公募研究A06
格子ＱＣＤによる重いクォークの物理の研究

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References
S. Aoki, Y. Kuramashi and S. Tominaga, hep-lat/0107009, hep-lat/0111025.
S. Aoki, Y.Kayaba and Y. Kuramashi in preparation

Introduction

Determination of Cabbibo-Kobayashi-Maskawa(CKM) matrix from experiments

$$\Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} \eta_{B_q} F_{B_q}^2 \hat{B}_{B_q} |V_{tq}|^2 \tag{1}$$

ΔM_q: mass difference of the neutral B_q meson ($q = d, s$)

F_{B_q}: decay constant of the neutral B_q meson ($q = d, s$)

$\hat{B}_{B_q}$: (RG invariant) mixing parameter of the neutral B_q meson

$$\langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B_q \rangle = \frac{8}{3} B_{B_q}(\mu) F_{B_q}^2 m_{B_q}^2$$

Theoretical (non-perturbative) estimates are required for F_{B_q}, B_{B_q}.

$\Rightarrow$ Lattice QCD !

Problem of lattice QCD for b quark

m_b = 4-5 GeV
$1/a$ = 2-4 GeV (quenched QCD), 1-2 GeV (full QCD)

$$m_b a \sim O(1) \Rightarrow \text{ large } O(a) \text{ errors !}$$

- Wilson/Clover quark action at $m_q a < 1$ + extrapolation
- lattice NRQCD, FNAL quark action $\rightarrow$ effective theory

New approach !?

Purpose of this research project

$O(a)$ improvement program for massive quarks

- remove $af(m_q a)$ lattice artifact, $f(m_q a)$ is an arbitrary function of $m_q a$
- for all on-shell quantities with $\vec{p} = O(\Lambda_{\rm QCD})$

Plan of our project

1. Propose $O(a)$ improvement scheme for massive quarks and determine improvement parameters at tree level (completed)
2. Determination of improvement parameters at 1-loop (in progress)
3. Numerical test of our proposal for charm quarks (in progress)
4. Extensive investigations for charm quark physics and CLEO-c (in future)
5. Non-perturbative determination of improvement parameters (in future)
6. Test and extensive investigation for bottom quark physics (in future)

On-shell improvement program

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1. Action

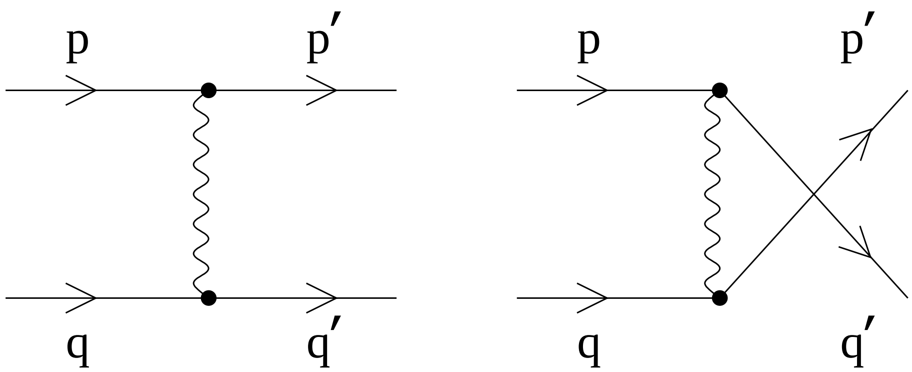
$$S_q^{\mathrm{imp}} = \sum_x \Bigg[\underbrace{m_0 \bar{q}(x)q(x)}_{\text{mass term}} + \nu_t \underbrace{\bar{q}(x)\gamma_0 D_0 q(x)}_{\text{time derivative}} + \nu_s \sum_i \underbrace{\bar{q}(x)\gamma_i D_i q(x)}_{\text{space derivative}}$$
$$-\frac{r_t a}{2} \underbrace{\bar{q}(x) D_0^2 q(x)}_{\text{remove doublers}} - \frac{r_s a}{2}\sum_i \bar{q}(x) D_i^2 q(x)$$
$$-\frac{iga}{2} c_E \sum_i \bar{q}(x)\sigma_{0i}F_{0i}q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x)\sigma_{ij}F_{ij}q(x)\Bigg],$$

