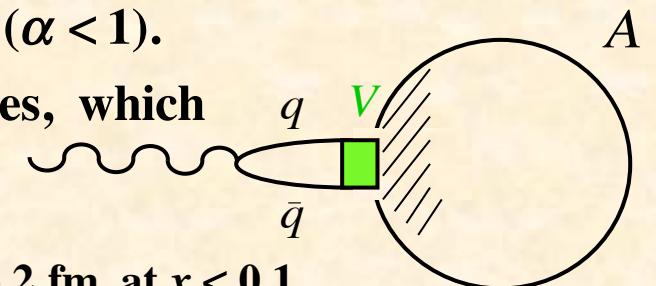
Ratio  $q_A(x)/q_N(x)$  is similar to the observed  
EMC data in 1983.



# Shadowing

- Shadowing means that internal constituents are shadowed due to the existence of nuclear surface ones, so that the cross section is smaller than the each nucleon contribution:  $\sigma_A = A^\alpha \sigma_N$  ( $\alpha < 1$ ).
- A virtual photon transforms into vector meson (or  $q\bar{q}$ ) states, which then interact with a target nucleus.

Propagation length of V ( $q\bar{q}$ ):  $\lambda = \frac{1}{|E_V - E_\gamma|} = \frac{2v}{M_V^2 + Q^2} = \frac{0.2 \text{ fm}}{x} > 2 \text{ fm at } x < 0.1$

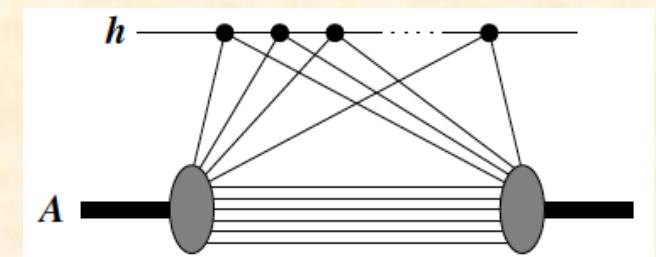


At small  $x$ , the virtual photon interacts with the target nucleus as if it were a vector meson (or  $q\bar{q}$ ).

- Shadowing takes place due to multiple scattering.

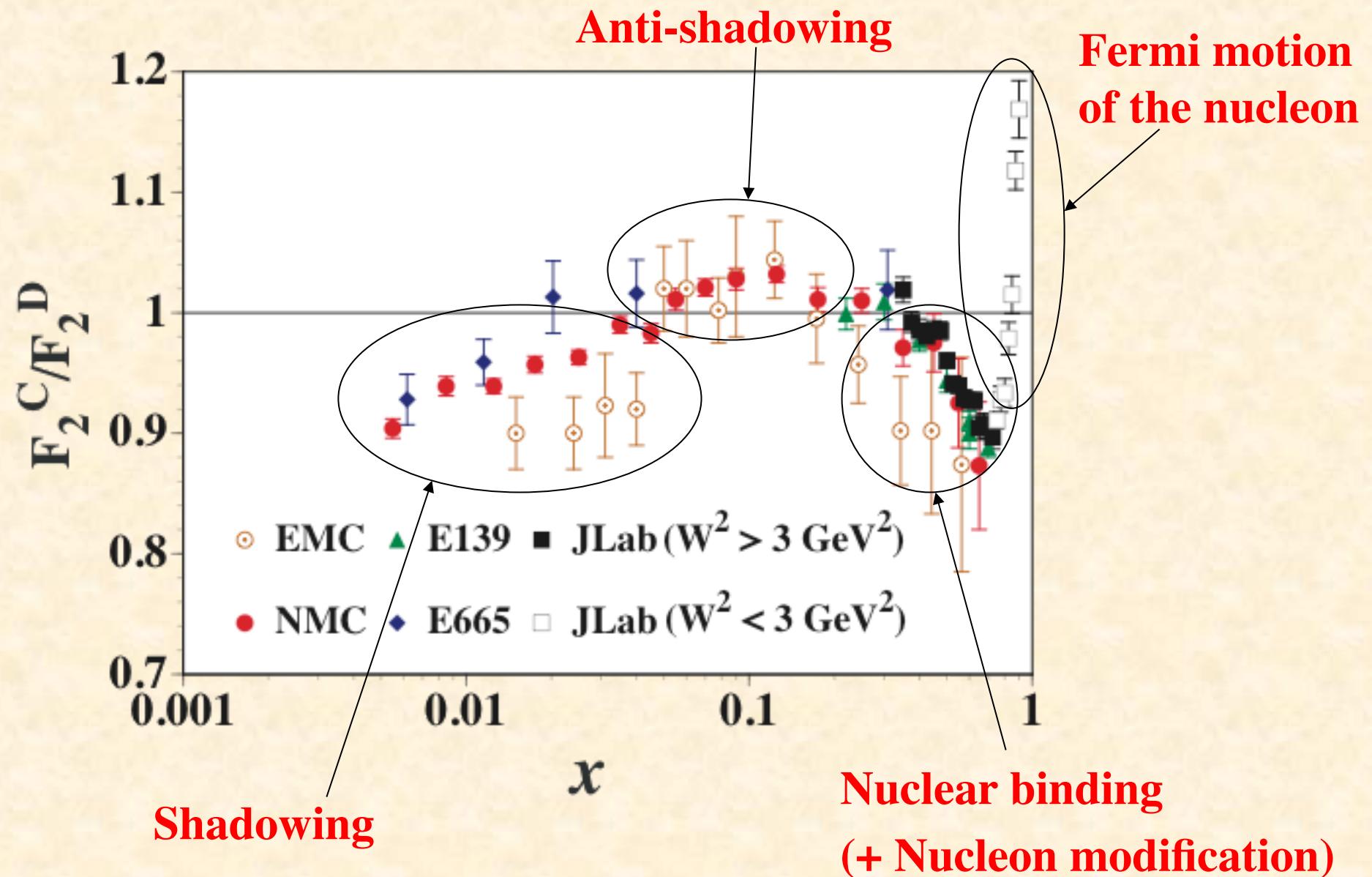
For example, the vector meson interacts elastically with a surface nucleon and then interacts inelastically with a central nucleon.

Because this amplitude is opposite in phase to the one-step amplitude for an inelastic interaction with the central nucleon, the nucleon sees a reduced hadronic flux (namely the shadowing).



# **Nuclear Parton Distribution Functions**

# Nuclear modifications of structure function $F_2$



# Experimental data

## (1) $F_2^A / F_2^D$

NMC: p, He, Li, C, Ca

SLAC: He, Be, C, Al,  
Ca, Fe, Ag, Au

EMC: C, Ca, Cu, Sn

E665: C, Ca, Xe, Pb

BCDMS: N, Fe

HERMES: N, Kr

+ JLab data

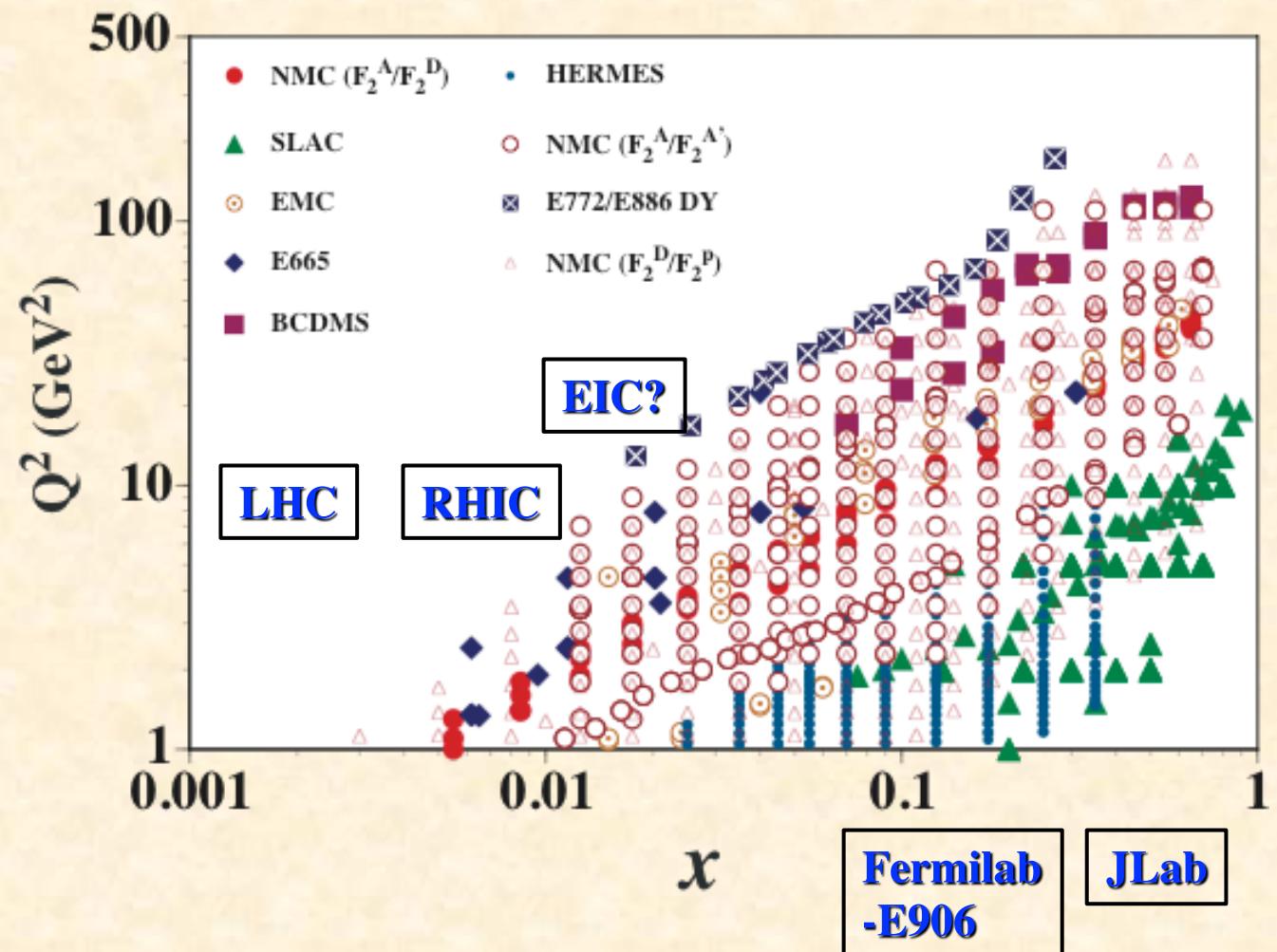
## (2) $F_2^A / F_2^{A'}$

NMC: Be / C, Al / C,  
Ca / C, Fe / C,  
Sn / C, Pb / C,  
C / Li, Ca / Li

## (3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$

E772: C / D, Ca / D,  
Fe / D, W / D

E866: Fe / Be, W / Be

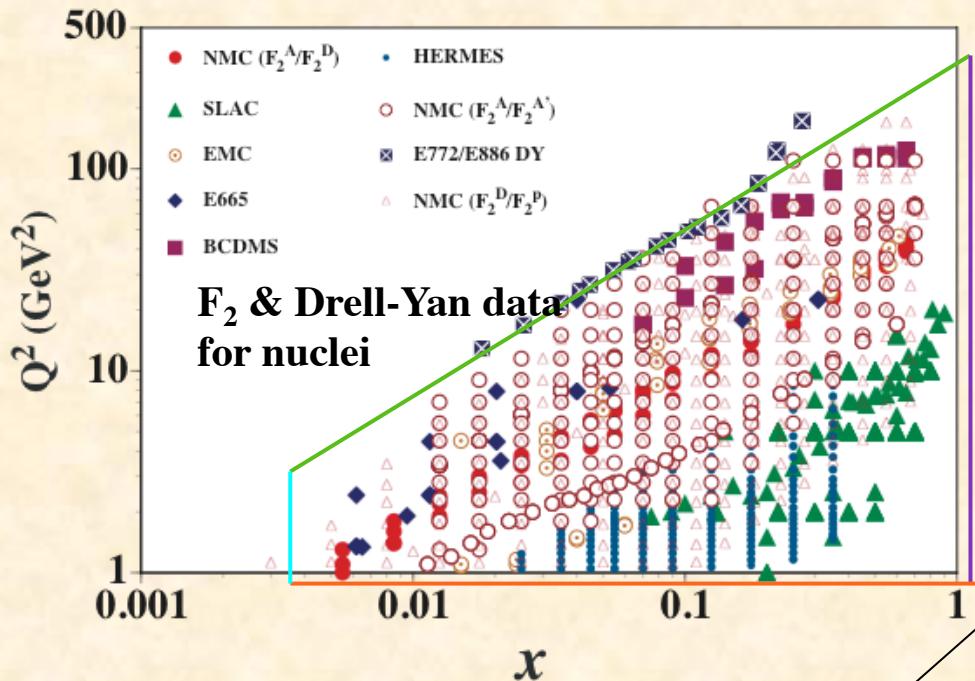


**Current nuclear data are kinematically limited.**

$$x = \frac{Q^2}{2p \cdot q} \approx \frac{Q^2}{ys}$$

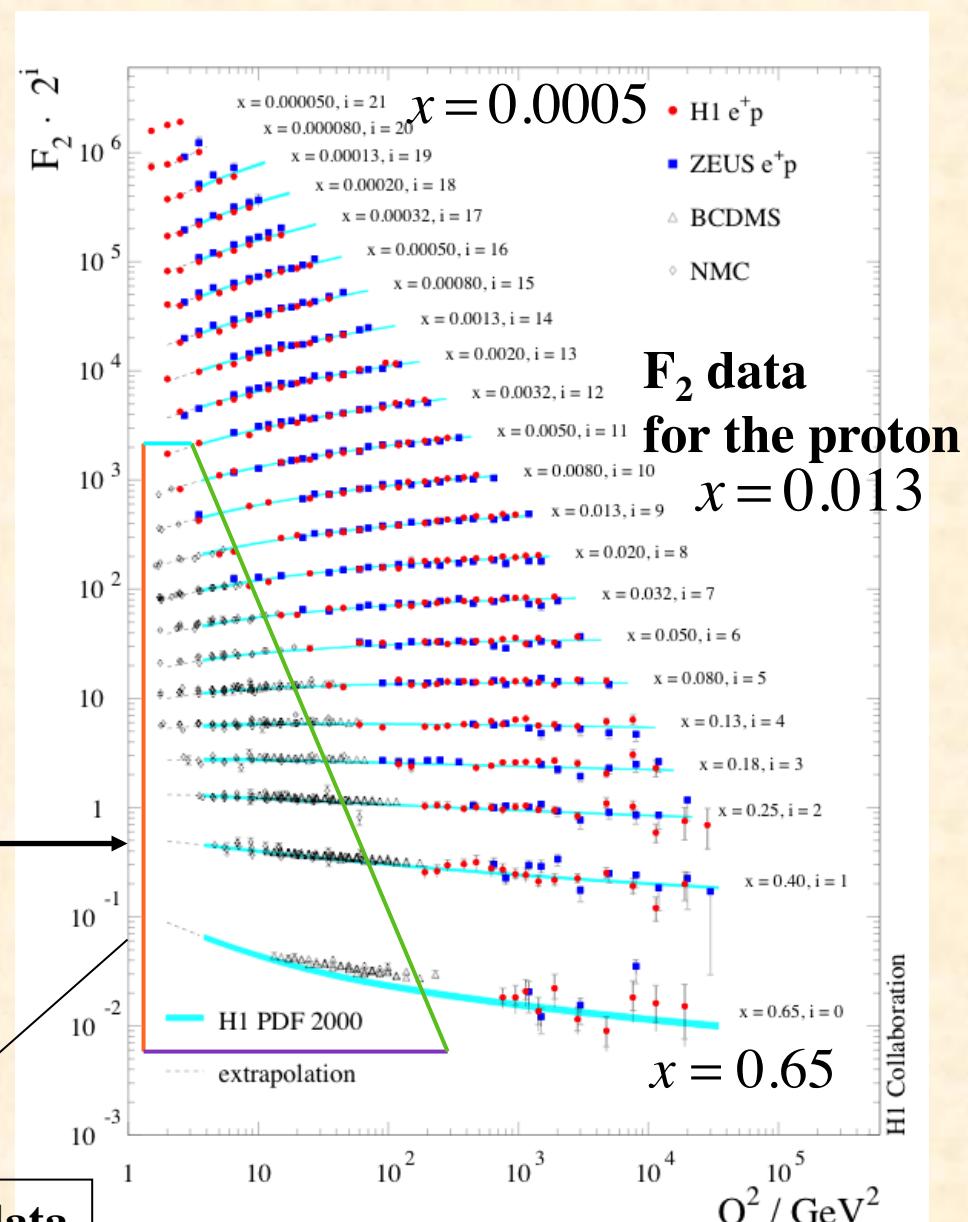
fixed target:  $\min(x) = \frac{Q^2}{2M_N E_{lepton}} \leq \frac{1}{2E_{lepton}(\text{GeV})}$   
if  $Q^2 \geq 1 \text{ GeV}^2$

for  $E_{lepton}(\text{NMC}) = 200 \text{ GeV}$ ,  $\min(x) = \frac{1}{2 \cdot 200} = 0.003$



region of nuclear data

(from H1 and ZEUS, hep-ex/0502008)



**$F_2$  data  
for the proton  
 $x = 0.013$**

H1 Collaboration

# Functional form Nuclear PDFs “per nucleon”

If there were no nuclear modification

$$Au^A(x) = Zu^p(x) + Nu^n(x), \quad Ad^A(x) = Zd^p(x) + Nd^n(x) \quad p = \text{proton}, \quad n = \text{neutron}$$

Isospin symmetry:  $u^n = d^p \equiv d, \quad d^n = u^p \equiv u$

$$\rightarrow u^A(x) = \frac{Zu(x) + Nd(x)}{A}, \quad d^A(x) = \frac{Zd(x) + Nu(x)}{A}$$

Take account of nuclear effects by  $w_i(x, A)$

$$u_v^A(x) = w_{u_v}(x, A) \frac{Zu_v(x) + Nd_v(x)}{A}, \quad d_v^A(x) = w_{d_v}(x, A) \frac{Zd_v(x) + Nu_v(x)}{A}$$

$$\bar{u}^A(x) = w_{\bar{q}}(x, A) \frac{Z\bar{u}(x) + N\bar{d}(x)}{A}, \quad \bar{d}^A(x) = w_{\bar{q}}(x, A) \frac{Z\bar{d}(x) + N\bar{u}(x)}{A}$$

$$\bar{s}^A(x) = w_{\bar{q}}(x, A) \bar{s}(x)$$

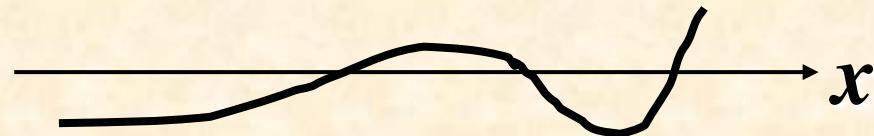
$$g^A(x) = w_g(x, A) g(x) \quad \text{at } Q^2=1 \text{ GeV}^2 (\equiv Q_0^2)$$

# Functional form of $w_i(x, A)$

$$f_i^A(x, Q_0^2) = w_i(x, A) f_i(x, Q_0^2) \quad i = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g$$

$$w_i(x, A) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^\beta}$$

Note: The region  $x > 1$  cannot be described by this parametrization.