2. Renormalization and Improvement condition

- Fermion propagator

$$S_F^{\mathrm{lat.}}(p_0, \vec{p}) = \frac{1}{Z_q}\frac{-i\gamma_0 p_0 - i\sum_i \gamma_i p_i + m_p}{p_0^2 + \sum_i p_i^2 + m_p^2} + (\text{no pole terms}) + O((p_i a)^2)$$

- Quark-quark scattering amplitude



(a) (b)

$$T = \text{ continuum amplitude } + O((p_i a)^2)$$

3. Results at tree level

- Renormalization

$$
\begin{aligned}
m_p &= \log\left|\frac{m_0 + r_t + \sqrt{m_0^2 + 2 r_t m_0 + 1}}{1 + r_t}\right|, \\
Z_m &= \frac{m_p}{m_0}, \\
Z_q &= \cosh(m_p) + r_t \sinh(m_p).
\end{aligned}
$$

- Improvement parameters

$$
\begin{aligned}
\nu_s &= \frac{\sinh(m_p)}{m_p}, \\
r_s &= \frac{\cosh(m_p) + r_t \sinh(m_p)}{m_p} - \frac{\sinh(m_p)}{m_p^2}, \\
c_E &= r_t \nu_s, \\
c_B &= r_s.
\end{aligned}
$$

4. 1-loop calculation has been almost completed.