A simple function = cubic polynomial

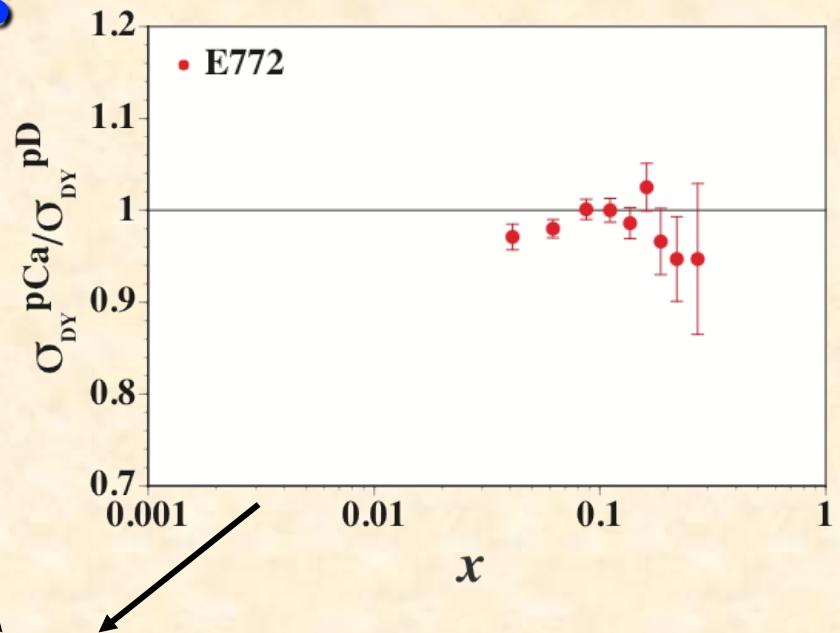
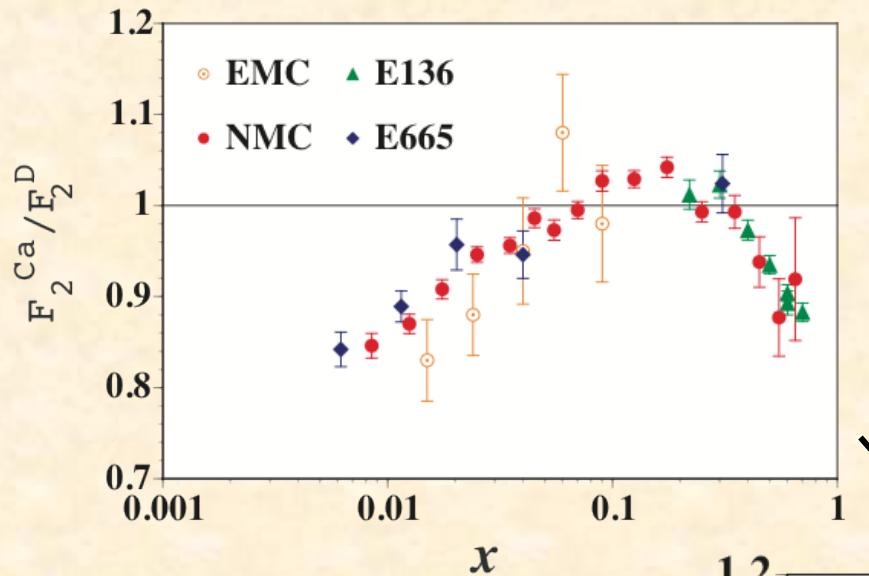
## Three constraints

**Nuclear charge:**  $Z = A \int dx \left[ \frac{2}{3} (u^A - \bar{u}^A) - \frac{1}{3} (d^A - \bar{d}^A) - \frac{1}{3} (s^A - \bar{s}^A) \right] = A \int dx \left[ \frac{2}{3} u_v^A - \frac{1}{3} d_v^A \right]$

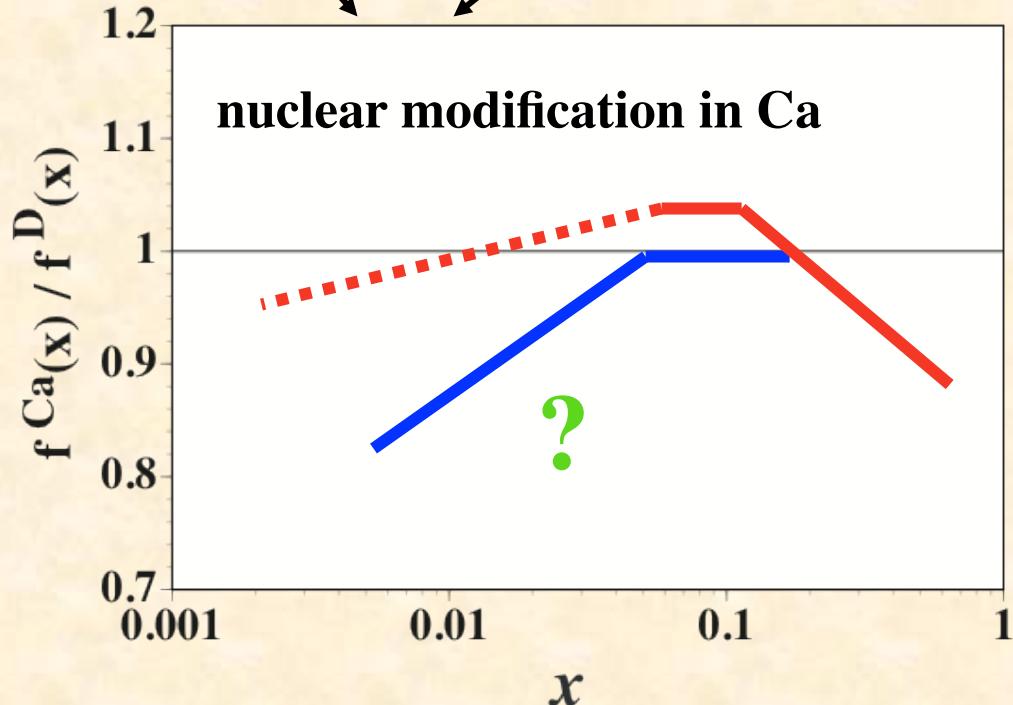
**Baryon number:**  $A = A \int dx \left[ \frac{1}{3} (u^A - \bar{u}^A) + \frac{1}{3} (d^A - \bar{d}^A) + \frac{1}{3} (s^A - \bar{s}^A) \right] = A \int dx \left[ \frac{1}{3} u_v^A + \frac{1}{3} d_v^A \right]$

**Momentum:** 
$$\begin{aligned} A &= A \int dx [u^A + \bar{u}^A + d^A + \bar{d}^A + s^A + \bar{s}^A + g] \\ &= A \int dx [u_v^A + d_v^A + 2(\bar{u}^A + \bar{d}^A + \bar{s}^A) + g] \end{aligned}$$

# Nuclear modification of PDFs

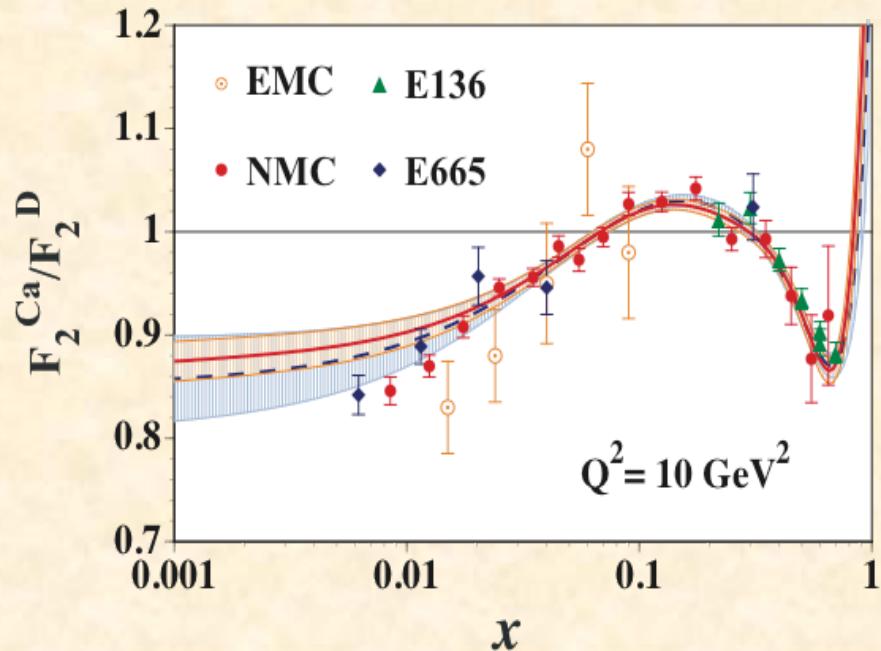


- valence quark
- antiquark
- gluon

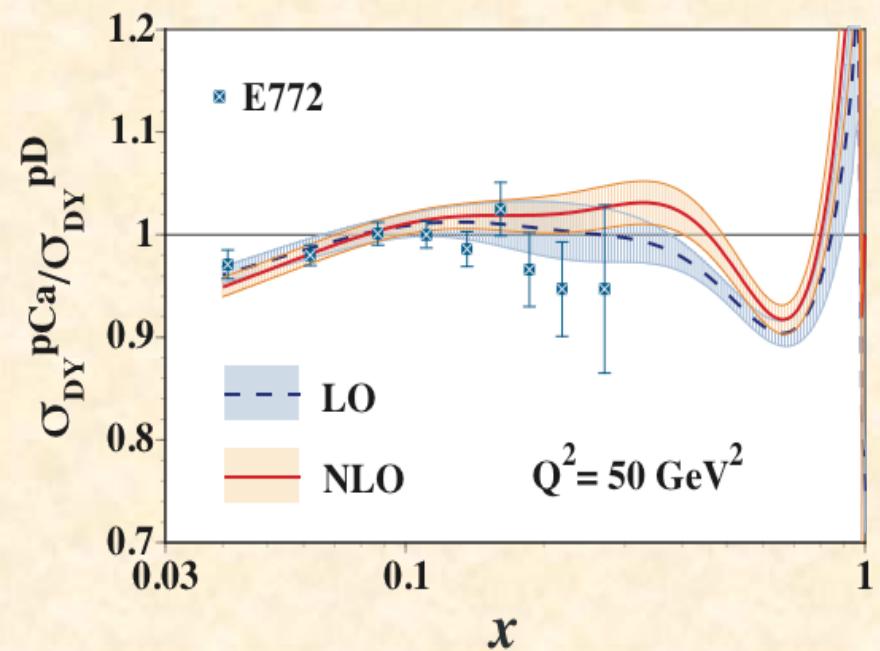


# Comparison with $F_2^{\text{Ca}}/F_2^{\text{D}}$ & $\sigma_{\text{DY}}^{\text{pCa}}/\sigma_{\text{DY}}^{\text{pD}}$ data

**LO analysis**

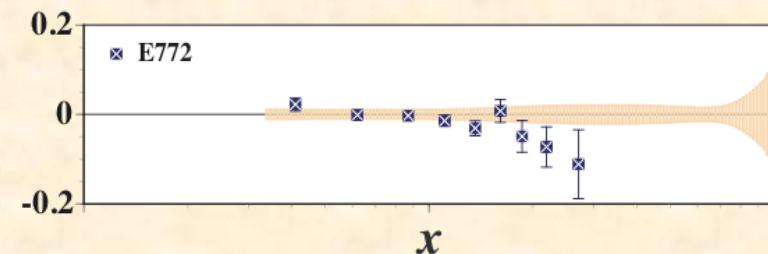
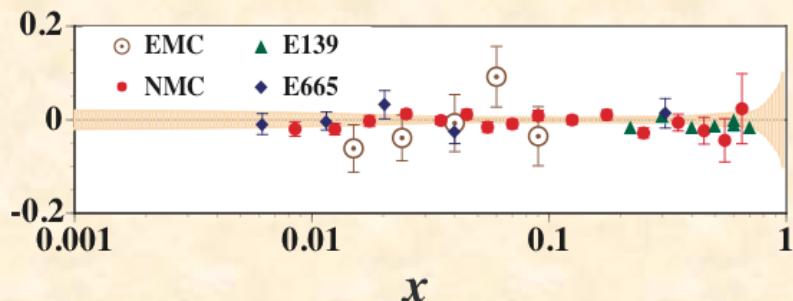


**NLO analysis**



$(R^{\text{exp}} - R^{\text{theo}})/R^{\text{theo}}$  at the same  $Q^2$  points

$R = F_2^{\text{Ca}}/F_2^{\text{D}}, \sigma_{\text{DY}}^{\text{pCa}}/\sigma_{\text{DY}}^{\text{pD}}$



# Scaling Violation and Gluon Distributions

$$\frac{\partial}{\partial \log Q^2} q_i^+(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[ \sum_j P_{q_i q_j}(x/y) q_j^+(y, Q^2) + \underline{P_{qg}(x/y) g(y, Q^2)} \right]$$

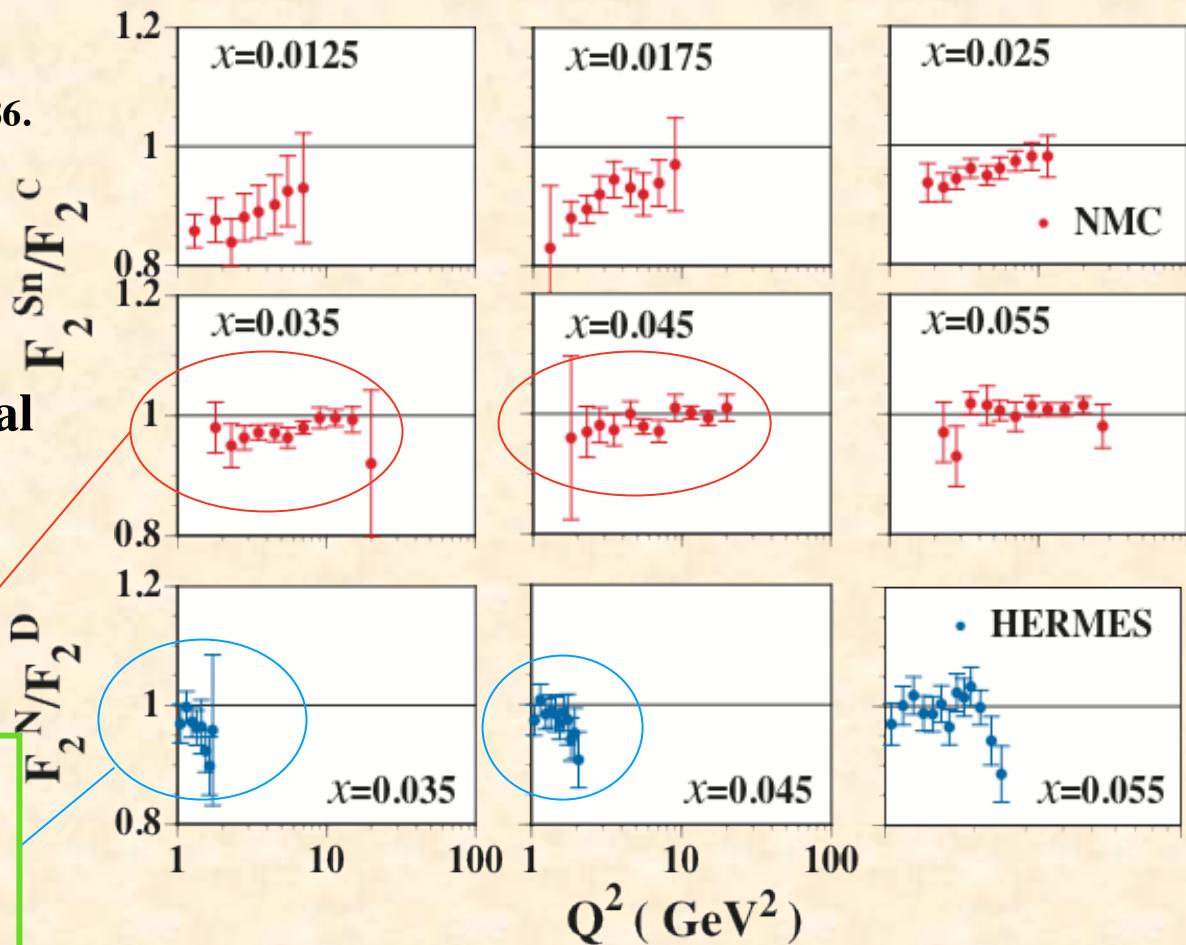
dominant term at small x

$$q_i^+ = q_i + \bar{q}_i$$

at small  $x$  K. Prytz, PLB 311 (1993) 286.

$$\frac{\partial F_2}{\partial (\ln Q^2)} \approx \frac{20 \alpha_s}{27\pi} x g$$

$Q^2$  dependence of  $F_2$  is proportional to the gluon distribution.

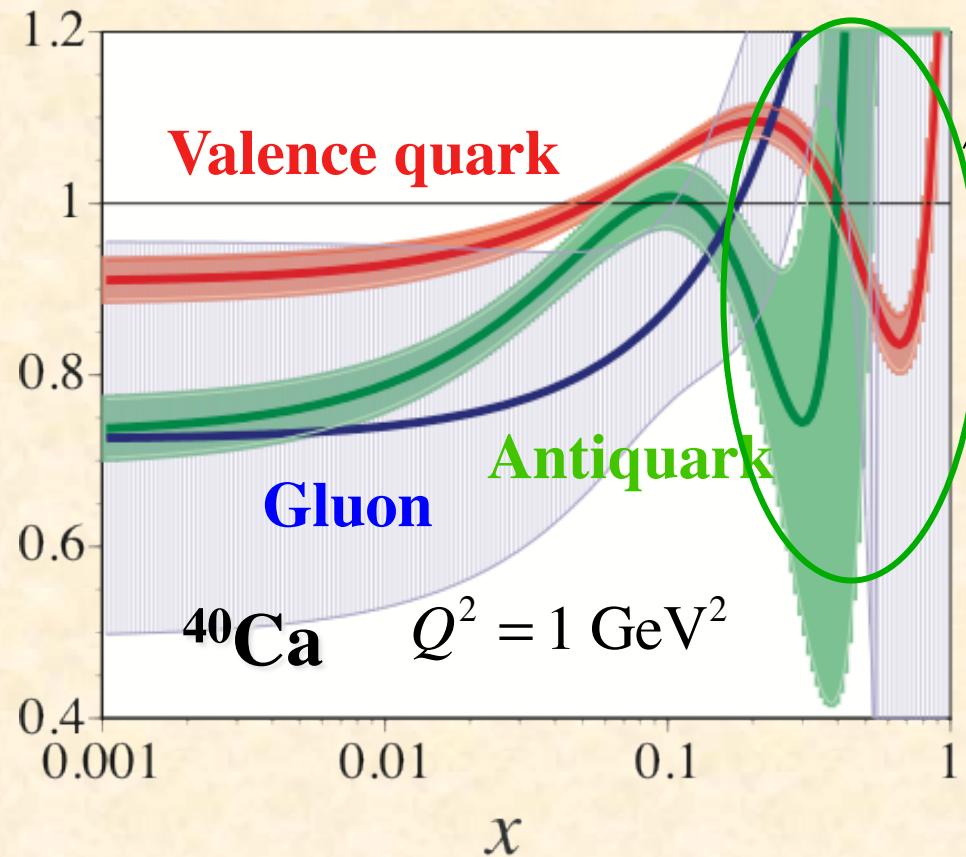


No experimental consensus of  $Q^2$  dependence!  
 →  $G^A(x)$  determination is difficult.