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Numerical simulations

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Test of Space-Time symmetry

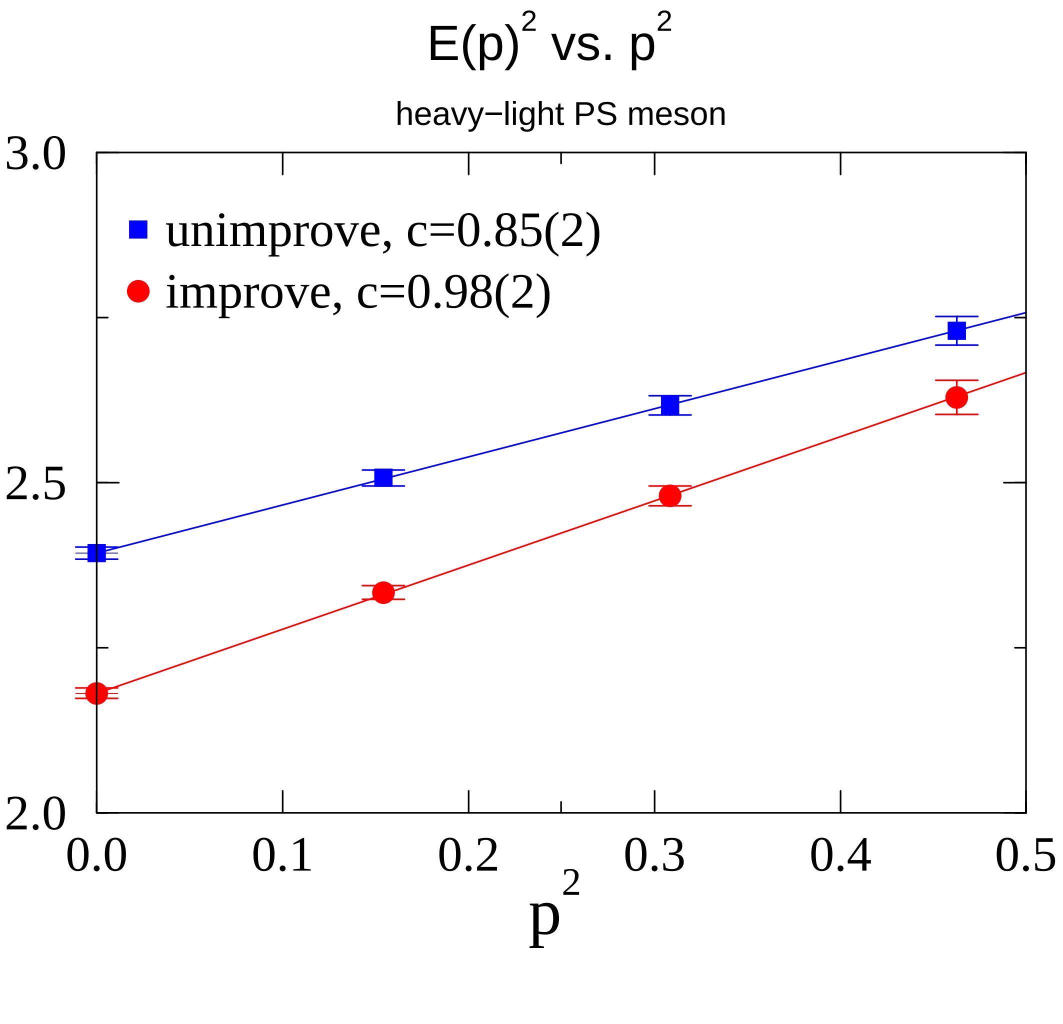
1. Dispersion relation

$$E(\vec{p}) = m_P^2 + c^2\vec{p}^2$$

c: "speed of light", $c = 1$ in the relativistic theory

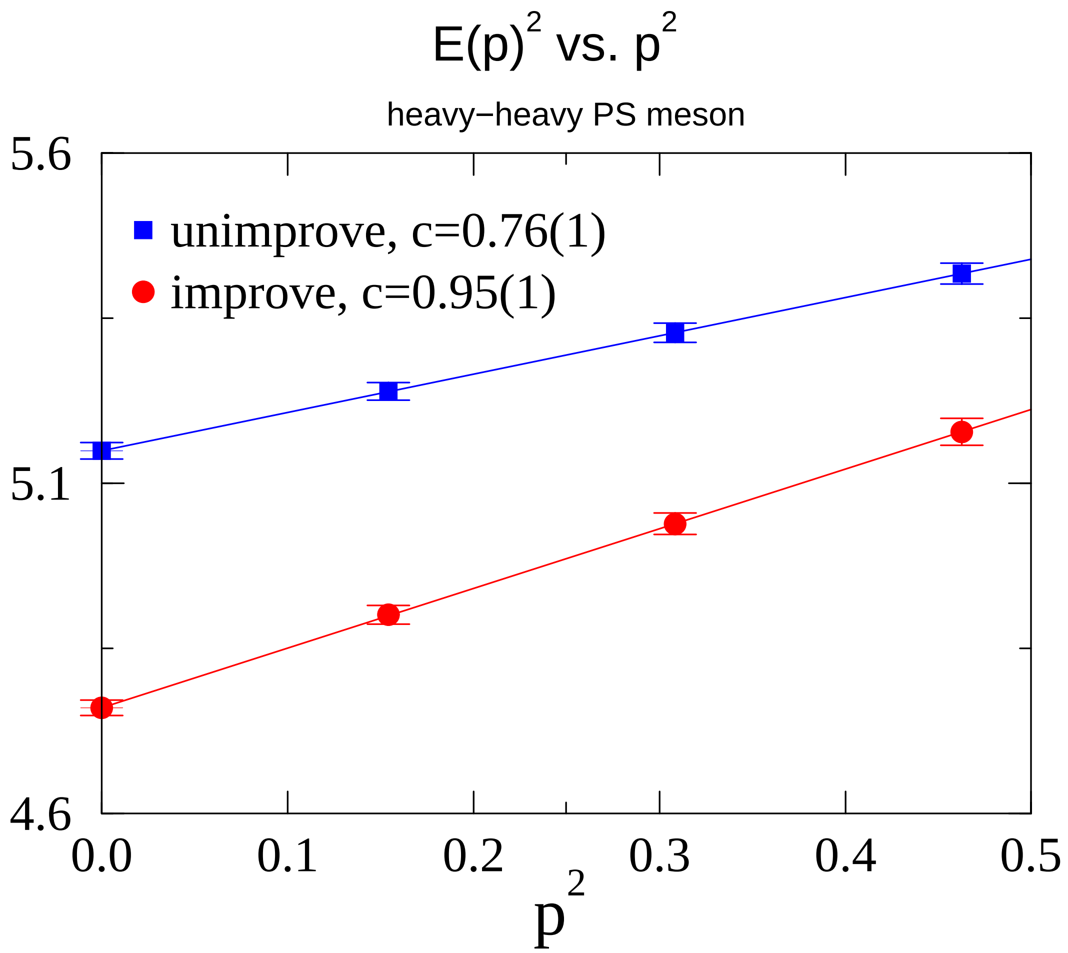
- Heavy-Light Pseudo-Scalar meson

$$\begin{aligned} c(\text{ unimproved}) &= 0.85(2) \\ c(\text{ improved}) &= 0.98(2) \end{aligned}$$



- Heavy-Heavy Pseudo-Scalar meson

$$c(\text{ unimproved}) = 0.76(1)$$
$$c(\text{ improved}) = 0.95(1)$$



Improved theory is much closer to the relativistic theory !