# Nuclear corrections on parton distribution functions

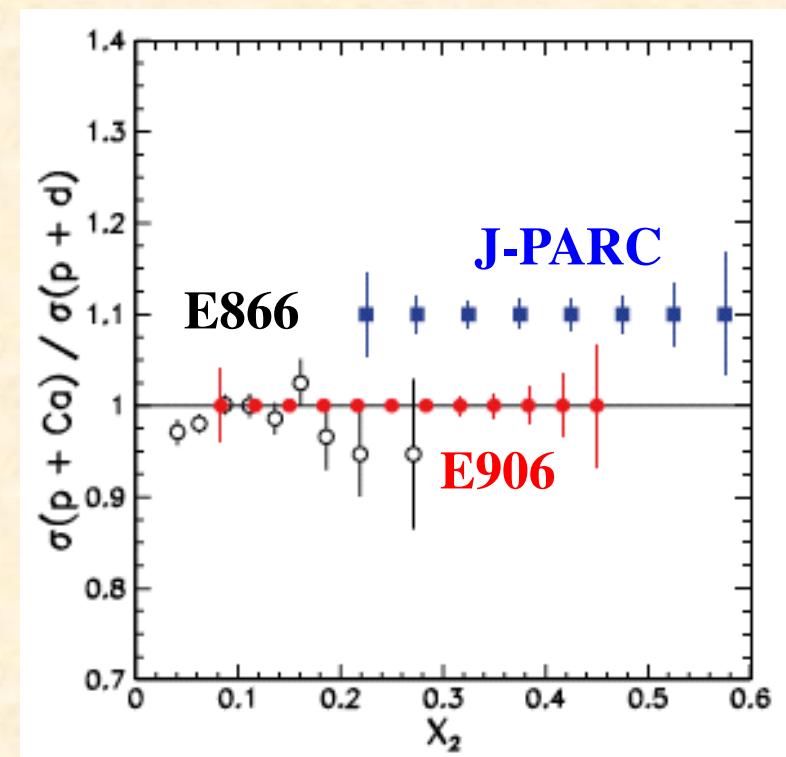
$$\frac{f^{Ca}(x, Q^2)}{f^N(x, Q^2)}$$

This region could be investigated by J-PARC.



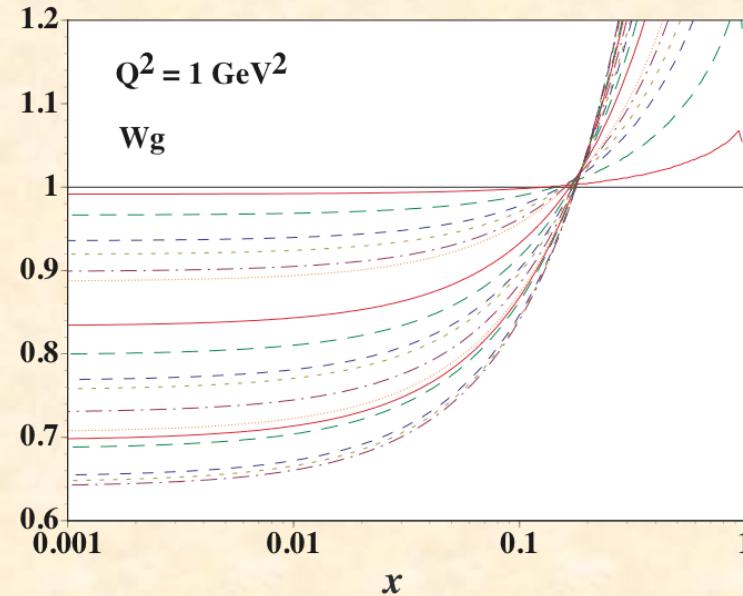
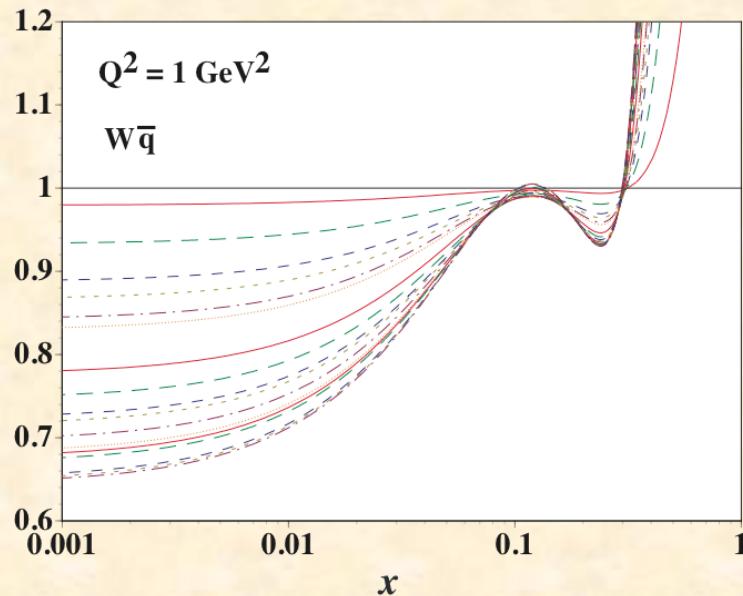
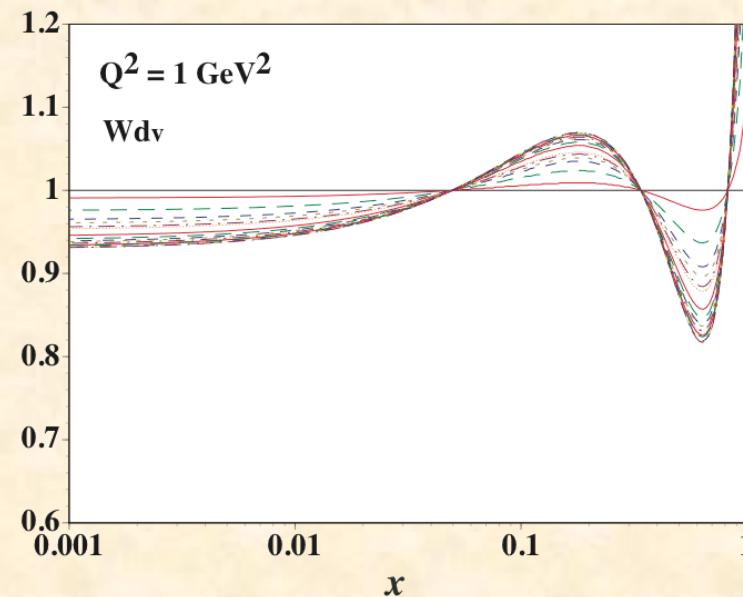
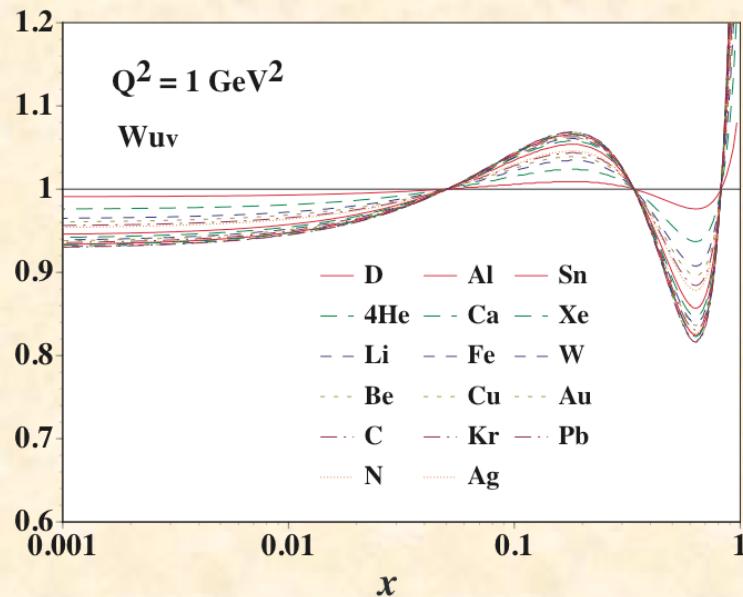
Global NPDF analysis result

J-PARC proposal P04



# Nuclear PDFs

M. Hirai, S. Kumano, T.-H. Nagai, PRC 76 (2007) 065207.  
<http://research.kek.jp/people/kumanos/nuclp.html>



# Recent global analyses on nuclear PDFs

## HKN07

- M. Hirai, S. Kumano, and T. -H. Nagai, Phys. Rev. C 76 (2007) 065207.
- Charged-lepton DIS, DY.

I may miss some papers.

## EPS09

- K. J. Eskola, H. Paukkunen, and C. A. Salgado, JHEP 04 (2009) 065.
- Charged-lepton DIS, DY,  $\pi^0$  production in  $dAu$ .

## CTEQ

- I. Schienbein, J. Y. Yu, C. Keppel, J. G. Morfin, F. I. Olness, J. F. Owens, Phys. Rev. D 77 (2008) 054013; D80 (2009) 094004;  
K. Kovarik *et al.*, PRL 106 (2011) 122301; PoS DIS2013 (2013) 274;  
PoS DIS2014 (2014) 047.
- Neutrino DIS, Charged-lepton DIS, DY.

## DSZS12

- D. de Florian, R. Sassot, P. Zurita, M. Stratmann, Phys. Rev. D85 (2012) 074028.
- Charged-lepton DIS, DY, RHIC- $\pi$

See also L. Frankfurt, V. Guzey, and M. Strikman, Phys. Rev. D 71 (2005) 054001;  
Phys. Lett. B687 (2010) 167; Phys. Rept. 512 (2012) 255; arXiv:1310.5879.  
S. A. Kulagin and R. Petti, Phys. Rev. D 76 (2007) 094023; C 82 (2010) 054614;  
arXiv:1405.2529.  
A. Bodek and U.-K. Yang, arXiv:1011.6592.

# Functional form of initial distributions at $Q_0^2$

Initial nuclear PDFs at

$$f_i^A(x) = \frac{1}{A} [Z f_i^{p/A}(x) + (A - Z) f_i^{n/A}(x)] \quad f_i^{N/A}(x): \text{PDF of bound nucleon in the nucleus}$$

Isospin symmetry is assumed:  $u \equiv d^n = u^p, d \equiv u^n = d^p$

## Functional forms

- HKN07 ( $Q_0^2 = 1 \text{ GeV}^2$ )

$$f_i^A(x) = w_i(x, A, Z) \frac{1}{A} [Z f_a^p(x) + (A - Z) f_a^n(x)], \quad w_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^{1/3}}\right) \frac{\mathbf{a}_i + \mathbf{b}_i x + \mathbf{c}_i x^2 + \mathbf{d}_i x^3}{(1-x)^{0.1}}$$

- EPS09 ( $Q_0^2 = 1.69 \text{ GeV}^2$ )

$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{\text{CTEQ6.1M}}(x, Q_0^2), R_i^A(x) = \begin{cases} \mathbf{a}_0 + (\mathbf{a}_1 + \mathbf{a}_2 x)[\exp(-x) - \exp(-\mathbf{x}_a)] & (x \leq x_a : \text{shadowing}) \\ \mathbf{b}_0 + \mathbf{b}_1 x + \mathbf{b}_2 x^2 + \mathbf{b}_3 x^3 & (x_a \leq x \leq x_e : \text{antishadowing}) \\ \mathbf{c}_0 + (\mathbf{c}_1 - \mathbf{c}_2 x)(1-x)^{-\beta} & (x_e \leq x \leq 1 : \text{EMC\&Fermi}) \end{cases}$$

- CTEQ-08 ( $Q_0^2 = 1.69 \text{ GeV}^2$ )

$$x f_i^{N/A}(x) = \begin{cases} \mathbf{A}_0 x^{\mathbf{A}_1} (1-x)^{\mathbf{A}_2} e^{\mathbf{A}_3 x} (1+e^{\mathbf{A}_4 x})^{\mathbf{A}_5} & : i = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s} \\ \mathbf{A}_0 x^{\mathbf{A}_1} (1-x)^{\mathbf{A}_2} + (1+\mathbf{A}_3 x)(1-x)^{\mathbf{A}_4} & : i = \bar{d} / \bar{u} \end{cases}$$

- DSZS12 ( $Q_0^2 = 1.0 \text{ GeV}^2$ )

$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{\text{MSTW2009}}(x, Q_0^2), R_v^A(x) = \mathbf{\epsilon}_1 x^{\alpha_v} (1-x)^{\beta_1} [1 + \mathbf{\epsilon}_2 (1-x)^{\beta_2}] [1 + \mathbf{a}_v (1-x)^{\beta_3}]$$

$$R_s^A(x) = R_v^A(x) \frac{\mathbf{\epsilon}_s}{\mathbf{\epsilon}_1} \frac{1 + \mathbf{a}_s x^{\alpha_s}}{1 + \mathbf{a}_s}, \quad R_g^A(x) = R_g^A(x) \frac{\mathbf{\epsilon}_g}{\mathbf{\epsilon}_1} \frac{1 + \mathbf{a}_g x^{\alpha_g}}{1 + \mathbf{a}_g}$$

# Comparison of nuclear PDFs

Different analysis results are consistent with each other because they are roughly within uncertainty bands.

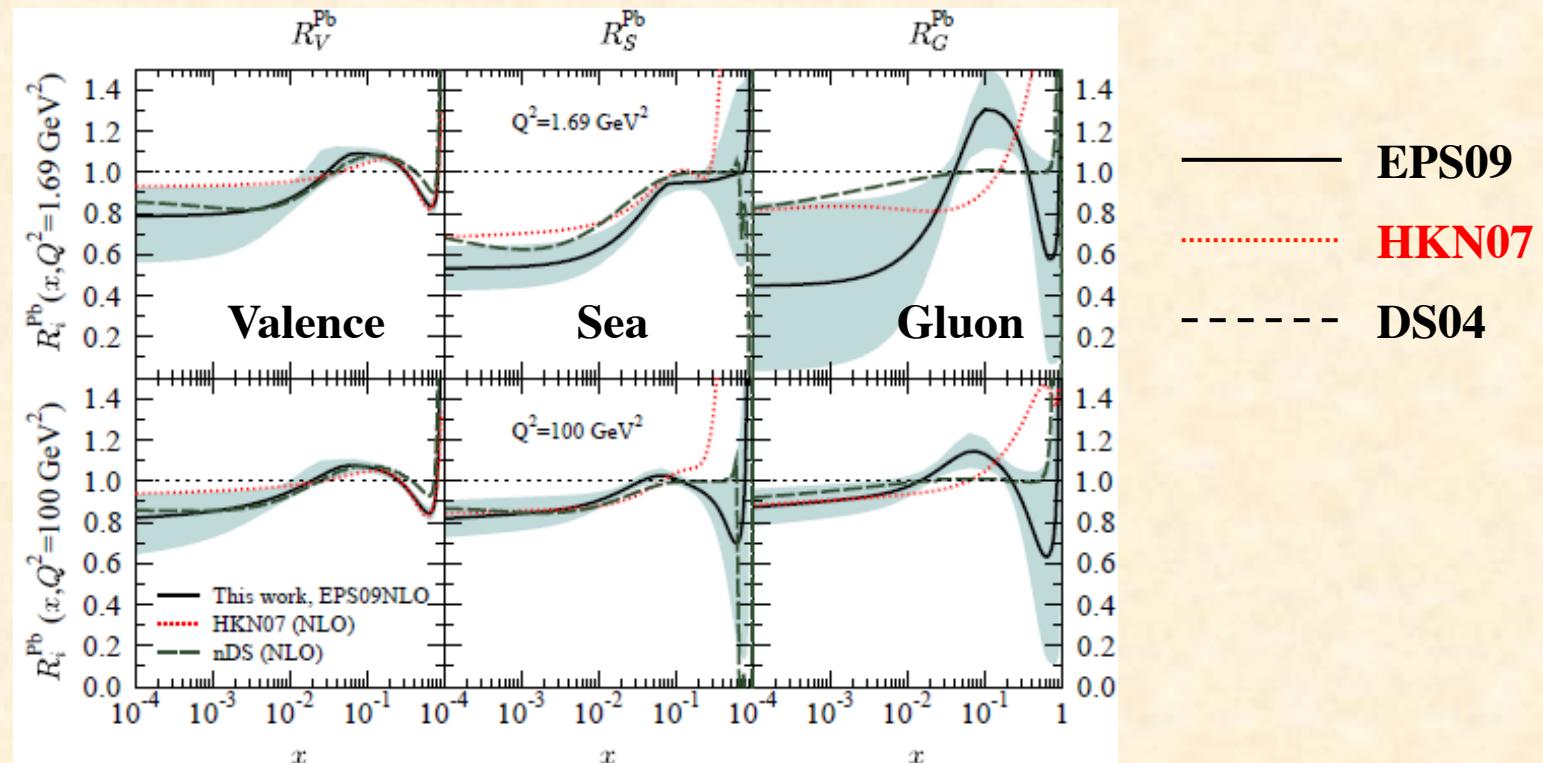
Valence quark: Well determined except at small  $x$ .

Antiquark: Determined at small  $x$ , Large uncertainties at medium and large  $x$ .

Gluon: Large uncertainties in the whole- $x$  region.

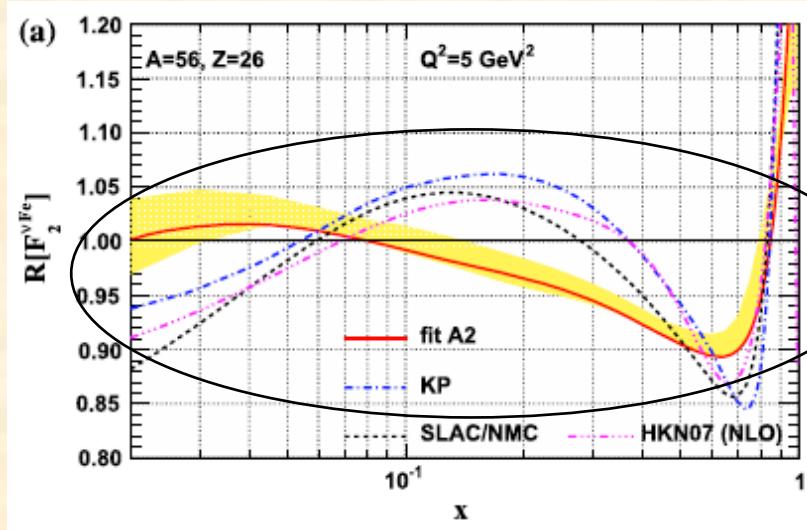
$$Q^2 = 1.69 \text{ GeV}^2$$

$$Q^2 = 100 \text{ GeV}^2$$



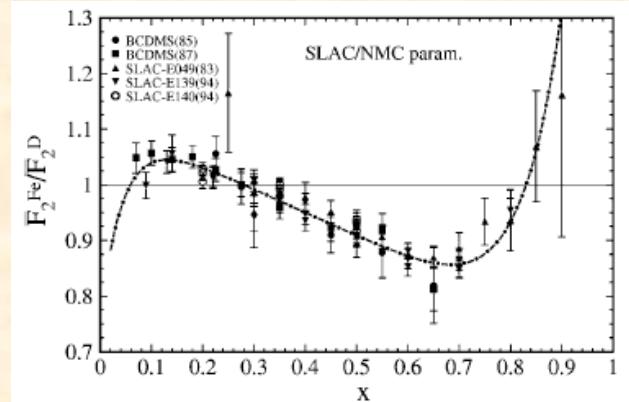
# Analysis of CTEQ-2008 (Schienbein *et al.*)

I. Schienbein *et al.*,  
PRD 77 (2008) 054013

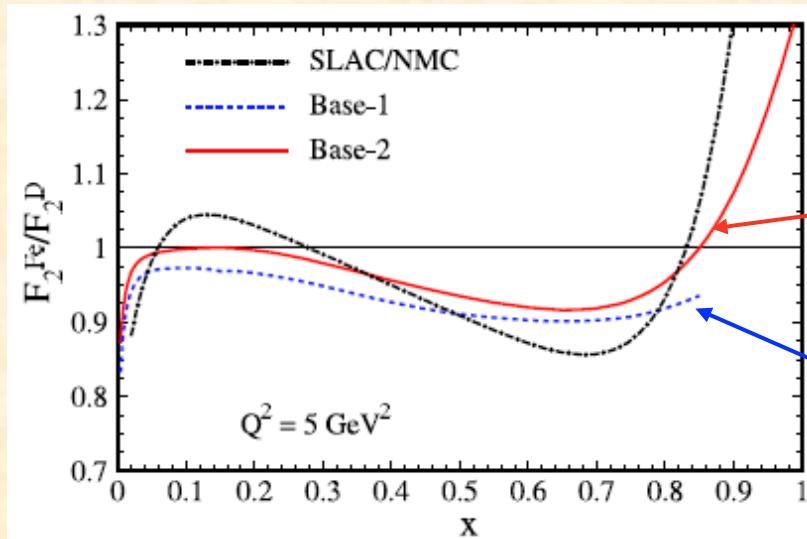


Differences  
from typical NPDFs.