2. Decay constant

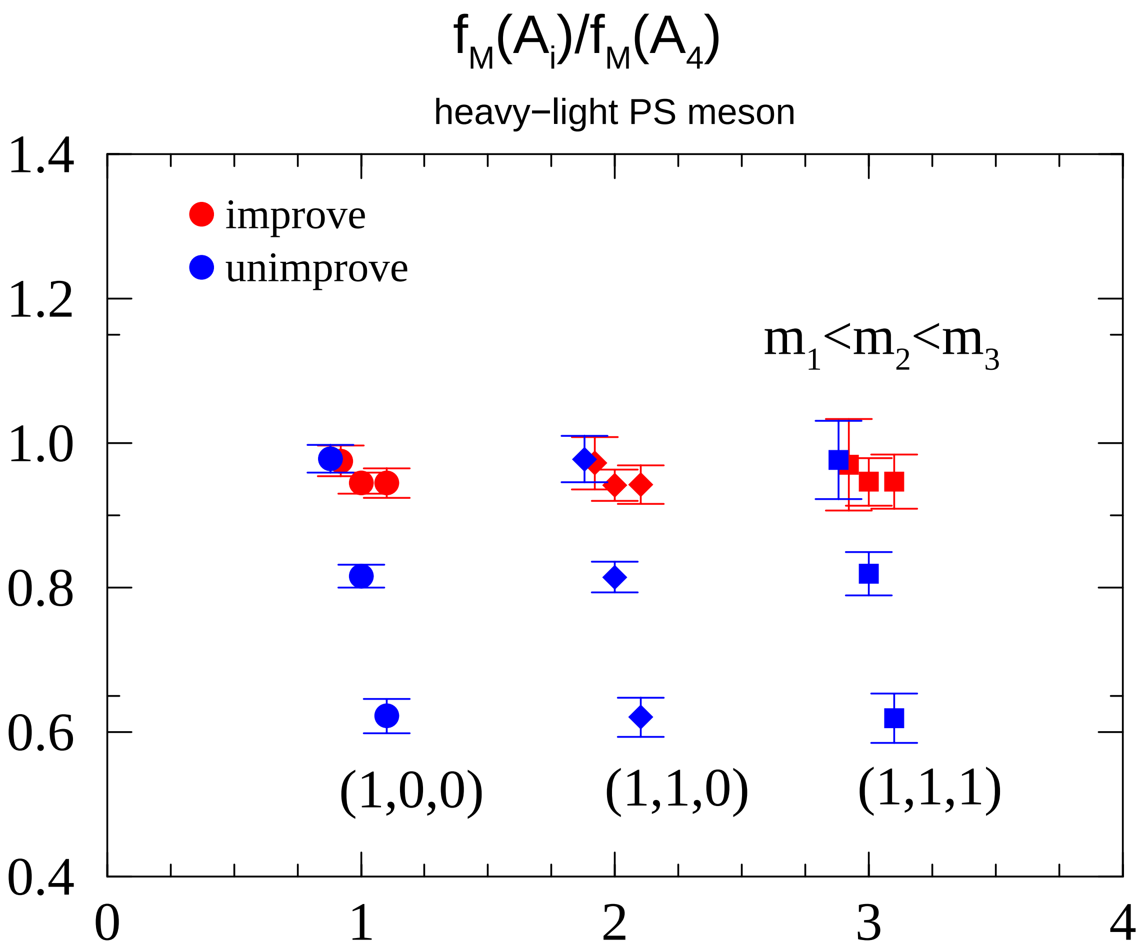
$$\langle 0|A_\mu|M(p)\rangle \;=\; ip_\mu f_M(A_\mu)$$

M: PS meson

<u>Ration of $f_M(A_i)$ and $f_M(A_4)$</u>

$$\vec{p} = (1,0,0), (1,1,0), (1,1,1)$$

$$m_{\rm heavy} \leftrightarrow m_1 < m_2 < m_3$$



$f_M(A_i)/f_M(A_4) \simeq 1$ for improved theory !

Conclusion

1. $O(a)$ improved theory we propose works well
   - speed of light $\simeq 1$
   - $f_M(A_i)/f_M(A_4) \simeq 1$
2. extensive calculations in this formulation for charm quark physics will be compared with results from CLEO-c
3. (future work) bottom quark physics, non-perturbative determination of improvement parameters