## Charged-lepton scattering



## Neutrino scattering



- Base-1**
  - remove CCFR data
  - incorporate deuteron corrections
- Base-2**
  - corresponds to CTEQ6.1M with  $s \neq s\bar{b}\bar{s}$
  - include CCFR data
  - Charged-lepton correction factors are applied.
  - $s \neq s\bar{b}\bar{s}$

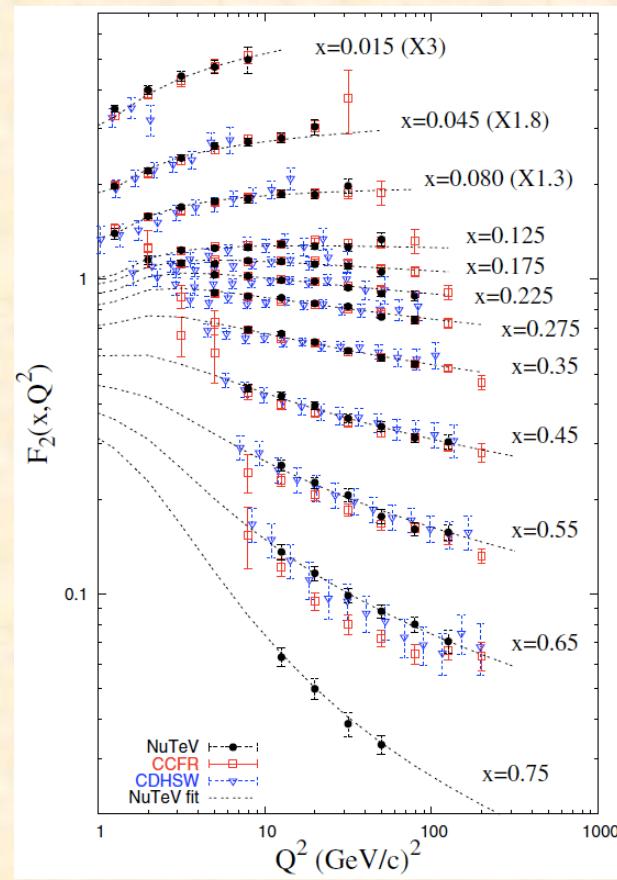
**Base-2:** Using current nucleonic PDFs,  
they (and MRST) obtained very different  
corrections from charged-lepton data.

**Base-1:** However, it depends on the analysis  
method for determining “nucleonic” PDFs.

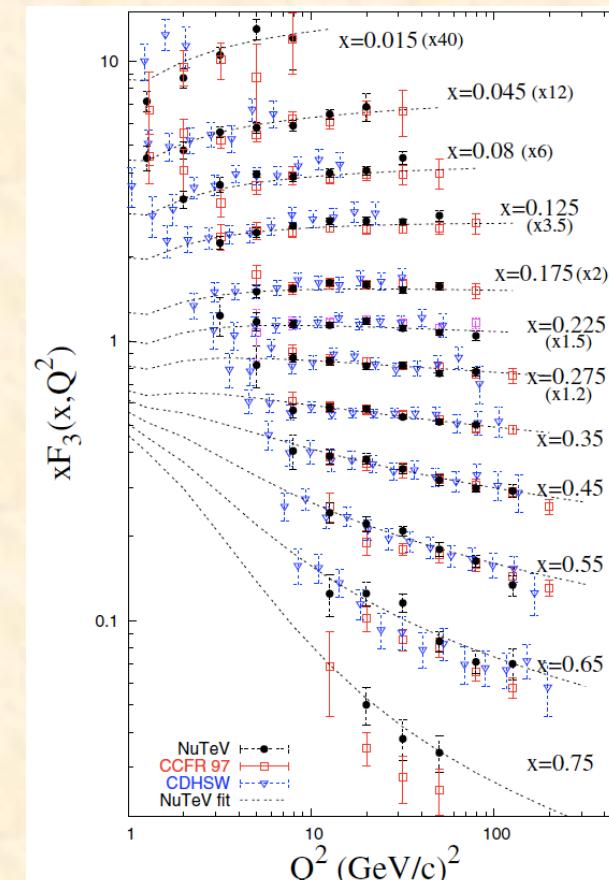
# Neutrino DIS experiments

Experiment	Target	energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

M. Tzanov *et al.* (NuTeV), PRD74 (2006) 012008.

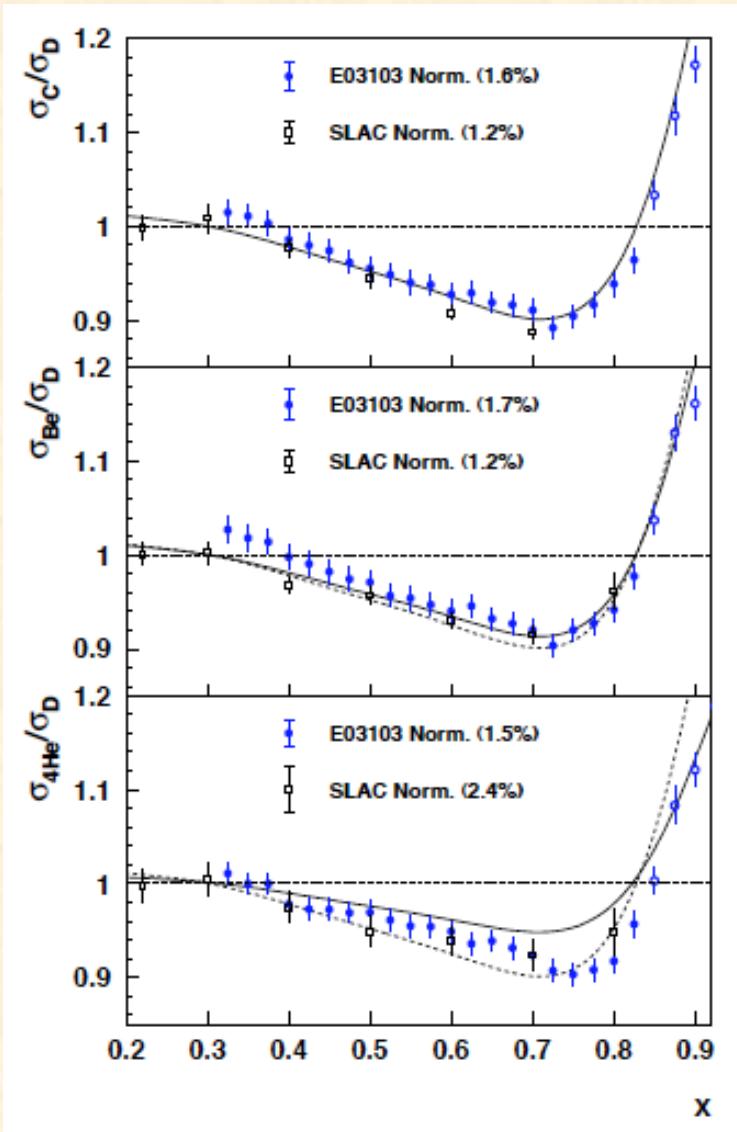


Future: MINERvA (He, C, Fe, Pb, ...)



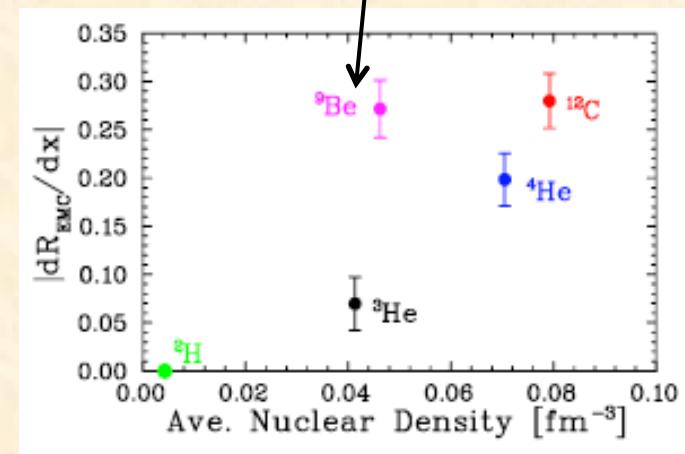
# Measurements at JLab

J. Seely *et al.*,  
Phys. Rev. Lett. 103 (2009) 202301.



Results indicate that nuclear modifications may not be described by usual  $A$  (and density) dependence for light nuclei.

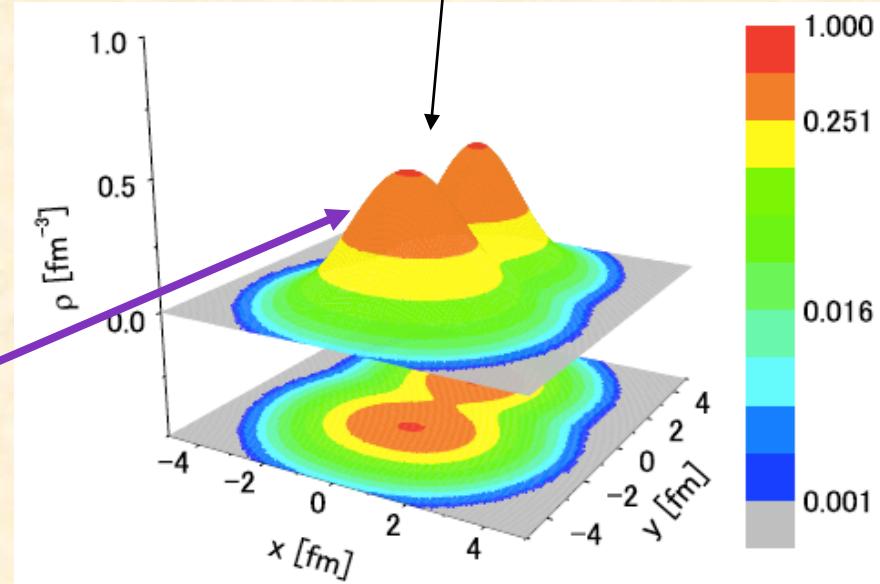
Anomalous (!?) for  ${}^9Be$



# Nuclear effects of ${}^9\text{Be}$

High-density regions  
= Something new?  
(Nucleon modifications,  
Short-range correlations, ...)

Typical nuclear clustering

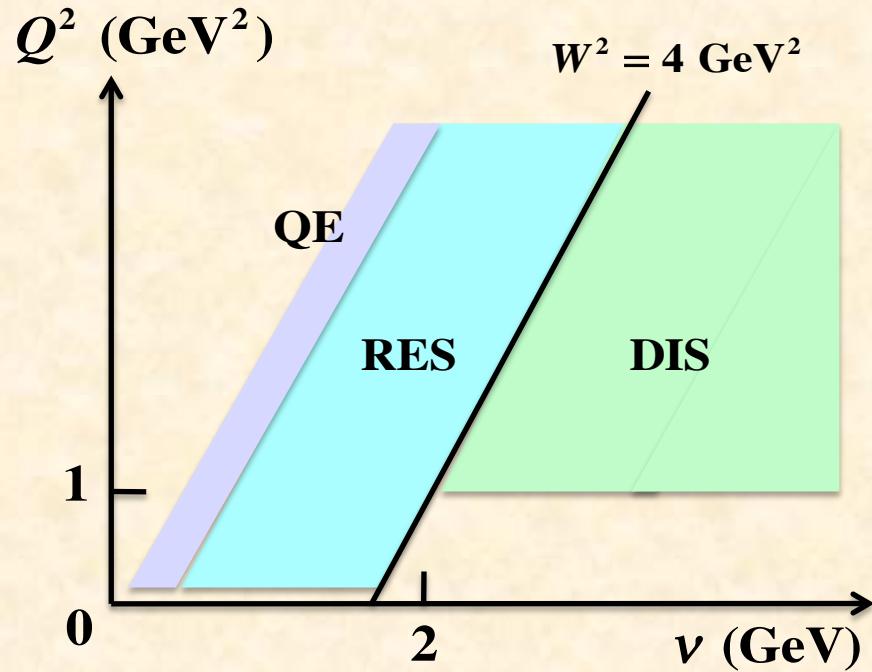


A theoretical-model density  
with cluster structure for  ${}^9\text{Be}$

A signature of nuclear clustering in high-energy processes,  
particularly in structure functions of deep inelastic scattering.

→ Internal nucleon modifications, Short-range correlations, ...

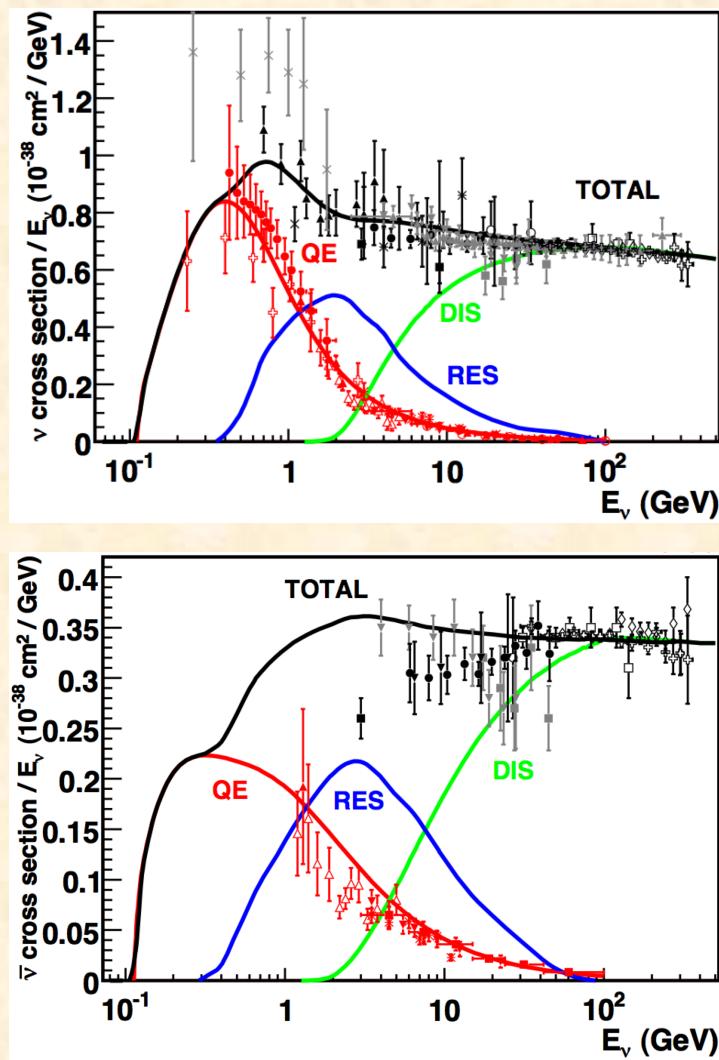
# Kinematical regions of neutrino-nucleus scattering



Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic)

Activities at the J-PARC branch, KEK theory center  
<http://j-parc-th.kek.jp/html/English/e-index.html>



J.L. Hewett *et al.*, arXiv:1205.2671,  
Proceedings of the 2011 workshop  
on Fundamental Physics at the Intensity Frontier

# **Summary on nuclear-PDF determination**

**Global analyses for the nuclear PDFs  
by using data of charged-lepton, neutrino DIS, pA, AA collisions**

**Valence quark: reasonably good,** in progress at JLab for large  $x$

**Antiquark:** good only at  $x = 0.1$ , in progress at Fermilab (E906)  $x = 0.1 \sim 0.4$ .

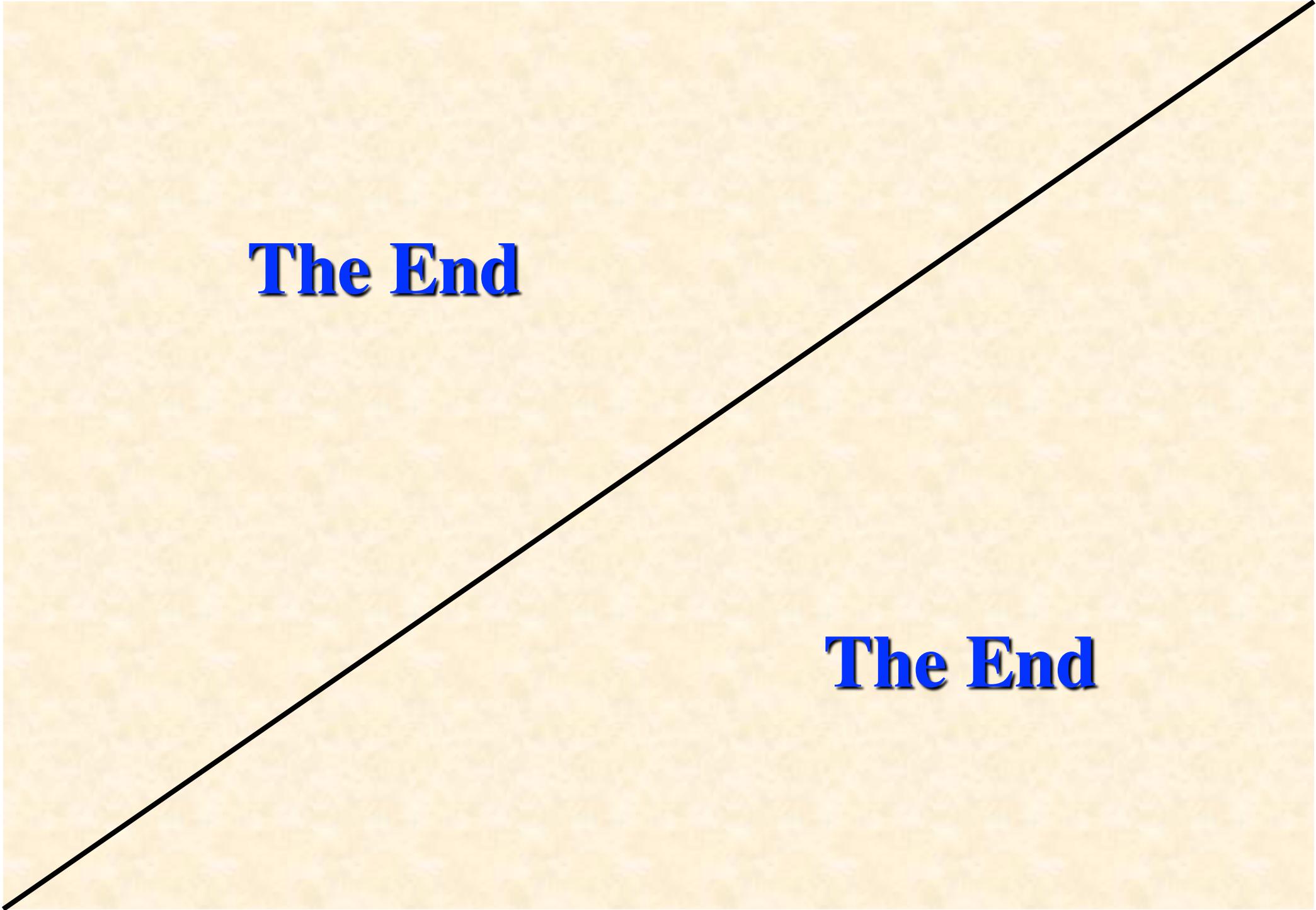
**Gluon:** large uncertainties in the whole- $x$  region, LHC

## **Issues**

- Charged-lepton DIS  $\Leftrightarrow$  Neutrino DIS
- Matching with resonance model
- Gluon distributions

## **New experimental information**

- JLab, Fermilab-DY, Minerva, LHC, ...



**The End**

**The End**